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A MODAL PUSHOVER ANALYSIS ON MULTI-SPAN CONCRETE BRIDGES TO ESTIMATE INELASTIC SEISMIC RESPONSES

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ABSTRACT

The performance of Modal Pushover Analysis (MPA) in predicting the inelastic seismic response of multi-span concrete bridges is investigated. The bridge is subjected to lateral forces distributed proportionally over the span of the bridge in accordance to the product of mass and displaced shape. The bridge is pushed up to the target displacement determined from the peak displacement of the n^{th} mode inelastic Single Degree of Freedom System derived from Uncoupled Modal Response History Analysis (UMRHA). The peak response from each mode is combined using Square-Root of Sum-of-Square (SRSS) rule. Although the use of SRSS rule is not appropriate in this bridge and the displaced pattern is shifted from the elastic shape due to yielding, MPA can predict well the total peak response of the bridge in inelastic range.

Keywords:

INTRODUCTION

Pushover analysis has been widely used for analyzing the seismic behavior of bridge structures [1,2,3]. It can be used as a method for determining the capacity of a bridge structure neglecting the higher mode effects. This approach may produce an error for long or irregular bridges, especially in cases where the bridge has a large scattered mass distribution in the transverse direction [1]. Nonlinear pushover analysis is shown able to predict the inelastic response of the Greveniotikos Bridge which was designed as continuous bridge decks with no intermediate movement joints [2].

At the same time, many researchers reported the successful of pushover analysis on building structures especially for low to medium-rise building, which is typically dominated by the first mode [4,5,6]. However, as the structure becomes higher, the participation of higher modes may increase. These higher mode effects may contribute to the structure's response significantly [4]. In this case, the single invariant force distribution used by pushover analysis cannot represent the potential range of loading experienced in dynamic response. Therefore several new analysis methods have been developed to overcome the limitations of conventional pushover analysis. One of them [5] is to perform pushover analysis using an invariant lateral force distribution for each mode independently, to consider the contribution of higher modes. The peak responses determined from every mode are combined using square-root of sum-of-square (SRSS) combinations. This procedure is termed as Modal Pushover Analysis (MPA). Chopra and Goel [5] claimed that as an improved pushover analysis, MPA offers conceptual simplicity but provides superior accuracy compared to the conventional pushover analysis in estimating seismic demands on buildings.

In the other hand, the structural behavior of bridges is different from that of other structures (i.e. building structures). Although bridge design also improved during the past ten years, in the field of seismic design, it is several years lacking behind the progress achieved in building design [3]. Therefore, the application of MPA on bridge structure can be considered as an alternative to bridge design improvement.

For elastic range, MPA has been proven consistent with Response History Analysis [7,8,9]. The following discussion will be drawn based on the investigation of MPA on multi-span concrete bridge especially in the inelastic range.

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MODAL PUSHOVER ANALYSIS

The governing equilibrium equations of the N-degree of freedom (N-DOF) system shown in Figure 1 to horizontal earthquake ground motion $\ddot{u}_g(t)$ are as follows:

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -\mathbf{m}\mathbf{i}\ddot{u}_g(t) \tag{1}$$

where, $\mathbf{u}$ is the vector of N lateral displacements relative to the ground; $\mathbf{m}$, $\mathbf{c}$ and $\mathbf{k}$ are the mass, damping, and lateral stiffness matrices of the system respectively; where i is an influence vector with every member equal to unity.

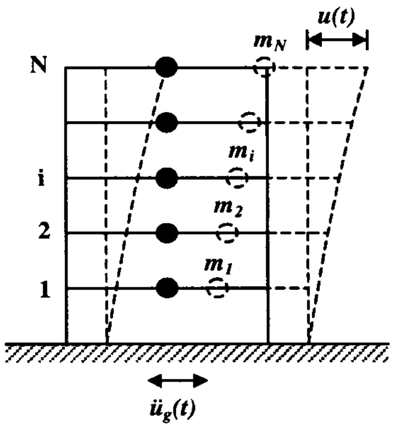


Figure 1. N-DOF System Under Ground Motion

In inelastic system, the relations between lateral forces f_s and the lateral displacements u are not single-valued, but depend on the history of the displacements:

$$\mathbf{f}_s = \mathbf{f}_s(\mathbf{u}, sign\dot{\mathbf{u}}) \tag{2}$$

Therefore for inelastic system Equation (1) can be rewritten as follows:

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{f}_s(\mathbf{u}, sign\dot{\mathbf{u}}) = -\mathbf{m}\mathbf{i}\ddot{u}_g(t) \tag{3}$$

Equation (3) consists of coupled equations. Solving these coupled equations directly, leads to the Nonlinear Time History Analysis (NLTHA).

