FROM ISOSCELES TRIANGLE TO NOBEL LAURATES COMPTON, DE BROGLIE, DAVISSON, AND THOMSON

Liek Wilardjo
Program Studi Teknik Elektro
Fakultas Teknik Elektronika dan Komputer – UKSW
Jalan Diponegoro 52-60, Salatiga 50711

1. Pengantar

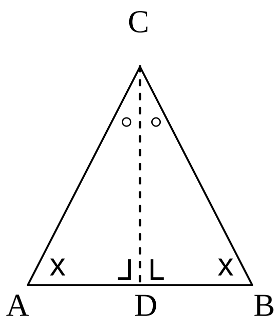
Artikel di bawah ini berisi tentang planimetri elementer, yakni segitiga samakaki dan dalil Pythagoras pada segitiga siku-siku. Dalam artikel ini ditampilkan contoh sederhana tentang geometri Euklides-an. Kedua jenis segitiga itu dan sifatnya dipakai untuk menunjukkan pembuktian sifat (*properties*) atau dalil (*propositions*) secara matematis.

Melalui planimetri, Bertrand Russell sewaktu masih bocah pertama kali mempelajari matematika dari guru privatnya[1]. Melalui dalil Pythagoras, Sokrates "memperagakan" bahwa matematika (menurut dia) ialah kebenaran abadi. Seorang bocah, anak budak belian, mampu menemukan sendiri bukti dalil Pythagoras[2]. Demonstrasi Sokrates itu menandai lahirnya *discovery* atau *inquiry method* yang sampai sekarang masih dipakai di bidang pendidikan. Anamnesis yang dilakukan dokter terhadap pasiennya juga didasarkan pada keyakinan yang sama.

Dalil Pythagoras dipakai Einstein untuk mengungkapkan kekararan (*invariance*) skalar berupa kuadrat vektor-4 pusa-tenaga (*momentum-energy four-vector*), yang "bermuara" pada "rumus"-nya yang terkenal, yakni $E = mc^2$. Louis Victor de Broglie kemudian menarik analogi dari cara Einstein itu, sehingga ia menemukan keseduaan zarah-gelombang (*particle-wave duality*), dengan persamaannya $p = h/l = \mathbf{h}k$. Dengan penemuannya itu Louis de Broglie bukan hanya memperoleh gelar doktor dari *Universite de Paris* (dengan Profesor Paul Langevin sebagai promotornya)[3] tetapi juga dianugerahi hadiah Nobel dalam fisika.

Hasil yang diperoleh Einstein, de Broglie, dan sebelumnya juga oleh Max Planck ($E = hv = \mathbf{h}w$, untuk foton) ternyata mengilhami karya-karya keilmuan lain, yang juga "menyabet" hadiah Nobel. Jadi dengan artikel ini saya hendak menunjukkan bahwa kebenaran yang tampaknya sederhana ternyata bisa membenih (*seminal*).

2. Isosceles Triangle



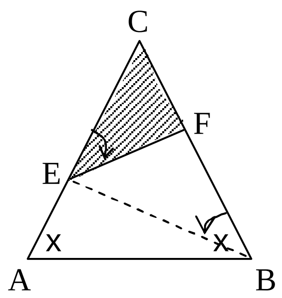
ABC is a triangle in which $\overline{AB}$ is the base and C the apex. If the base angles, $\angle$ A and $\angle$ B, are equal, then side $\overline{AC}$ is equal to side $\overline{BC}$, and thus ABC is an isosceles triangle.

Proof :

(1) Draw the triangle in a piece of clear / transparent plastic. Draw the bisector of the apex angle $\angle$ C until it intercepts base AB at a point, D. It is obvious that $\overline{CD}$ is perpendicular to $\overline{AB}$. Now fold the paper along $\overline{CD}$. You will see that A and B coincide and the right triangles ACD and BCD overlap perfectly, "proving" that $\overline{AC}$ is indeed equal to $\overline{BC}$.

What we have just done is actually not proving that ABC is an isosceles triangle. It is a "platonic" way of demonstrating experimentally or observationally that $\overline{AC}$ and $\overline{BC}$ are of equal length. To Plato, geometry is real. i.e., it deals with concrete entities.

(2) To prove the theorem mathematically, assume that $\overline{AC} \neq \overline{BC}$, say, that $\overline{AC}$ is longer than $\overline{BC}$. Then there is a point, call it *E*, on $\overline{AC}$ such that $\overline{BC} = \overline{EC}$. Draw line $\overline{EF}$ with the point F on side $\overline{BC}$, angle $\angle$ CEF being equal to angle $\angle$ CBE. Then the triangles CEF and CBE identical in

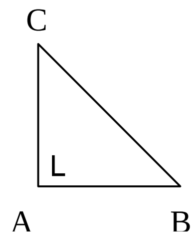


shape, two, and consequently all three, of their corresponding angles being equal. Therefore there is a proportionality between the corresponding sides of the two triangles, i.e., $\overline{CE} : \overline{EF} : \overline{FC} = \overline{CB} : \overline{BE} : \overline{EC}$, or $\overline{CE} \times \overline{EC} = \overline{EF} \times \overline{BE} = \overline{FC} \times \overline{CB}$. This is a three-side equation with the left, the middle and the right sides. From the equality of the left and the right side and noting that $\overline{CE} = \overline{CB}$ (according to our starting assumption), we conclude that $\overline{EC} = \overline{FC}$. But if $\overline{CE} = \overline{CB}$, $\overline{EC}$ couldn't possibly be equal to $\overline{FC}$, F being a point somewhere in between the vertices B and C.

