Mathematical Model of Free Convection Boundary Layer Flow on Solid Sphere with Viscous Dissipation and Thermal Radiation

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Abstract—Present study considers the mathematical modeling of free convection boundary layer flow and heat transfer on a solid sphere with viscous dissipation and thermal radiation effects. The transformed partial differential equations are solved numerically by using the Keller-box method. Numerical solutions are obtained for the reduced Nusselt number, the local skin friction coefficient, the velocity and temperature profiles. The features of the flow characteristics for various values of the Prandtl number, radiation parameter and Eckert number are discussed. It is worth mentioning that the results are obtained until $x = 180°$. This is contrary to the previous report where the separation boundary layer flow occurs after $x = 120°$. The results in this paper is important for the researchers working in the area of boundary layer flow and this can be used as reference and also as complement for comparison purposes in the future.

Index Terms—Mathematical modeling, free convection boundary layer flow, solid sphere, viscous dissipation, thermal radiation.

I. Introduction

CONVECTION boundary layer flow on a solid sphere becomes an important topic due to numerous engineering and industrial applications such as the spherical storage tanks and turbocharged ball bearing in automotives segment. The efficiency of lubricating and heat removes process (convection) in turbocharged unit definitely depends on design of the turbocharged itself, the flow, conductivity, viscosity and the characteristic of the fluid used.

The exact analysis on laminar free convection from a sphere was first investigated by Chiang et al. [1]. Amato and Tien [2] have worked on the experimental studies on free convection heat transfer from isothermal spheres in water. The experimental result obtained shows a very good agreement with predictions of Acrivos theory. Geoola and Cornish [3]

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have solved the steady-state free convective from a solid sphere using an extrapolated Gauss-Seidel method and obtained solutions for Grashof number less than 50. Next, Huang and Chen [4] have investigated this topic with the effects of suction and blowing while Jafarpur and Yovanovich [5] introduced a new analytical method in solving the laminar free convective from isothermal sphere. This problem has been extended to micropolar fluid by Nazar et al. [6], [7] by considering constant wall temperature and constant surface heat flux. Shafie et al. [8], Molla et al. [9] and Miraj et al. [10] studied the effect of g-jitter on double diffusion, magnetohydrodynamic, heat generation and thermal radiation on natural convection flow on sphere, respectively. Salleh et al. [11] considered the Newtonian heating as boundary conditions in this topic before extended it with micropolar fluid [12]. Other type of fluids considered including viscoelastic fluid which have been covered by Kasim et al. [13] and recently by Abdul Gaffar et al. [14] by considering the thermal radiation and heat generation/absorption effects in porous media with constant surface heat flux, respectively.

In considering the viscous dissipation effects, we know that the viscous dissipation is the induced kinetic energy converted into thermal energy. It is usually present in free convection subjected to large deccelaration from high rotating speeds [15]. Soundalgekar [16] has solved the viscous dissipation effects on unsteady free convective flow past an infinite, vertical porous plate with constant suction analytically. Vajravelu and Hadjinicolaou [17], Murthy and Singh [18], Tunc and Bayazitoglu [19] and Chen [20] have investigated the viscous dissipation effects on viscous fluid over a stretching surface, non-Darcy regime, microtubes and vertical surface, respectively. Many investigations have been made regarding the viscous dissipation effects including from the works of Olanrewaju et al. [21], Kameswaran et al. [22], RamReddy et al. [23] and recently by Qasim and Noreen [24] and Sheikholeslami et al. [25].

Based on the above contribution, the aim of present study is to investigate the free convective boundary layer flow on a solid sphere with viscous dissipation and thermal radiation effects.