In developing MPA for inelastic structures, Equation (3) will be transformed to the modal coordinates of the corresponding linear system. Although it is not proper because modal analysis is not valid for inelastic system, it can be assumed that at initial state of inelastic condition, the inelastic system has the same properties (e.g. stiffness, mass, and damping) with the elastic system [7]. Expanding the displacements of the inelastic system in terms of the natural vibration modes of the corresponding linear system one will obtain [8]:

$$\mathbf{u}(t) \cong \sum_{n=1}^{N} \boldsymbol{\varphi}_n q_n(t) \tag{4}$$

where, φ_n and $q_n(t)$ are the n^{th} natural vibration mode of the structure, and the modal coordinate respectively. Then, using Equation (4) and premultiplying by $\boldsymbol{\varphi}_n^T$, Equation (3) can be rewritten as [8]:

$$\ddot{q}_n + 2\zeta_n\omega_n\dot{q}_n + \frac{F_{sn}}{M_n} = -\Gamma_n\ddot{u}_g(t) \quad for\ n = 1,2,3,...,N \tag{5}$$

where:

$$\Gamma_n = \frac{L_n}{M_n} \qquad L_n = \boldsymbol{\varphi}_n^T\mathbf{m}\mathbf{i} \qquad M_n = \boldsymbol{\varphi}_n^T\mathbf{m}\boldsymbol{\varphi}_n \tag{6}$$

in which ω_n is the natural circular frequency and ζ_n is the damping ratio for the n^{th} mode. The solution $q_n(t)$ can readily be obtained by comparing Equation (5) to the equation of motion for the n^{th} mode elastic SDF system subjected to $\ddot{u}_g(t)$:

$$\ddot{D}_n + 2\zeta_n\omega_n\dot{D}_n + \omega_n^2 D_n(t) = \ddot{u}_g(t) \tag{7}$$

Comparing Equation (5) and (7) gives:

$$q_n(t) = -\Gamma_n D_n(t) \tag{8}$$

and substituting in Equation (4) gives the floor displacements:

$$\mathbf{u}_n(t) = \Gamma_n\boldsymbol{\varphi}_n D_n(t) \tag{9}$$

The preliminary step in developing modal pushover analysis for inelastic systems is performing uncoupled modal response history analysis (UMRHA). The UMRHA neglects the coupling of the N-equations in modal coordinates in Equation (5) to obtain the maximum displacement (Equation (9)) in each mode in the modal coordinate.

To represent the relation between lateral forces f_s and the lateral displacements u (Equation 2), structure is pushed to a maximum value determined in Equation (9) using lateral forces distributed over the building height in accordance to s_n*:

$$\mathbf{s}_n^* = \mathbf{m}\boldsymbol{\varphi}_n \tag{10}$$

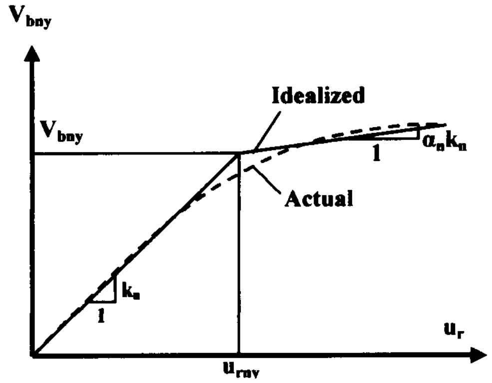
The base shear V_{bn} can be plotted against displacement u_{rn}. A bilinear idealization of this pushover curve for the n^{th} mode is shown in Figure 2(a).

The relation between forces and displacement follows [8]:

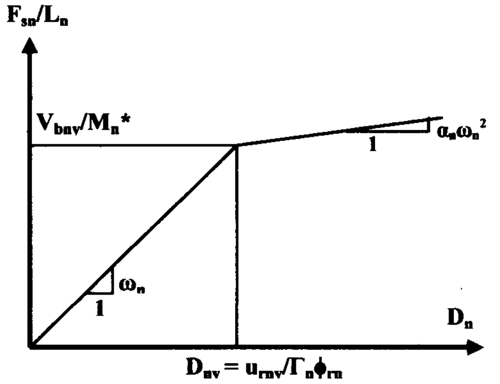
$$F_{sn} = \frac{V_{bn}}{\Gamma_n}, \qquad D_n = \frac{u_{rn}}{\Gamma_n\phi_{rn}} \tag{11}$$

By these relationships, pushover curve can be converted into the F_{sn}/L_n - D_n relation as shown in Figure 2(b). The yield value of F_{sn}/L_n and D_n are:

$$\frac{F_{sny}}{L_n} = \frac{V_{bny}}{M_n^*}, \qquad D_{ny} = \frac{u_{rny}}{\Gamma_n\phi_{rn}} \tag{12}$$



(a) Idealized pushover curve



(b) F_{sn}/L_n-D_n relationship

Figure 2. Properties of the n^{th}-mode inelastic SDOF system from pushover curve [8]

in which $M_n^* = L_n \Gamma_n$ is the effective modal mass. The two equations are related through

$$\frac{F_{sny}}{L_n} = \omega_n^2 D_{ny} \tag{13}$$

The peak displacement for each mode is given by:

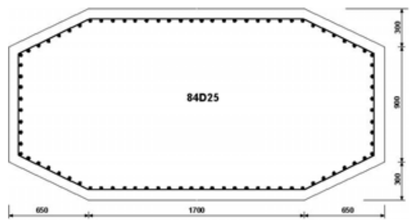
$$u_{rno} = \Gamma_n \varphi_n D_n \tag{14}$$

where D_n, the peak value of $D_n(t)$ can be determined by solving Equation (7) or from the inelastic response (or design) spectrum. The other peak response (e.g. shear, moment, etc.) can be derived statically from this pushover analysis. The peak modal responses are combined according to the square-root-of-sum-of-squares (SRSS) rule. Then, the SRSS rule provides an estimate of the peak value of the total response:

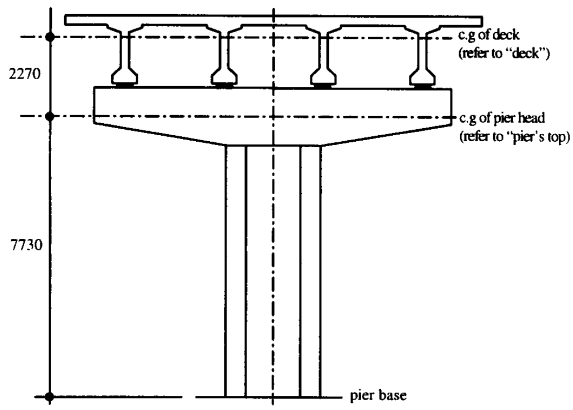
$$r_o \approx \left[\sum_{n=1}^{N} r_{no}^2 \right]^{1/2} \tag{15}$$

BRIDGE SELECTION AND MODELING

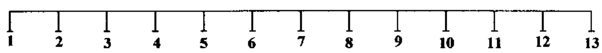
A multi-span concrete bridge in Surabaya area is chosen as the study case. The bridge deck is supported by a single-span prestressed concrete girders. The girders are placed on the concrete pier head through the bearing and locked in the transverse direction. The supporting piers are in various heights, but in this study equal height of 7.7 m is selected. The width of the bridge is 10.5 m with 30 m length of equal span. After doing comparison on the dynamic properties of three- to twenty-spans of the bridge, the twelve-span bridge is considered to be able to represent the behavior of multi-span bridge as the whole [9]. Some basic structural properties of the bridge are shown in Figure 3.



(a) Piers' section



(b) Elevation view (yz-direction)



(c) Pier's numbering (xz-direction)

Figure 3. The Bridge Properties

The deck is supported by four prestressed girders of 30 m span, connected by stringer beams at 6 m interval span. The deck structure is modeled as an assemblage of linear elements in SAP2000 Non-linear program [10,11]. The deck is assumed to be rigid in x- and y-direction. All node lies at the same elevation in which at the centre of gravity of the girder and stringers, and mass is lumped at both ends of element.

Each pier is modeled as an element with an elastic-plastic behavior. It is assumed that the piers will fail in flexural mode where a plastic hinge formed at the pier's base. The moment-rotation capacity of the plastic hinge is defined based on the stress-strain relationship of the section considering the confine-ment effect from transverse reinforcement [12]. The flexural rigidity of pier section is taken $0.7E_cI_g$ as recommended by ATC-40 [13].

Bearing is modeled using link element in SAP2000 Nonlinear program [10,11]. It is composed of six separate "springs", one for each of the six deformational degrees-of freedom (axial, two-way shears, two-way bendings, and torsion).

The bridge pier is supported by twenty five driven piles with dimension 0.45 x 0.45 x 35 m. The 6.9 x 6.9 x 1.5 m pile cap is modeled as a shell element in SAP2000 Nonlinear program. To accommodate the soil-structure interaction, each pile is modeled as spring with six degree of freedom to represent translational and rotational support. As recommended by ATC-40 [13], for a purely friction pile which implies that the force at the tip is zero, the vertical stiffness of pile, $K_v = 3.42\text{x}10^5$ kN/m for each pile. While, the horizontal stiffness of pile is assumed to be $0.05K_v = 1.71\text{x}10^4$ kN/m [14].

GROUND MOTION

Indonesian seismic zoning was based on the peak ground acceleration (PGA) induced by the design earthquake with 500-years return period [15]. This zoning defines Surabaya in zone 2. The response spectrum of structures in this zone is shown in Figure 4. The earthquake record needs to be modified prior to the analysis so that it can represent the ground motion for Surabaya area. The modification has been done using RESMAT [16], a program developed at Petra Christian University, Indonesia, especially to match with the response spectra curve for soft soil. The modified ground excitation resulted by RESMAT program is shown in Figure 5.

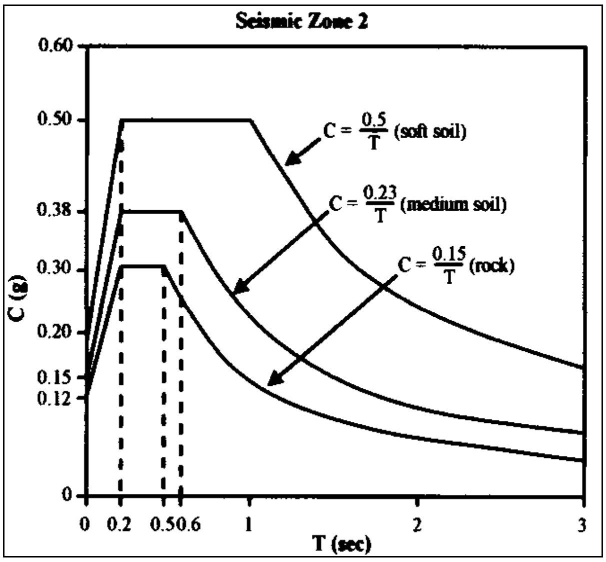


Figure 4. Response Spectrum for Surabaya Area [15]