The conclusion that $\overline{EC} = \overline{FC}$ leads to an absurd situation. Thus the opposite of our assumption, i.e., that $\overline{AC} = \overline{BC}$, is true by the logic of "*reductio ad absurdum*". *Quod erat demosntrandum* (*Q.E.D.*)

3. Pythagorean Theorem

ABC is a right-angle triangle. The angle $\angle$ A is 90°. $\overline{AB}$ and $\overline{AC}$ are the right sides, while $\overline{BC}$ the hypotenuse. Calling $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$ *c*, *b*, and *a*, respectively, Pythagorean theorem states that $a^2 = b^2 + c^2$, i.e., the square of the hypotenuse is equal to the sum of the squares of the right sides.

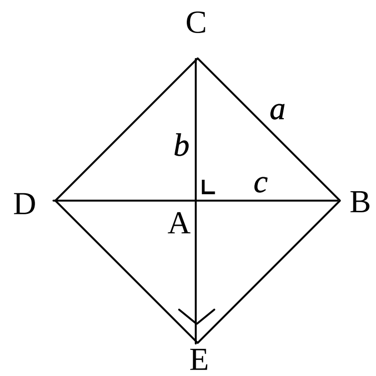


Proof:

(1) For an isosceles. right triangle, $b = c$, so we have to prove that

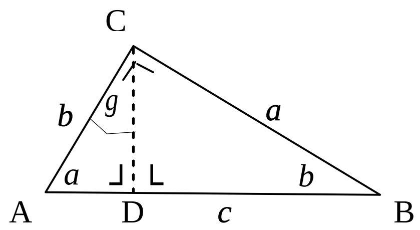
$a^2 = b^2 + c^2 = 2\,b^2$ (or $a = b\sqrt{2}$.

Draw three more right triangles of the same shapes and size to that of triangle ABC, so that we have a square, named BCDE, whose centre is A. The area of square BCDE is $L_1 = a^2$, a being its side. The area of triangle ABC is $L_2 = \frac{1}{2}bc = \frac{1}{2}b^2$, since $b = c$. From the figure it is



obvious that $L_1 = 4L_2$, therefore $a^2 = 4\times\frac{1}{2}b^2 = 2b^2$, giving $a = b\sqrt{2}$, thus proving the theorem for the special case of an isosceles, right-angle triangle.

According to Plato (in "*Dialogue with Meno*")[1] this proof was taken by Socrates as a supporting evidence of the truth of his belief that mathematics is eternal truth. In front of his friend, Meno, Socrates demonstrated that a boy who was the son of a slave could rediscover the Pythagorean theorem by himself, Socrates merely guiding him with a series of questions. This is what we call the discovery or the inquiry method of teaching.



(2) For any right-angle triangle, the proof goes as follows.[2] Consider the right-angle triangle ABC with the right angle at the vertex C, subtended between the right sides $\overline{CB} = a$ and $\overline{CA} = b$ and the hypotenuse, *c*, as its base, $\overline{AB}$. Name the base angles a and *b*, at vertex A and vertex B, respectively. The right-angle triangle ABC is defined, i.e., it is fully-specified, when its base and either one of its base angles is given. Thus its area, *L*, is completely determined by *c* and b. As angles or functions of angles can be considered as dimensionless, *L* must be proportional to the square of the base, *c*, times some function of a base angle, say *b*.

Thus $L \propto c^2 f(b)$.

Draw $\overline{CD}$ with *D* on $\overline{AB}$ and $\overline{CD} \perp \overline{AB}$. Since the sum of the three interior angles in any triangle is equal to 180°, the angle *g* at the vertex C of the right-angle triangle ADC must be equal to the angle *b* at the vertex B of the right-angle triangle ACB. Then the area of triangle ADC, L_1, is proportional to $b^2 f(g) = b^2 f(b)$, while that of triangle CDB, L_2, is proportional to $a^2 f(b)$. But from the figure we see that: $L_1 + L_2 = L$

Hence: $b^2 f(b) + a^2 f(b) = c^2 f(b)$,

or $b^2 + c^2 = a^2$ (*Q.E.D.*)

I leave it as an exercise for you to find $f(b)$.

