Understanding the combinatorial thinking through the strategy used by students cognitive reflective in solving permutation

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Abstract. Combinatorial thinking is useful for training students in the concept of enumeration and others. Therefore, it is important to understand the students' combinatorial thinking. The strategy were used to be able to know the combinatorial way of thinking. This research aimed to determine the strategy and describe the process of using the strategy of the research subject. The data collection was done by giving the permutation problem and interview. The validation was done in time triangulation. The technique of data analysis was done through three steps namely data reduction, data presentation, and conclusion. The students with reflective cognitive style were chosen based on MFFT to 140 students. There were 49 students with reflective cognitive style were asked to do the permutation problem. There were two groups namely 92% of the students used one strategy and 8% of the students used two kinds of strategy. The research subjects chosen were only two students. The results showed that two types of strategies used are the filling slot and the formula. The students in general used filling slot. For convincing the truth, some students used another way that is the formula of permutation. Finally, the students matched the answers from the two strategies.

1. Introduction

This research was inspired from the benefit of learning combinatorial way. The benefit of learning combinatorial was training the students in enumeration, making prediction, generating, and systematic thinking; they could help to develop many concepts, such as equivalency and the relation of order, the function, sample, and many others [1]. Therefore combinatorial was good both in Indonesian and in abroad as the required material to be taught at school. One of the materials related to the combinatorial was permutation. The permutation represented the more difficult understanding for the students than combinatorial [2]. When they learnt permutation or combinatorial, the students must have done thinking activity. Thinking activity in this research was called as combinatorial thinking because it had relation with combinatorial material.

The definition of combinatorial thinking according to the experts was as the following: Combinatorial thinking was the specific thinking way related with combinatorial concepts[3]. Combinatorial thinking is a process to find some alternative solution of discrete problems[4]. Combinatorial thinking needs critical thinking procedure and continuous reason when solve the

problem [4]. Combinatorial thinking is the ability to consider all possible alternatives in a given situation [5]. So, combinatorial thinking was a process to find all possibilities which was connected with combinatorial concept.

Combinatorial thinking was abstract, because thinking was in human's brain. For knowing it, it could be seen from the way they solved the task finished. Solving problem was the way of thinking, analyzing, and giving reason by using experience and knowledge related with the question [6]. When the students solved the combinatorial problem, they needed accuracy. In fact, accuracy and caution of the students was influenced by cognitive style of the students. It was because of cognitive style which basically emphasized in the characteristic of individual consistency in thinking, remembering, and solving the problems [7]. The accuracy related with the reflective cognitive style. The students with reflective cognitive style had individual characteristics that needed a long time to respond because of considering all possible alternatives [7]. Therefore, they were accurate or careful in doing, gave the right answer and rechecked the answers. Besides cognitive style, the most important in solving the task was a strategy.

Choosing the right strategy, it would ease to do and get the right solution. Strategy was defined as scientific approach in solving the problem [8]. Strategy could be used to understand the students' way in combinatorial thinking, by finding out whether they understood the strategy chosen and knew why they chose the strategy [9]. Understanding the strategy meant that the students thought about the accurate procedure in using strategy chosen and gave logical reason in each step done. Observing the students' combinatorial thinking through strategy in this research was 1). Students gave the logical reason why they chose the strategy [9]; 2) Students described the accurate procedure when they used the strategy dan students gave logical reason in each step when they solved the problem [4]. There were three strategies in solving combinatorial task, namely: systematic listing, principle counting, or combinatorial operation. Systematic listing meant that they listed all possibilities which could happen by applying certain variables; Principle counting referred to the principle of addition, multiplication principle, and inclusive exclusive principle; while combinatorial operation referred to the combinatorial formula [10].

Based on the description above, it was important to understand the students' combinatorial thinking. This is because it can to know whether the students understand the material given and whether they got benefit from learning combinatorial. therefore, it is necessary to conduct research to understanding the combinatorial thinking through the strategy used by students cognitive reflective in solving permutation. So, this research aimed to determine the strategy and describe the process of using the strategy of the research subject.

2. Experimental Method

This research was qualitative research. The subjects were chosen from the students of eleventh grade students in Senior High School in Jombang that they had reflective cognitive style. There were 140 students as the respondents. The researcher did the cognitive style test by using instrument of MFFT (*Matching Familiar Figure Test)* to get the students with reflective cognitive style. The result of the test showed that there were 49 students who had reflective cognitive style. The permutation problem was given to 49 students. The permutation problem was given after getting the validation from the experts.

