# ANALYSIS OF THE CHARACTERISTIC IMPEDANCE OF A U-STRIPLINE USING A FIELD-CELL TECHNIQUE

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#### ABSTRACT

A simple approach of calculating the characteristic impedance of a U-stripline is developed. The procedure is straight forward based on field-cell concept. The results obtained are very close to the experimental results.

## INTRODUCTION

The stripline technique has been used widely at SHF and UHF frequencies in the design and realization of two-port networks such as amplifiers and filters 12.3. There seems to be a dearth of information on the use of stripline for conveying a signal from one point to another, especially at the lower end of the frequency spectrum. As an alternative it is suggested that a U-stripline (using U-section) can be used for these purposes since it is simple to construct and easy to mount, with low cost. Although it is advantageous to use a flexible coaxial cable at the lower end of the spectrum and waveguide at the upper end of the spectrum, the cable tends to have a very high attenuation at high frequencies and the waveguide tends to have a very large structure at the lower end of the spectrum4. In stripline, the height (h) and the width (w) (refer to Figure 1) of the stripline depend only on the dielectric material that is placed between the two conductors for a given characteristic impedance. The U-stripline is more rigid even though without any dielectric material when compared to normal striplines. Thus, it is more practical for a long transmission line. It can be constructed from a U-section placed at constant height above a wide metallic sheet as ground. This arrangement is shown in Figure 1. The most important parameter of stripline is its characteristic impedance. Therefore, the analysis and the experimental studies of the characteristic impedance of a U-stripline is presented here.

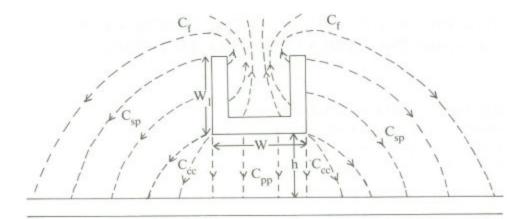


Fig. 1 Electric field of a U-stripline

# THEORETICAL FORMULATION FOR THE CHARACTERISTIC IMPEDANCE OF A U-STRIPLINE

The characteristic impedance of a stripline can be derived using a field-cell concept<sup>4</sup>, thus

$$Z_{o} = \frac{N_{s}}{N_{p}} \sqrt{\frac{\mu}{\epsilon}}$$
 (1)

if  $N_s$  and  $N_p$  can be determined, where  $N_s$  = number of cells in series and  $N_p$  = number of cells in parallel,  $N_s$  and  $N_p$  can also be related to the inductance and capacitance of the stripline. Thus inductance per unit length is given by

$$L = \frac{N_s}{N_p} \mu \tag{2}$$

and capacitance per unit length is

$$C = \frac{N_p}{N_s} \epsilon$$
 (3)

Before considering these cells, the capacitance and inductance can be divided into several components<sup>2</sup>. The capacitance is divided into parallel plate capacitance  $C_{pp}$ , side plate capacitance  $C_{sp}$ , corner capacitance  $C_{c}$  and fringe capacitance  $C_{f}$  as shown in Figure 1. Similarly the inductance is divided into  $L_{pp}$ ,  $L_{sp}$ ,  $L_{c}$  and  $L_{f}$  respectively.

The parallel plate capacitance can be calculated directly using Equation (3), thus

$$C_{pp} = \varepsilon \frac{W}{h}$$
 (4)

where h = the height of the stripline above the ground and w = the width of the stripline

At the corner of the stripline, the cells are determined by considering the area of fringing field at the corner divided by the area of one cell (i.e. h<sup>2</sup>).

$$C_{c} = \varepsilon \frac{\frac{1}{4} \pi h^{2}}{h^{2}} = \frac{\varepsilon \pi}{4}$$
 (5)

The side capacitance  $C_{sp}$  is determined using the  $z = \exp(w)$  transformation (Figure 2)<sup>5</sup>. The side plate is first transformed into a parallel plate in the w-plane coordinate. This forms a parallel capacitor with different parameters. Using Equation (3) we arrive at

$$C_s = \frac{2\varepsilon}{\pi} \log_e \left( \frac{W_1}{h} + 1 \right) \tag{6}$$

where w<sub>1</sub> is the width of the side plate (Figure 1).

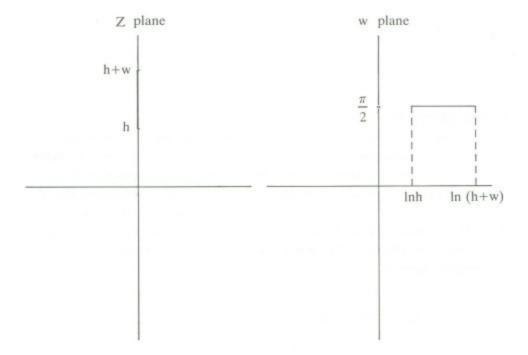


Fig. 2  $Z = e^{W}$  transformation

The inside of the U-stripline plays no part in the calculation because the current normally flows on the outside of the conductor at high frequencies. Thus the fringe capacitance can be neglected. Similarly the components of inductances of the U-stripline can be determined in the same manner. By summing up all the capacitances ( $C_{pp} + 2C_c + 2C_{sp}$ ) and the inductances ( $L_{pp} + 2L_c + 2L_{sp}$ ) respectively, the characteristic impedance of the U-stripline can be determined using equations (1), (2) and (3). Thus, for U-stripline without dielectric material (i.e.  $\epsilon = \epsilon_0$ ;  $\mu = \mu_0$ ), we obtain

$$Z_o = \frac{377}{\frac{W}{h} + 1.57 \frac{4}{\pi} \log_e \left(\frac{W_1}{h} + 1\right)}$$
(note that  $\sqrt{\frac{\mu_o}{\epsilon_o}} = 377 \ \Omega/\Box$ )