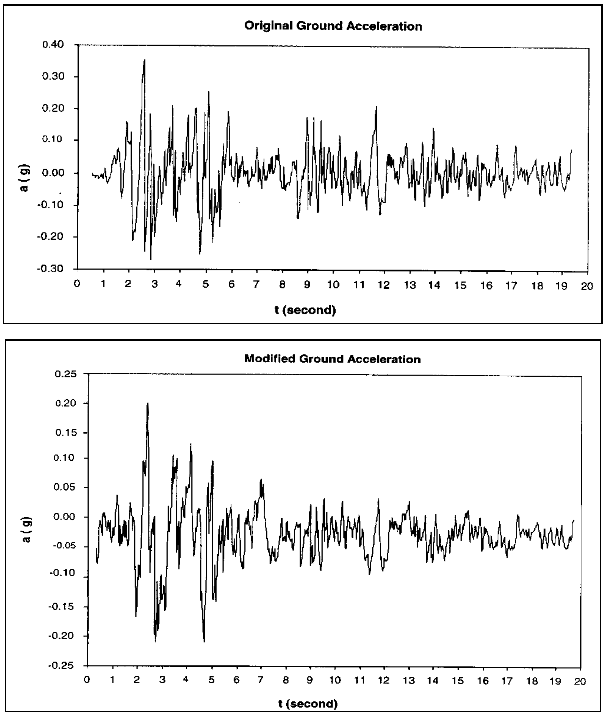


Figure 5. Ground Excitation Record

RESULTS AND DISCUSSION

The first three vibration modes of the bridge for linearly elastic vibration are shown in Figure 6. All mode shapes are normalized to the centre pier (i.e. pier 7). The elastic dynamic properties of three modes are shown in Table 1. These three modes are selected based on the highest modal mass participating factor and natural periods among forty modes resulted by modal analysis [9]. The spatial force distributions, s_n^* (Equation 10), for the first three modes are shown in Figure 7. These force distribution will be used in the modal pushover analysis to be presented later.

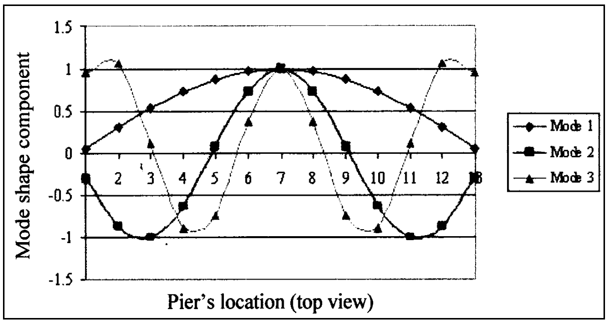


Figure 6. First three Modes of The Bridge, Normalized to Pier's Top of Pier 7

Firstly, this study conducts UMRHA and MPA, then the results will be compared with Nonlinear Time History Analysis (NLTHA). To ensure that this bridge responding beyond the inelastic range, the peak ground acceleration is scaled up to 0.5g. The force-displacement relationships of each mode which will be used to solve Equation (7) can be seen in Figure 8.

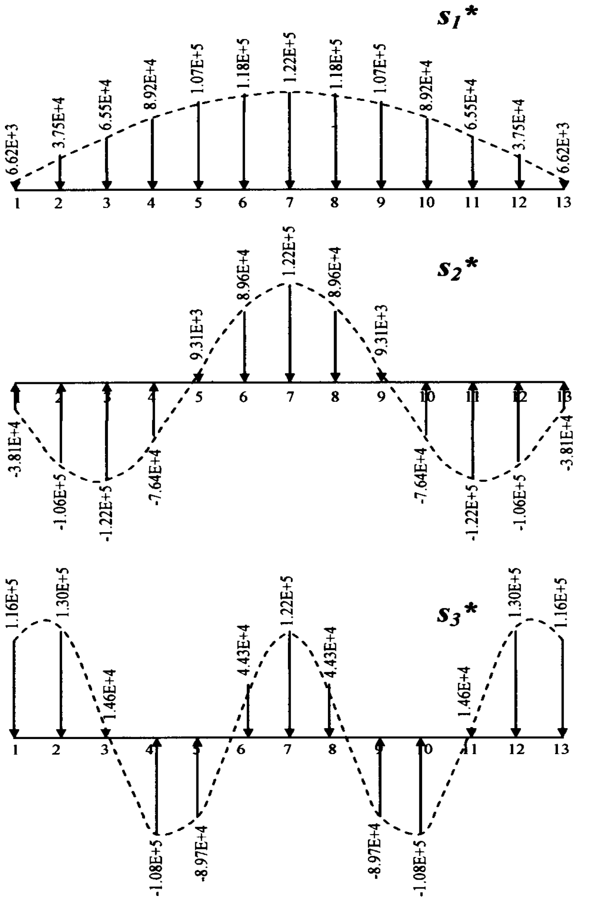


Figure 7. Force Distribution $s_n^* = m\phi_n$, n = 1, 2, 3 (at Pier's Top Elevation)