The permutation problems:

Refa wanted to keep her cellular phone security by using passwords. The passwords were in four numbers and they might not be doubled. Refa used her date of birth namely 1,9, 8 dan 3.

How many orders of the passwords which could be made by Refa? Explain it!

Choosing the subjects by considering the research that the male was better than female in conversation, had combinatorial thinking, and variable control, the researcher used random sampling for the male students [11]. The researcher only took two students to represent the data from the strategy used because the two students almost had the same combinatorial ability. It made the researcher easier to describe their strategy in solving permutation task, so that the researcher could understand the way they used combinatorial thinking. After getting the subject of the research, they were interviewed. The validity of the data used time triangulation. After stating that the data was valid,

then the data was analyzed. There were three techniques of data analysis namely data reduction, data presentation, and conclusion.

3. Result and Discussion

This research involved 49 students with reflective cognitive style. They were given a permutation problem, the result was 92 % of the students used one strategy (filling slot) and 8% of the students used two strategies (filling slot and permutation formula). Based on the result, it could be stated that in general the students used multiplication principle. However, there were students who used permutation formula to convince their answers. besides determine strategy, this research also describe the process of using the strategy of the research subject.

This shows the process of using a strategy from a subject that uses one type of strategy.
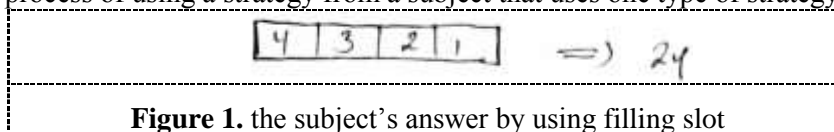
Figure 1. the subject's answer by using filling slot

This was the interview between the researcher and the subject of the research:

Researcher: what do you think about this problem?
Subject : reva wanted to make password for her cellular phone by using date of her birthday namely 198 and 3, but the key was that the numbers might not be the same.
Researcher: Then, to solve it, what did you do?
Subject : I used the filling slots directly
Researcher: Why did you use filling slots?
Subject : Because it was easier
Researcher: Easier? What did you mean?
Subject : It was easier in indentifying the possibilities from the numbers.
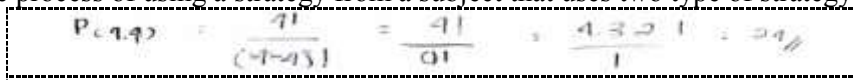Researcher: Like what? Could you explain it?
Subject : First there were four numbers which became the passwords, it meant that there were four boxes, the first box could be in four numbers because all numbers had possibilities. There were four boxes, so the second box was filled with 3 numbers that might not be the same, it meant that 4 - 1 was 3. Then, the third box was filled with 2 (meant 3-1), the fourth box filled with 1. Then 4 x 3 x 2 x 1=24
Researcher: Was there another way?
Subject : no , because this task was to determine the ways in determining the numbers of filling the place for numbers as the passwords.

Based on figure 1 and the result of the interview. Students gave the logical reason why they chose the strategy. It could be seen that The subject used this strategy because it eased them to identify the possibilities and made them sure that the task given was to determine the ways in filling slot [9]. Students described the accurate procedure when they used the strategy dan students gave logical reason in each step when they solved the problem, can be seen when the subject used the strategy started from the subject understood the problem by revealing the information and finding the key words where the information was used as the data in arranging the strategy. The information was the password consisted of 4 numbers and the password formed from the numbers 1, 9, 8 and 3. The key word is the number should not be the same in creating a password. By using the information, the subject determined the strategy. Strategy used in the multiplication principle (filling slot). The subject drew 4 boxes because the passwords were 4 numbers. The subject used the key words when they used the filling slots, each box had one difference, so that the subject filled the first box was filled with 4 possibilities, the second box was filled with 3 possibilities (the way was 4-1=3), the third box was filled with 2 possibilities (3-1=2), and the fourth box was filled with one possibility. (2-1=1). Then the subject multiplied the possibilities namely 4 x 3 x 2 x 1 was 24 [4].

This shows the process of using a strategy from a subject that uses two type of strategy

Figure 2. The subject's answer used permutation formula

This was the interview between the researcher and the subject of the research:

Researcher: What did you understand about the problem?

Subject   : The task meant that the subject would look for the order of the passwords which consisted of 4 numbers and the important note here was there might not be the same numbers, there were 4 numbers here. Because it could not use the same numbers, so the I used permutation, the order would be different such as 1983 and 1389 were counted differently.

Researcher: Why did you say so?

Subject   : Because it avoided the same numbers.