Equation (7) above can be applied only when the height h of the U-stripline above the ground is smaller or equal to the width of the stripline<sup>4</sup>. For the case of h greater than w, the formula for a single conductor above a ground plane (with image below) can be used by taking the impedance as half that of the corresponding two conductor lines; the U-stripline is considered equivalent to a circular conductor with radius half of the width w of the U-stripline.

Hence for h > w

$$Z_0 = 60 \log_e \frac{4h}{w} \tag{8}$$

# MEASUREMENTS

The setup using reflectometer as shown in Figure 3 is used. The values of the characteristic impedance  $Z_o$  of a 12.7 mm  $\times$  12.7 mm U-stripline for various values of height h have been conducted, as shown in Table 1. The U-stripline size used in this paper is based on the requirement that the width must be very small compared to the wavelength. In this experimental work the incoming signal  $E_i$  is set to be 2.55 volts and the reflected signal  $E_r$  is then measured. The actual value of  $Z_o$  is determined using the following relationship<sup>6</sup>

$$\frac{E_r}{E_i} = \frac{Z_o - Z_c}{Z_o + Z_c} \tag{9}$$

where  $Z_c$  is the characteristic impedance of the link cable (50 ohm). The calculated values from Equation (7) is compared with the practical values as shown in Figure 4.

 $\label{eq:table 1} MEASUREMENT OF ~Z_0 FOR ~VARIOUS ~HEIGHTS ~USING \\ REFLECTOMETER, ~WITH ~INCOMING ~SIGNAL~E_i = 2.55 ~VOLTS$ 

h (mm)	E <sub>r</sub> (volts)	Z <sub>0</sub> (ohm)	
1.25	- 1.0	21.8	
2.25	- 0.4	36.4	
3.50	0.0	50.0	
4.50	0.15	56.3	
6.25	0.45	71.4	
8.25	0.6	80.8	
11.0	0.8	95.7	
14.25	0.9	104.6	

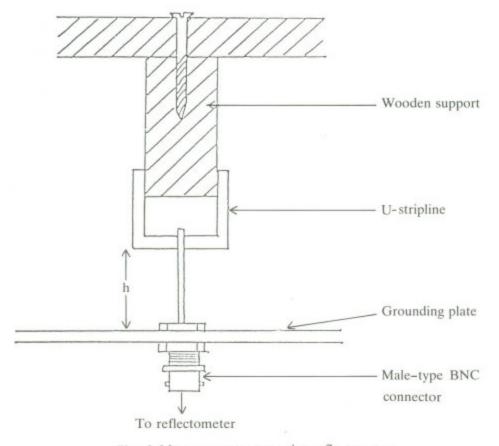
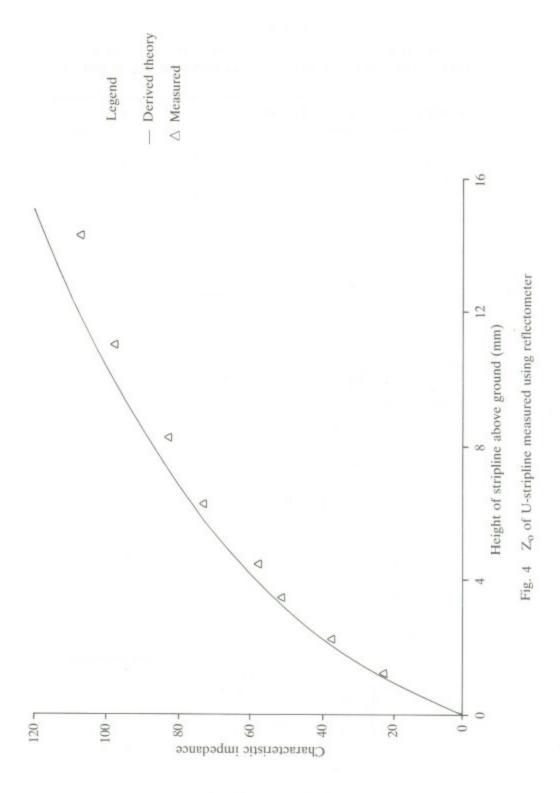