II. Mathematical Formulations

A solid sphere with radius a, which is heated to a constant temperature T_w embedded in a viscous fluid with ambient tem-

perature T_∞ is considered. The model and physical coordinate system is shown in Fig. 1.
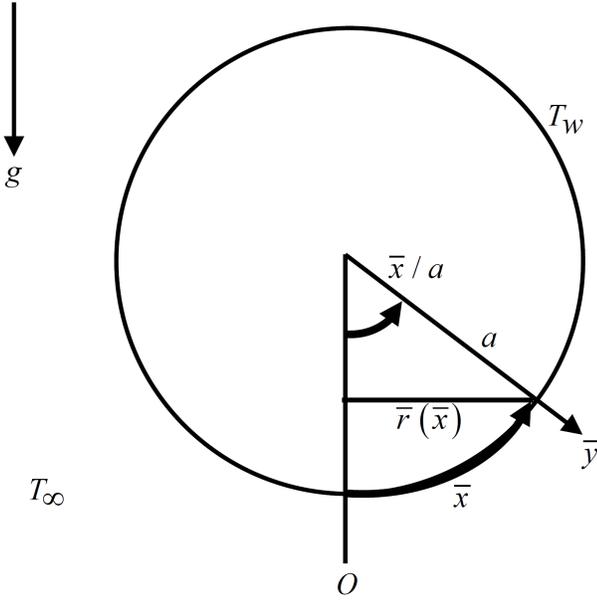
Fig. 1. Coordinate system of the physical model.

The orthogonal coordinates of $\bar{x}$ are measured along the sphere surface, starting from the lower stagnation point ($\bar{x} = 0$) and $\bar{y}$ measures the distance normal to the surface of the sphere. We define $\bar{r}(\bar{x}) = a\sin(\bar{x}/a)$ as the radial distance from the symmetrical axis to the surface of the sphere. Based on Nazar et al. [6] and Salleh et al. [11], the dimensional governing equations are

$$\frac{\partial}{\partial \bar{x}}(\bar{r}\bar{u}) + \frac{\partial}{\partial \bar{y}}(\bar{r}\bar{v}) = 0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta(T - T_\infty)\sin(\bar{x}/a), \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial \bar{y}^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial \bar{y}} \tag{3}$$

subject to the boundary conditions

$$\begin{aligned}\bar{u}(\bar{x},0) = \bar{v}(\bar{x},0) = 0, \qquad & T(\bar{x},0) = T_w, \\ \bar{u}(\bar{x},\infty) \to 0, \qquad & T(\bar{x},\infty) \to T_\infty,\end{aligned} \tag{4}$$

where $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$ and $\bar{y}$ axes, respectively. μ is dynamic viscosity, ν is the kinematic viscosity, g is the gravitt acceleration, k is the thermal conductivity, β is the thermal expansion, T is the local temperature, ρ is the fluid density and C_p is the specific heat capacity at a constant pressure. The radiative heat flux q_r can be simplified as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial \bar{y}}. \tag{5}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. Using Rosseland

approximation [26], (3) is reduced to

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \left(\frac{k}{\rho C_p} + \frac{16\sigma^* T_\infty^3}{3k^* \rho C_p}\right)\frac{\partial^2 T}{\partial \bar{y}^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2. \tag{6}$$

From the above equation, we can see that the effect of radiations is to enhance the thermal diffusivity. If we define $N_R = \frac{4\sigma^* T_\infty^3}{kk^*}$ as the radiation parameter, then (6) becomes

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{k}{\rho C_p}\left(1 + \frac{4}{3}N_R\right)\frac{\partial^2 T}{\partial \bar{y}^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2. \tag{7}$$

Note that thermal radiation effects are absent when $N_R = 0$. Next, we introduce the governing non-dimensional variables

$$\begin{aligned} r = \frac{\bar{r}}{a}, \; x = \frac{\bar{x}}{a}, \qquad & y = Gr^{1/4}\frac{\bar{y}}{a}, \qquad u = \frac{a}{\nu}Gr^{-1/2}\bar{u}, \\ v = \frac{a}{\nu}Gr^{-1/4}\bar{v}, \quad & \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \end{aligned} \tag{8}$$

where $Gr = \frac{g\beta(T_w - T_\infty)a^3}{\nu^2}$ is the Grashof number. Using (8), (1), (2) and (7) becomes the following dimensionless governing equations

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0, \tag{9}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta\sin x, \tag{10}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\left(1 + \frac{4}{3}N_R\right)\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2, \tag{11}$$

subject to the boundary conditions

$$\begin{aligned} u(x,0) = v(x,0) = 0, \qquad & \theta(x,0) = 1, \tag{12} \\ u(x,\infty) \to 0, \qquad & \theta(x,\infty) \to 0 \tag{13} \end{aligned}$$