Researcher: Why did you use permutation?

Subject   : Because, the problem is concerned with the arrangement.

Researcher: What kind of permutation did you use?

Subject   : Permutation with different items

Researcher: Like what?

Subject   : It used the permutation with the different item. The item was different, the formula was p (n, r) the next formula was n!/r!, per n-r n-r!, the number of n was 4, n was the total number of the data was 198 and there was 3 in 4 then r was the question to be answer in 4 numbers, 4 numbers to be passwords and it became 4!/ n became 4-4! The result was 4!/0!, because 0! =1 the result was 24

Researcher: From the formula above, what kind of term was in this permutation?

Subject   : Factorial

Researcher: What did you mean with factorial?

Subject   : Factorial was the multiplication which was counted backward for example 4! = 4×3×2×1.

Based on figure 2 and the result of the interview. Students gave the logical reason why they chose the strategy. it could be seen when the subject explained the understanding by giving the example that 1983 was not same with 1389. By understanding the different position, the subject could determine the first strategy was permutation. The kind of permutation chosen by the subject was permutation with different item because avoided the same numbers [9]. Students described the accurate procedure when they used the strategy dan students gave logical reason in each step when they solved the problem. It could be seen first, the subject identified the information in the problem, the information was the order of passwords consisted of four numbers and might not use the same numbers. The subject arranged the formula namely $Pn,r = n!/n-r!$. The subject determined n, n was the total numbers of for making passwords. The subject determined the r, where r was the numbers of the passwords namely 4. The subject used formula of permutation with different item namely $n!/n-r! = 4!/4-4!$, The result was $4!/0!$. The subject changed $0! = 1$ as the basic determination from factorial. The result was $4!/1$ was $4!$. The subject explained 4! It became 4 x 3 x 2 x 1, with the basic factorial of multiplication backward and operated to be 24 [4].

Figure 3. The subject's answer by using filling slot
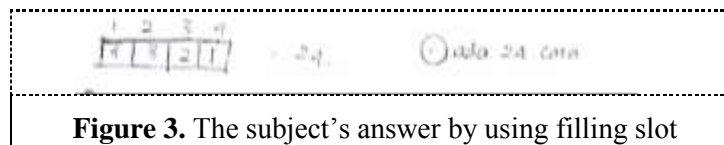
This was the interview between the researcher and the subject of the research:

Researcher: Did you use another way to find the solution besides this formula?

Subject: I used the boxes so that I was sure with my answer

Researcher: what is boxes?

Subject: Filling slots

Researcher: Why did you use filling slots ?

Subject: Because it was easier to put the possible to make password.

Researcher: Could you try to explain the way?

Subject: I only drew four boxes because the passwords consisted of four numbers. I named the boxes namely the first box, the second box, the third box, and the fourth box. The first box filled with the first possibility namely four possible answers (1,9,8,3), the second possibility was 3 possible numbers because it might not be allowed to be repeated, and so on until the third box and the fourth with number 2 and number 1. All numbers were multiplied 4 x 3 x 2 x 1 and the result was also 24. So, I was sure that the answer was 24 possible.

Based on figure 3 and the result of interview. It could be seen that the subject used different strategies to find solution. The subject did this to convince the answer. Students gave the logical reason why they chose the strategy. It looked when the subject chose the filling slot strategy because it was easier to put the possible to make a password, the subject preferred to say the box way[9]. Students described the accurate procedure when they used the strategy dan students gave logical reason in each step when they solved the problem, can be seen when the subject made four boxes because the password consisted of four numbers. subject named these boxes namely the first box, the second box, the third box, and the fourth box. The first box was filled with four because all number possible (1,9,8,3), the second possibility was 3 possible numbers that might not be the same and so on till the third and the fourth box with number 2 and number 1. All numbers were multiplied 4 x 3 x 2 x 1 and the result was also 24 [4]. So the subject was sure that the answer was 24. After seeing the same answer between the permutation formula and the strategy of filling slot, then subject was sure that the answer was correct.

4. Conclusion

Based on the description above, it could be concluded that from 49 reflective cognitive students, there was 92 % used one strategy (filling slot) and 8% of the students used two strategies (filling slot and permutation formula). In general, the students used multiplication principle or filling slot. However, there were students who used another strategy namely permutation formula to convince his work. The description of the process used by the subject of the research was: both subjects had a different reason in choosing the strategy. Both subjects described the accurate procedure when they used. In each step in doing the filling slot strategy and the strategy of permutation formula in each step to give logic reasons.

5. Acknowledgments

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