Fig. 3 Measurement setup using reflectometer

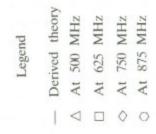


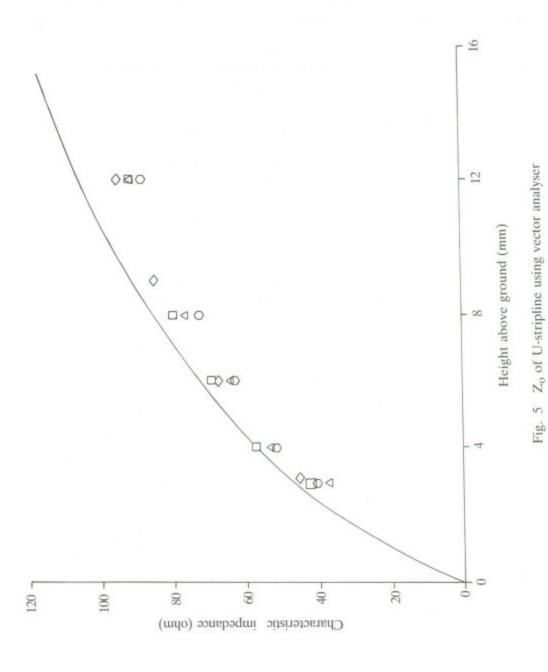
The above method of measurement has a limited range of frequencies since the reflectometer generates a single frequency of 1 kHz only. However, using a Rhode and Schwarz vector analyser an open and short circuited method is employed to determine the characteristic impedances of the U-striplines. The results of measurement for various frequencies of importance at the upper limit are tabulated in Table 2 and plotted in Figure 5. It is also interesting to note that the frequencies are chosen such that to give an equivalent to a multiple of a half wavelength of a feeding cable in order to reduce error due to stray inductances at the connector.

TABLE 2  $\label{eq:measurement} \text{MEASUREMENT OF } Z_o \text{ OF U-STRIPLINE AT UHF USING } \\ \text{VECTOR ANALYSER}$ 

Frequency (MHz)	Height h (mm)	$Z_{closed}$	Z <sub>open</sub>	$Z_o$
	3	2.1 -j120	1.0+j14.2	41.3-j0.9
500	4	2.3 -j160	1.2+j17.3	52.7-j1.4
	6	2.0 -j179	1.6+j23.2	64.5-j1.9
	8	3.8 -j192	2.2+j30.4	76.5-j2.0
	12	2.1 -j192	2.7+j43.1	91.0-j2.4
625	3	3.0 -j104	2.5+j17.8	43.2-j2.4
	4	5.5 -j153	2.0 + j21.1	56.9-j1.6
	6	7.2 -j168	2.9+j27.9	68.6-j2.1
	8	13.8 -j199	3.3 + j31.3	79.0-j1.4
	12	17.4 -j158	5.6+j51.4	90.6+j0.2
750	3	6.2 -j13.2	25.9+j142	45.5+j5.9
	4	0.75-j12.6	44.3+j207	51.5-j3.9
	6	13.4 -j126	5.5+j35.8	67.7-j1.6
	8	17.4 -j132	5.9+j43.6	76.5-j0.1
	9	25.0 -j142	7.5+j49.4	84.9+j1.0
	12	27.0 -j127	11.1+j67.7	94.4+j2.1
875	3	5.0 -j62.0	4.0+j29.0	42.7-j1.2
	4	5.7 -j77.4	4.5+j34.0	51.6-j1.5
	6	11.0 -j86.3	7.7+j45.1	63.1-j1.3
	8	15.1 -j93.7	11.6+j57.2	74.4-j1.5
	12	24.5 -j91.8	19.8+j79.3	88.0+j0.8

Note that the j value in  $Z_o$  should be 0. This is because the shorting of the stripline is not perfect.





## CONCLUSIONS

Even though the expression for the characteristic impedance for U-stripline derived using field-cell concept is very simple but it gives an appreciable estimation of characteristic impedance,  $Z_{\rm o}$ . The measured values using the reflectometer as plotted in Figure 4 and open and close circuit method in Figure 5 are found to be in good agreement with the calculated values using the suggested approach. The formula is also valid for high frequency as long as the wavelength is very much greater than the height and the width w of the U-stripline. The upper limit of the frequency range is around 875 MHz which is wide enough for high frequency transmission. For a better performance it is suggested that the parameters, height (h) and width (w) (similarly  $W_1$ ) should be less than  $\lambda/30$  of the upper limit of the operating frequency.

# REFERENCES

- Sayre, C.L. Microstrip Circuits for the Classroom Laboratory. IEEE Trans. Education, 1985, E28 (1).
- Michaelides, M. Microstrip Transmission Lines; Impedances Matching, Coupling and Filtering. Mulhard Technical Communication, no. 116, 1972, pp. 170–176.
- 3. Harvey, A.F. Microwave Engineering. Academic Press, 1963.
- 4. Kraus, J.D. Electromagnetics. 2<sup>nd</sup> ed., McGraw-Hill, 1984, pp. 397-398.
- Nussbaum, A. Electromagnetic Theory for Engineers and Scientists. Prentice—Hall, 1965.
- 6. Packard, H. Application Note 62, Section 3, 3-1 to 4-4.