Table 1. Elastic Dynamic Properties of The Bridge

Mode	Period (sec)	Mass Participating Ratio	Frequency (cyc/sec)	Circular Frequency (rad/sec)	Eigenvalue (rad²/sec²)
1	0.66	0.54	1.52	9.57	91.55
2	0.63	0.06	1.58	9.92	98.38
3	0.56	0.03	1.80	11.29	127.40

Observe that only the first mode experience inelasticity (for peak ground acceleration 0.5g). The figures in the right side show the force-displacement relation which should be used to solve Equation (7). By making of this relationship, there is additional approximation in UMRHA in addition to neglecting coupling among each modal equation.

By solving Equation (7), we can obtain the individual modal responses. The combined response due to the three modes from UMRHA, and the exact response from NLTHA for the pier's top displacement of pier 7 are shown in Figure 9. The peak values of response are noted; in particular, the peak displacement due to each of the three modes is u_{r1o} =39.10 mm, u_{r2o} =12.91 mm, u_{r3o} = 6.81 mm. All peak values of the bridge are presented in Figure 10 and 11 respectively; also included the combined responses due to one, two, and three modes, as well as the exact results.

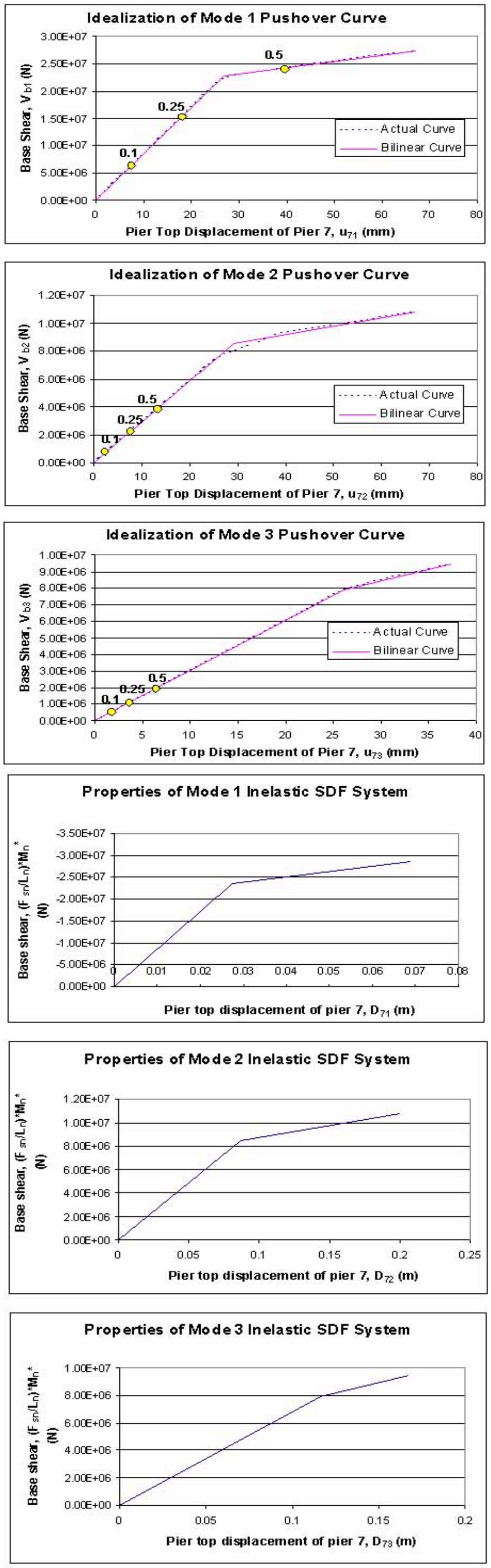


Figure 8. Pushover Curve and F_{sn}/L_n-D_n Curve for Mode 1, 2, and 3. Peak Ground Acceleration are Noted as 0.1, 0.25, and 0.5.

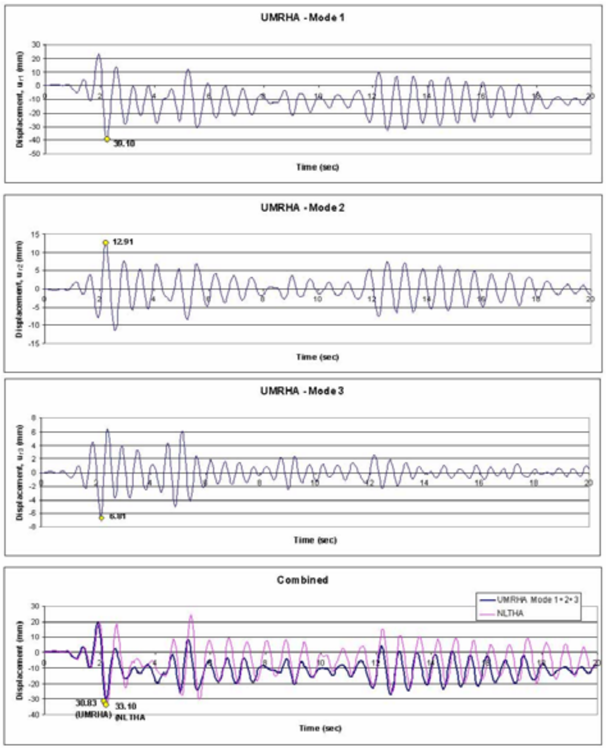


Figure 9. Pier's Top Displacement History Due to Peak Ground Acceleration 0.5g

Observe that errors tend to decrease as response contributions of more modes are included, although the trends are not systematic as when the system remained elastic [9].