Notice that $Pr = \frac{\nu k}{\rho C_p}$ is the Prandtl number and $Ec = \frac{\nu^2 Gr}{a^2 C_p(T_w - T_\infty)}$ is an Eckert number. In order to solve (9)-(11), the following functions are introduced

$$\psi = xr(x)f(x,y), \qquad \theta = \theta(x,y), \tag{14}$$

where ψ is the stream function, which is defined as $u = \frac{1}{r}\frac{\partial \psi}{\partial y}$ and $v = -\frac{1}{r}\frac{\partial \psi}{\partial x}$ that satisfies (9) and θ is the rescaled dimensionless temperature of the fluid. If we substitute (14) into (10)-(11), the following transformed partial differential equations are obtained

$$\begin{aligned}\frac{\partial^3 f}{\partial y^3} + \left(1 + \frac{x}{\sin x}\cos x\right)f\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \\ \frac{\sin x}{x}\theta = x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right),\end{aligned} \tag{15}$$

$$\begin{aligned}\frac{1}{Pr}\left(1 + \frac{4}{3}N_R\right)\frac{\partial^2 \theta}{\partial y^2} + \left(1 + \frac{x}{\sin x}\cos x\right)f\frac{\partial \theta}{\partial y} = \\ x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial y} - xEc\left(\frac{\partial^2 f}{\partial y^2}\right)^2\right),\end{aligned} \tag{16}$$

with boundary conditions

$$\begin{aligned} f(x,0) = 0, \frac{\partial f}{\partial y}(x,0) = 0, \; & \theta(x,0) = 1 \\ \frac{\partial f}{\partial y}(x,\infty) \to 0, \qquad & \theta(0,\infty) \to 0. \end{aligned} \tag{17}$$

The physical quantities of interest are the skin friction coefficient $C_f = \frac{\tau_w}{\rho u_\infty^2}$ and the local Nusselt number $Nu_x = \frac{aq_w}{k(T_w - T_\infty)}$ where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu \left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}, \qquad q_w = -k \left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0}. \qquad (18)$$

From (14) and (18), we obtain

$$C_f Gr^{1/4} = x \frac{\partial^2 f}{\partial y^2}(x,0) \text{ and } Nu_x Gr^{-1/4} = -\frac{\partial \theta}{\partial y}(x,0). \qquad (19)$$

III. NUMERICAL METHOD

The partial differential equations (15) and (16) subject to boundary conditions (17) are solved numerically using the Keller-box method. This method is an implicit finite-difference method in conjunction with Newton's method for linearization. This is a suitable method to solve parabolic partial differential equations. The previous studies which use the Keller-box method in solving boundary layer problems including Ishak et al. [27], [28], Nazar et al. [29], [30] and Salleh et al. [31], [32]. As described in the books by Na [33] and Cebeci and Bradshaw [34], the Keller-box method consists of four main steps:

1) reduce (15) and (16) to a first-order system,
2) write the difference equations using central differences,
3) linearize the resulting algebraic equations by Newton's method, and write them in the matrix-vector form,
4) solve the linear system by the block tridiagonal elimination technique.

IV. RESULTS AND DISCUSSIONS

Equations (15)-(16) with boundary conditions (17) are solved with three parameters considered namely the Prandtl number Pr, radiation parameter N_R and Eckert number Ec. Keller-box method is used to solve this problem. The boundary layer thickness $y_\infty = 12$ and step size $\Delta y = 0.02$ are used in obtaining the numerical results. It is worth mentioning that the numerical results obtained are not limited until $x = 120°$ as reported previously by Huang and Chen [4], Nazar et al. [6] and Salleh et al. [11]. The solution is obtained to the end of sphere ($x = 180°$). Table 1 shows the comparison values with previous published results. We found that the results are in a good agreement. Furthermore we believe that Keller-box method is proven to be very efficient to solve the convective boundary layer problems involving reduced partial differential equations.

Figures 2 and 3 show the variation of reduced skin friction coefficient $C_f Gr^{1/4}$ against x with various values of Pr, Ec and N_R, respectively. From both figures, we can see that the increment of x results in the increment of $C_f Gr^{1/4}$ in the middle of sphere surface before decreasing $C_f Gr^{1/4} \approx 0$ at the end of sphere.