[1] Plato (c. 427 – c. 347 B.C.) *from his dialogue Meno, 81 E – 86 C, Guthic, W.K.C., trans., Harmondsworth, Penguin, 1956*

[2] Wilardjo L., "Dalil Pythagoras", Desiderata, Vol. 4, No. 2, Juli- Des 2003, pp 59 - 64

4. Seminality of Pythagorean Theorem

Using Einstein's special Theory of Relativity, we have, from the invariance of the square of the momentum-energy four-vector,

$$p_x^2 + p_y^2 + p_z^2 + (j\frac{E}{c})^2 = p_x'^2 + p_y'^2 + p_z'^2 + \left(j\frac{E'}{c}\right)^2 \quad \mathbf{L} \qquad (1)$$

or
$$p^2 - \frac{E^2}{c^2} = p'^2 - \frac{E'^2}{c^2} \quad \mathbf{L} \qquad (2)$$

Here *p* and *E*, and *p'* and *E'*, are the relativistic momenta and total energies of a particle in the frames of reference *K* and *K'*, respectively. Writing *E'* as the sum of rest-mass energy $E_0' = m_0 c^2$ and kinetic energy $(g-1)E_0'$, and assuming that the particle is at rest in *K'*, we have:

$$p^2 - \frac{E^2}{c^2} = 0 - \frac{(m_0 c^2)^2}{c^2}, \quad \mathbf{L} \quad (3)$$

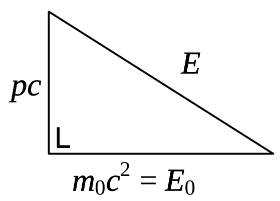
which can be rewritten as

$$p^2 c^2 - E^2 = -m_0^2 c^4 \quad \mathbf{L} \qquad (4)$$

Thus we have

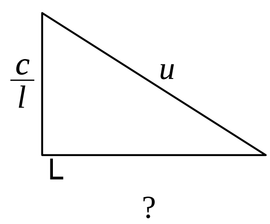
$$E^2 = p^2 c^2 + m_0^2 c^4 \quad \mathbf{L} \qquad (5)$$

Equation (5) is Pythagorean in form and can be depicted by a right-angle triangle with the right sides *pc* and $m_0 c^2$, and hypotenuse *E*.



Analogously, Louis-Victor de Broglie drew a right triangle as follows:

The hypotenuse of the triangle is frequency, *u*, and one of the right sides



is the velocity of light, *c*, divided by wavelength, l. The other right side is, by analogy, an invariant of wave which, at the time, was as yet unnamed.

As the consequence of the Pythagorean relation among the three sides of his right-angle triangle, de Broglie concluded that

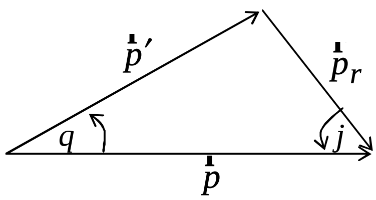
$$p = \frac{h}{l} = \mathbf{h} k \quad \mathbf{L} \qquad (7)$$

In equation (7) h is Plank's constant (6.625×10^{-34} Js), h is Dirac's constant (= $h/2\,p$), and k (= $2\,p\,/\,l$) is wave member or the phase constant of the wave representing the particle.

$p = \frac{h}{l}$ had been known to be true for photons, but de Broglie stated boldly that the relation held for massive particles as well. Hence we have the so-called particle-wave duality. Not only did de Broglie get the degree of doctor of philosophy in Physics from the University of Paris (1924), but he was also awarded the 1929 Nobel prize in Physics.

Arthur H. Compton used Planck's quantization of photons, $E = hu$ (= $\mathbf{h}\,w$), and de Broglie particle-wave duality, $p = \frac{h}{l}(= \mathbf{h} k)$, in the analysis of his experiment of The scattering of photons by a free electron in a substance, and won the 1927 Nobel prize in Physics.

The principle of conservation of momentum in Compton scattering can be depicted with a triangular vector diagram, the sides of which are $\vec{p}'$ ($= h/l'$, the momentum of the scattered photon) and $\vec{p}_r$ ($= g\ m_0 v$, the momentum of the recoiled electron), and the base of which is $\vec{p}'$ ($= h/l$ the momentum of the incident photon).



Together with the principle of conservation of energy,

$$hu = hu' + (g - 1)\, m_0 c^2 \quad \mathbf{L} \qquad (8)$$

the conservation of momentum,

$$p_r^2 = p^2 + p'^2 - 2\,p\,p' \cos q \quad \mathbf{L} \qquad (9)$$

results in the equation of Compton effect,

$$\Delta\ l = l_c (1 - \cos q) \quad \ldots \qquad (10)$$

From the equation of Compton effect, it is recognized that the unnamed invariant in de Broglie's right-angle triangle is c times the reciprocal of Compton wavelength,

$$c(1/l_c) = c(m_0\, c/h) = m_0\, c^2 / h \quad \mathbf{L} \quad (11)$$

Two other physicists, Clinton J. Davisson and George P. Thomson, shared the 1937 Nobel prize for experimentally proving the de Broglie's duality in the scattering of electrons by crystalline lattices.

DAFTAR PUSTAKA

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(2) Plato: "**Mathematics as Eternal Truth**", dalam Stuart Brown, *et al.*: **Conceptions of Inquiry**, Methuen, London, 1981, p. 3 – 11

(3) R.H. March: **Physics for Poets**, McGraw-Hill, New York, 1970, p. 207 – 212