Pushing the bridge into the target displacement derived from UMRHA; in particular u_{r1o} =39.10 mm, u_{r2o} =12.91 mm, u_{r3o} =6.81 mm, for the first, second, and third mode will produce the peak values as presented in Figure 12 and 13, respectively. The figures also present the combined responses due to one, two, and three modes, as well as the exact results from Nonlinear Time History Analysis (NLTHA).

In the present study the peak displacement determined from UMRHA is used to determine the target displacement in MPA. If this target displacement can be taken directly from inelastic response spectrum, it will be very efficient to do the analysis because we do not need to conduct UMRHA.

However, MPA is only good in predicting the peak displacement at the target point (i.e. pier's top of pier 7). In general, both methods overestimate the pier's top displacement of piers in the middle span (i.e. pier 5 to 9) as shown in Figure 10 (for UMRHA) and Figure 12 (for MPA). At the same time, both of them failed to predict the peak displacement of node at the different elevation from the target's elevation (i.e. pier's base elevation shown in Figure 10(d) and 12(d)).

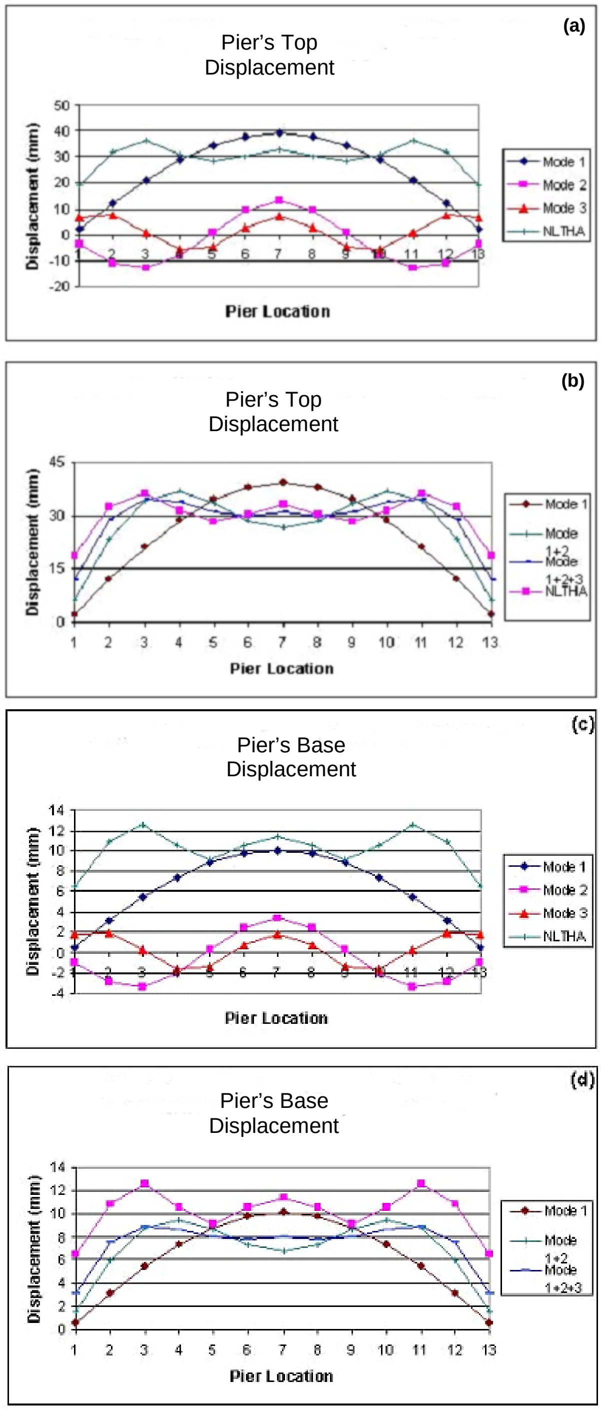


Figure 10. Pier's Top and Base Displacement from UMRHA

Observed, there is a shift pattern of mode 1 for pier's top displacement in Figure 12(a) and (b) compared to the pattern in the elastic range. This shift is caused by yielding at the pier's base. For clarification, Figure 14(a) shows the mode 1 pushover curve. Pier 7 is starting to yield at displacement 26 mm. Subsequent yielding occurs at pier 6 and 8 at displacement 27 mm. It is clear that when the peak displacement at pier's top of pier 7 reach 39.10 mm, three piers already yield. As the result, bridge can not maintain the elastic pattern of mode in the elastic range.