Figures 4 and 5 illustrate the variation of reduced Nusselt number $Nu_x Gr^{-1/4}$ against x with various values of Pr, Ec and N_R, respectively. We can see that $Nu_x Gr^{-1/4}$ in Figs. 4 and 5 is decreasing w.r.t. x. From Fig. 4, it seems that

TABLE I
COMPARISON VALUES OF $Nu_x Gr^{-1/4}$ WITH PREVIOUS PUBLISHED
RESULTS FOR VARIOUS VALUES OF x.

x	Huang and Chen [4]	Nazar et al. [6]	Salleh et al. [11]	Present
0°	0.4574	0.4576	0.4576	0.4576
10°	0.4563	0.4565	0.4565	0.4565
20°	0.4532	0.4533	0.4533	0.4533
30°	0.4480	0.4480	0.4481	0.4480
40°	0.4407	0.4405	0.4406	0.4406
50°	0.4312	0.4308	0.4310	0.4310
60°	0.4194	0.4198	0.4195	0.4195
70°	0.4053	0.4046	0.4053	0.4053
80°	0.3886	0.3879	0.3886	0.3886
90°	0.3694	0.3684	0.3692	0.3692
100°			0.3470	0.3469
110°			0.3216	0.3215
120°			0.2925	0.2925
130°				0.2594
140°				0.2216
150°				0.1795
160°				0.1265
170°				0.0712
180°				0.0045
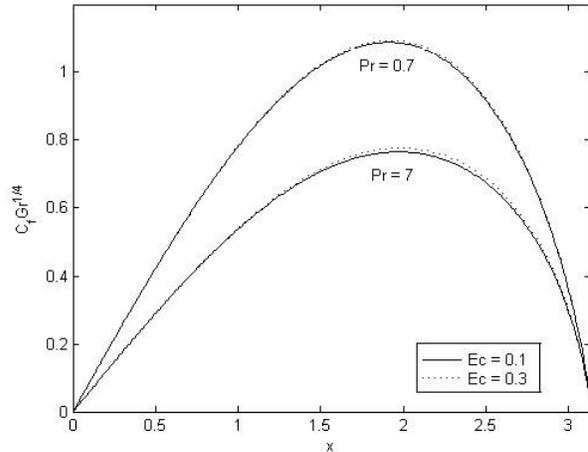
Fig. 2. Variation of $C_f Gr^{1/4}$ against x with various values of Pr and Ec when $N_R = 1$.

when Pr increases, the value of $Nu_x Gr^{-1/4}$ also increases, which physically means that bigger value of Pr contributes to the enhancement of convective heat transfer process. Next, the increment on Ec implies the reduction on the value of $Nu_x Gr^{-1/4}$. Also, the Ec effect is more pronounced with bigger value of Pr. This is due to the large kinetic energy offered from large Pr then converted to the thermal energy which enhance the conduction effects. From Fig. 5, the increase of N_R gives the reduction in $Nu_x Gr^{-1/4}$. The effect of changes in N_R is the most pronounced at the stagnation region ($x = 0$). Furthermore, we found that all set of parameters give $Nu_x Gr^{-1/4} \approx 0$, which physically means that no convection or the conduction process occurs at the end of sphere.

Finally, Figs. 6, 7, 8 and 9 display the temperature $\theta(y)$ and velocity profiles $f'(y)$ for various values of Pr and N_R, respectively. From Fig. 6, observe that the increase of Pr results in the decrease of boundary layer thickness. This is
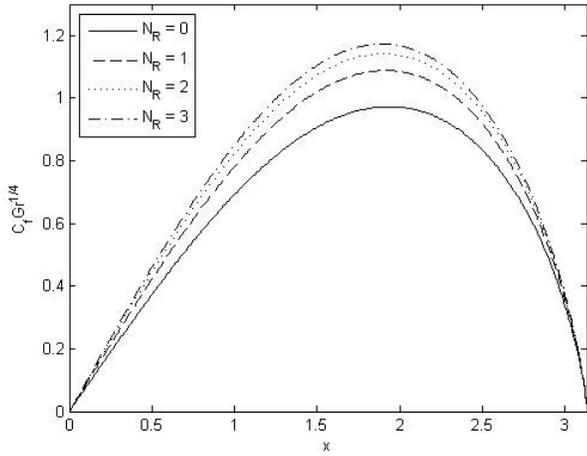
Fig. 3. Variation of $C_f Gr^{1/4}$ against x with various values of N_R when $Pr = 0.7$ and $Ec = 0.1$.
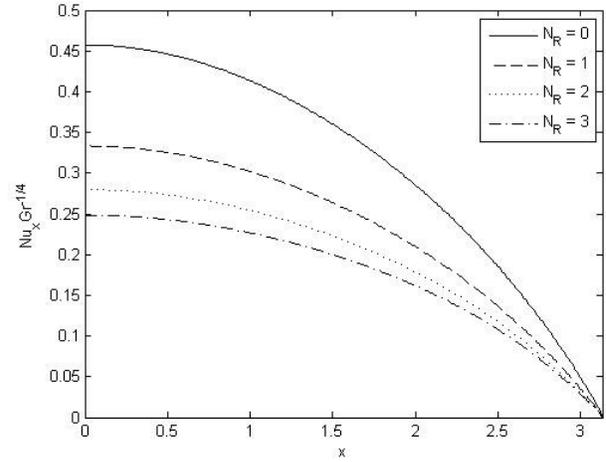
Fig. 5. Variation of $Nu_x Gr^{-1/4}$ against x with various values of N_R when $Pr = 0.7$ and $Ec = 0.1$.
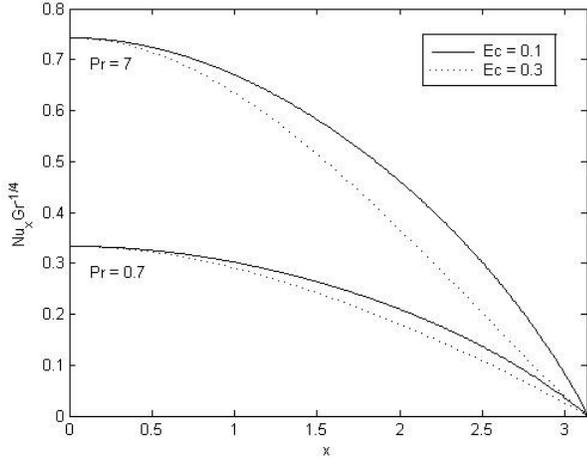
Fig. 4. Variation of $Nu_x Gr^{-1/4}$ against x with various values of Pr and Ec when $N_R = 1$.
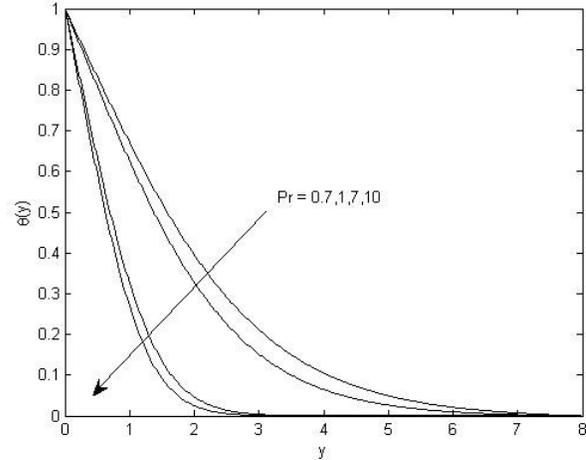
Fig. 6. Temperature profiles $\theta(y)$ for various values of Pr when $N_R = 1$ and $Ec = 0.1$.

due to a decrease of thermal diffusivity, which leads to the reduction in energy ability that reduces the thermal boundary layer thickness as Pr increases. Meanwhile, the increase of N_R enhances the boundary layer thickness. The velocity profiles in Fig. 8 suggest that the increase of Pr reduces the velocity of the fluid and the velocity gradient which results in the decrease of skin friction coefficients. The situation is contrary with N_R in Fig. 9. Boundary layer thickness increases as N_R increases. Furthermore, it is worth mentioning that the Ec does not have any effect on $\theta(y)$ and $f'(y)$. It is clear from (15) and (16) that at stagnation region $(x = 0)$, Ec is terminated from the equations.