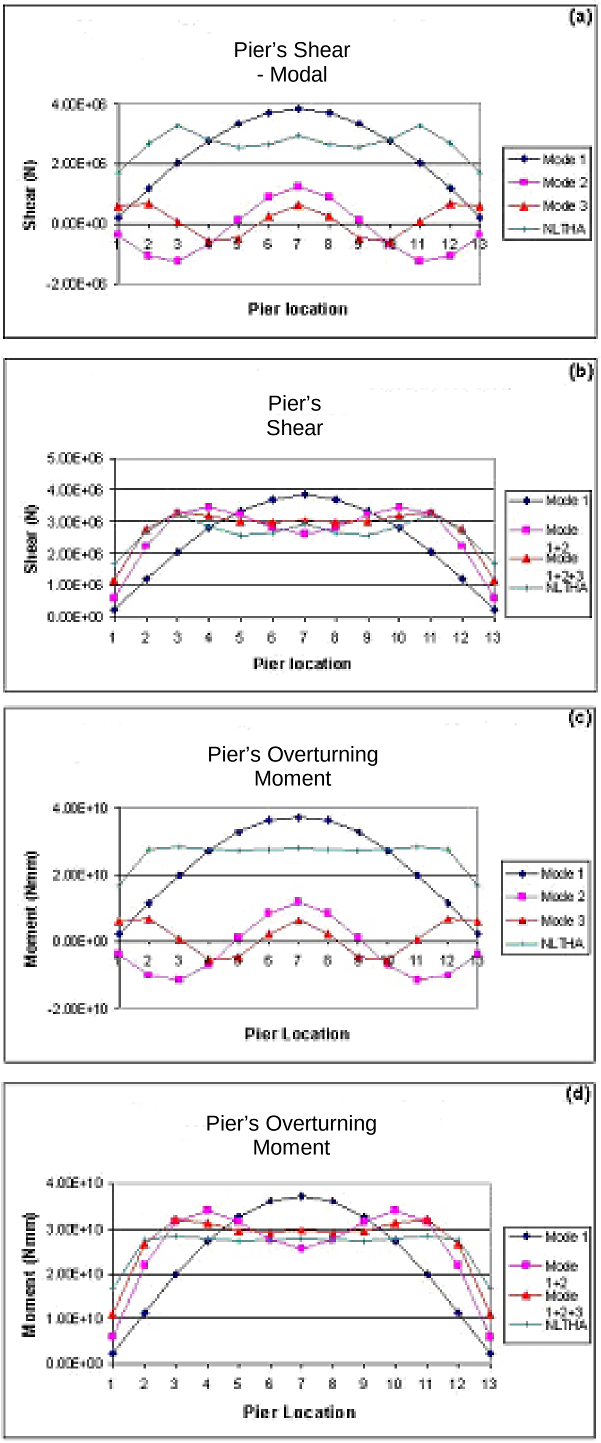


Figure 11. Pier's Shear and Overturning Moment from UMRHA

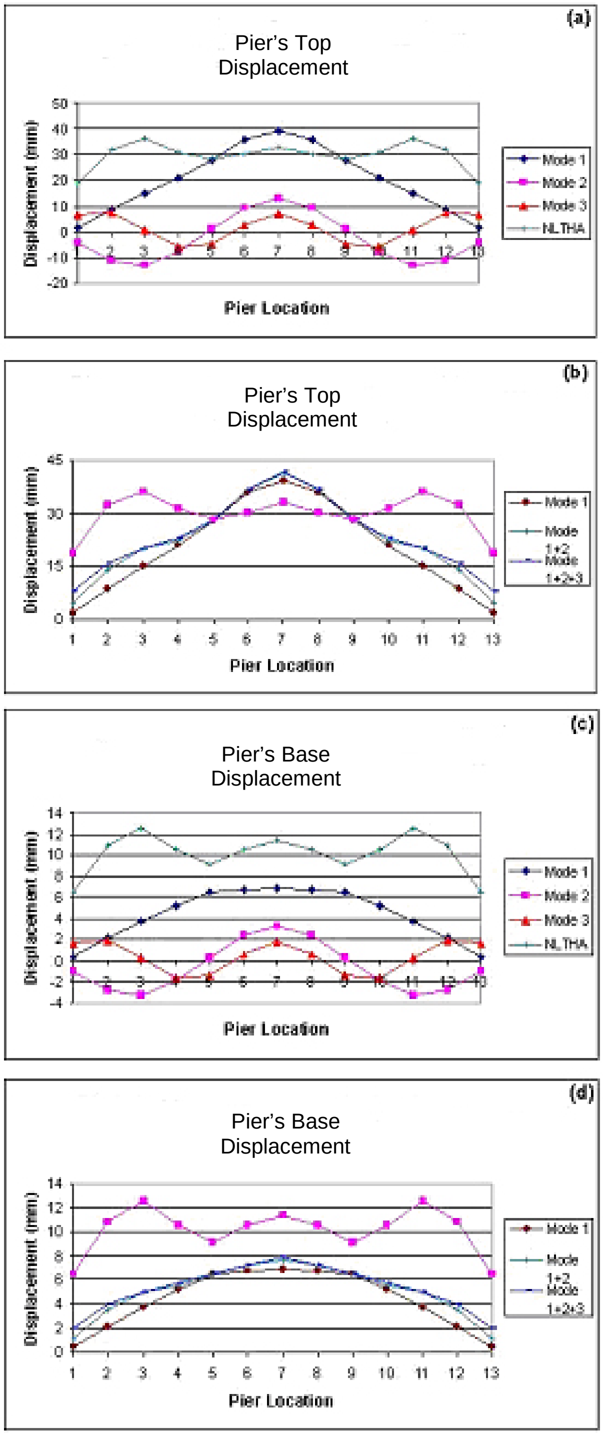


Figure 12. Pier's Top and Base Displacement from MPA

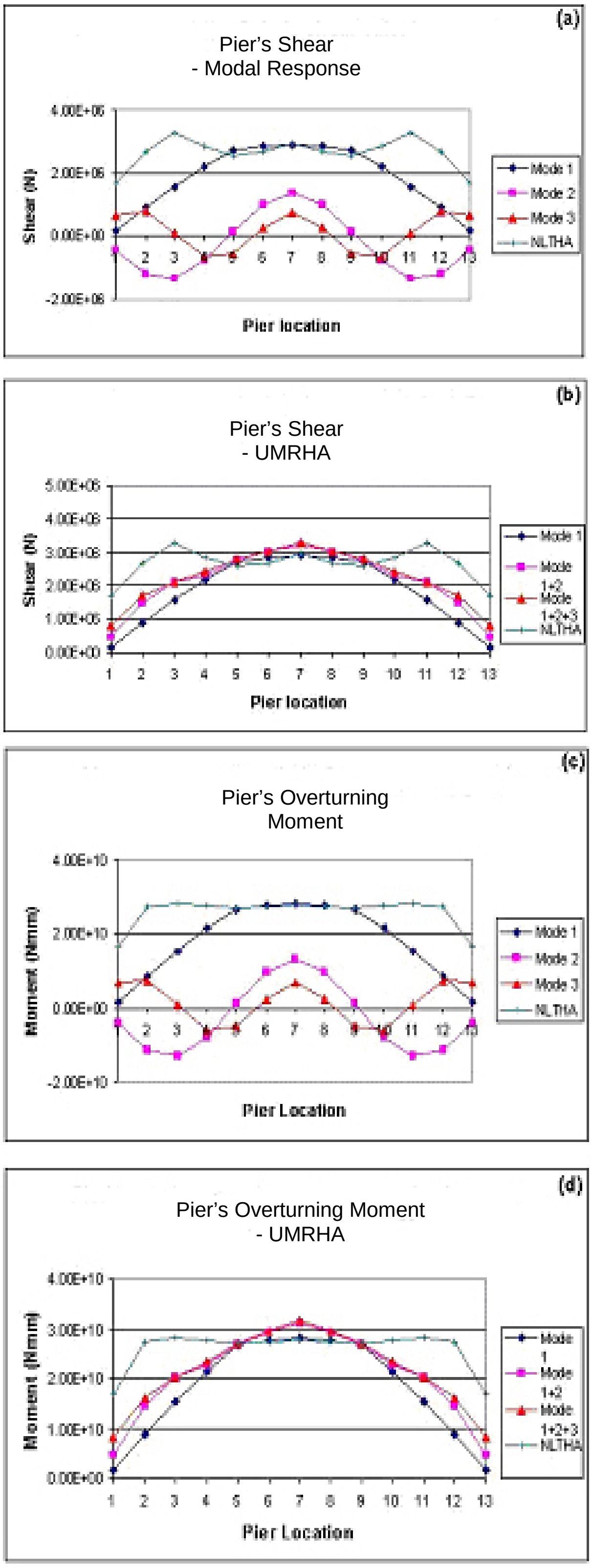
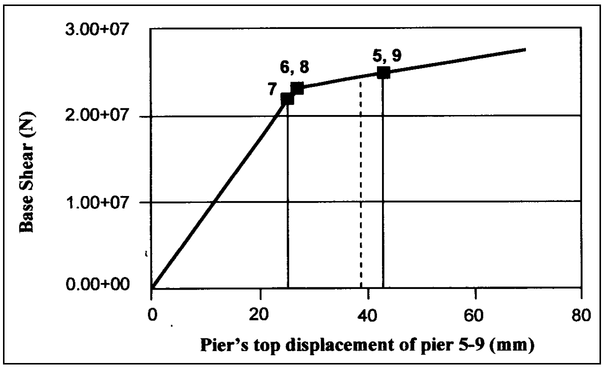


Figure 13. Pier's Shear and Overturning Moment from MPA



Note: 5-9 is the number of pier

Figure 14. Hinge Formation History Due to 0.5g Peak Ground Acceleration.

CONCLUSIONS

1. Although MPA can predict well the maximum displacement in the elastic range, it fails to estimate the maximum displacement of each pier especially piers at the quarter end span of the bridge.
2. For the case of multi-span bridge used in this study, the performance of MPA in nonlinear range shows a similar tendency with MPA in linear range. Although the mode shape changes due to yielding, the maximum displacement still can be predicted, as well as shear and overturning moment at the piers.
3. Being an approximate method, MPA gives an acceptable accuracy beside of simplicity and efficiency in calculation. Therefore, the performance of MPA needs to be investigated in more various structures.

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