V. CONCLUSIONS

In this paper, the mathematical modeling of free convection boundary layer flow on solid sphere with viscous dissipation and thermal radiation effects has been solved numerically. We have shown how the Prandtl number Pr, radiation parameter

N_R and Eckert number Ec affect the values of the reduced Nusselt number and reduced skin friction coefficient as well as the velocity and temperature profiles.

As a conclusion, the increase of Pr results in the increase of reduced Nusselt number while reduced skin friction coefficient decreases. Pr gives huge effects to the flow compared to Ec. Conversely, the increase of N_R enhances the reduced skin friction coefficient while the reduced Nusselt number decreases. Furthermore, the increase of Ec gives small increment on the reduced skin friction coefficient but decrease the reduced Nusselt number. Note that Ec is more pronounced with the large value of Pr.

Next, the increase in Pr reduces the thermal boundary layer thickness, the velocity of the fluid and the velocity gradient which results in the decrease of skin friction coefficients. Meanwhile, the thermal boundary layer thickness and the velocity of the fluid increase as N_R increases.
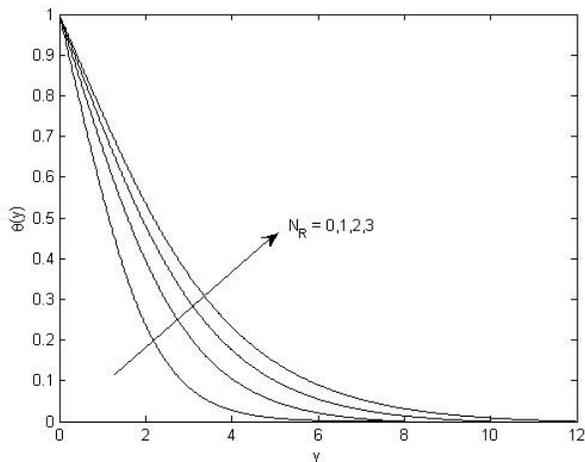
Fig. 7. Temperature profiles $\theta(y)$ for various values of N_R when $Pr = 0.7$ and $Ec = 0.1$.
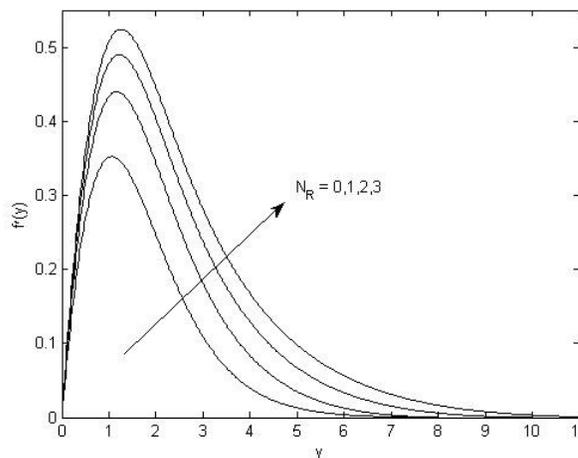
Fig. 9. Velocity profiles $f'(y)$ for various values of N_R when $Pr = 0.7$ and $Ec = 0.1$.
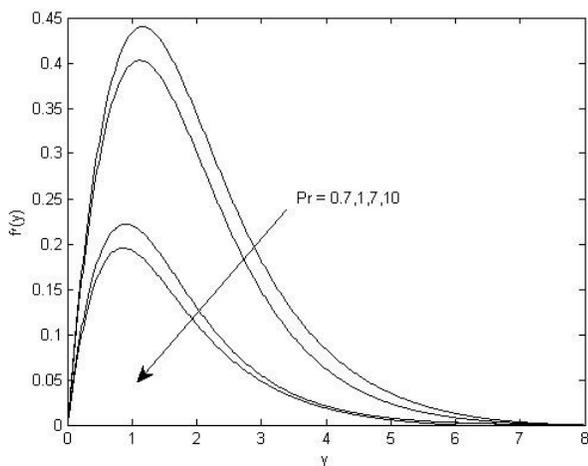
Fig. 8. Velocity profiles $f'(y)$ for various values of Pr when $N_R = 1$ and $Ec = 0.1$.

ACKNOWLEDGMENT

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