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EVALUATION OF FINITE ELEMENT MESH ARRANGEMENTS AND STRESS INTENSITY FACTOR CALCULATION METHODS FOR OPENING MODE FRACTURE OF CRACKED-CEMENTED MATERIALS

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ABSTRACT

Fracture mechanics is a branch of mechanics, which deals with the cracked body. Every construction material that currently in use inevitably is not flawless. The pre-existing crack may grow to cause structure failure due to low stress, which acts to a structure. Stress intensity factor (K) is a single parameter in fracture mechanics, which can be used to examine if a crack, would propagate in a cracked structure under particular loading condition. Finite element method is used to analyze the cracked body to provide the displacements data around the crack tip (at quarter point elements) due to load prescribed, for stress intensity factor determination. Two methods of stress intensity factor calculation, Quarter Point Displacement Technique (QPDT) and Displacement Correlation Technique (DCT), were evaluated. A series of standard fracture testing were undertaken to provide the fracture load data (P_f), which coupled with the stress intensity factor analytical formula to calculate fracture toughness. The results showed that under a particular mesh arrangement, the result of finite element analysis could deviate from the analytical formula calculation result. The QPDT method is suitable for compact tension specimen but DCT seemed to be not. For cracked beam analysis, the QPDT and DCT calculations were in good agreement with the analytical formula as long as coupled with the appropriate mesh arrangement around the crack tip.

Keywords: *finite element, stress intensity factor, fracture toughness, quarter point element, crack, fracture mechanics.*

INTRODUCTION

Many accidents in past were caused by the collapse of structures, such as bridges, multi-storey buildings, ships and dams. Failure of structure was not only caused by poor design, but it was eventually found that it was also triggered by material deficiencies in the form of pre-existing flaws which could initiate cracks and fractures. Low stress fractures induced by small cracks are very similar to the brittle fractures of welded low-strength steel structures. This phenomenon has triggered the emergence of engineering fracture mechanics.

In structural design, the conventional design is mainly based on the tensile strength, yield strength and buckling stress, this design will completely fail to work if there is likelihood of cracks in the material used which can cause failure due to crack propagation. Regarding this, conventional design needs to be combined with engineering fracture mechanics, which provides a framework to overcome the inadequacies in the conventional design to deal with the defect in the material.

The most important parameter in engineering fracture mechanics is stress intensity factor (SIF). It is the single parameter, which controls the stability of the crack. The tendency of a crack to grow is analyzed using stress intensity factor at one cracked body.

For a simple problem, stress intensity factor can be calculated very easily, but problem might arise when one is dealing with complicated cracked structures. Regarding this problem, finite element analysis is one of the most powerful means to provide some parameters for examining the stability of the cracked structures (stress intensity factor). To process the data to calculate the stress intensity factor, some finite element methods were developed in the past such as quarter point element displacement

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technique [1], *J*-integral [2], fictitious crack model [3], two-parameter fracture model [4], and the energy-based cohesive crack propagation model [5]. Most of these methods are viable tools in dealing with concrete structures. These methods are properly applicable in linear elastic fracture mechanics (LEFM) framework, as long as the size of the aggregate is much less than the structures [6].

In this paper, the study is limited to Mode I fracture mode (opening mode) and consists of two targets; the first is to find out the typical finite element mesh arrangement in dealing with opening mode of fracture problem and the second is to compare two techniques; Displacement Correlation Technique (DCT) and Quarter Point Displacement Technique (QPDT), for stress intensity factor calculation based on the data generated from the finite element analysis.

LITERATURE REVIEW

Concept of Stress Intensity Factor (K)

There are three types of fracture mode, which may be encountered in the field, Mode I is opening mode, Mode II is sliding mode and the Mode III is tearing mode (Figure 1).

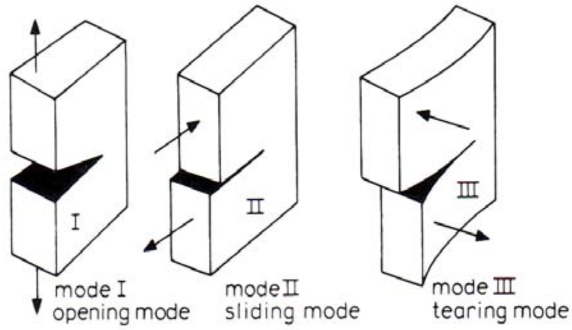

Figure 1. Typical Modes of Fracture

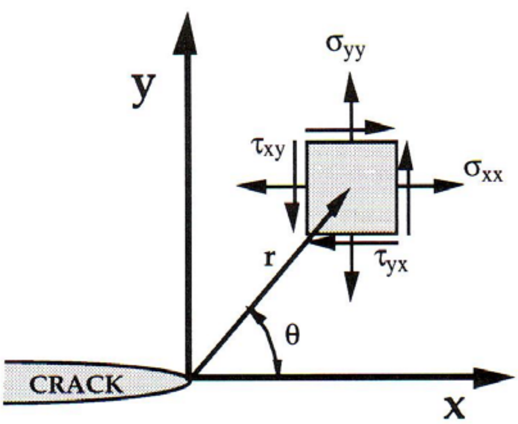

Figure 2. Stresses Near The Tip of a Crack in an Elastic Material

Stress intensity factor (K) is the quantity that gives the magnitude of elastic stress field. The stress at the crack tip can be expressed as (Figure 2):

$$s_{ij} = \frac{K_I}{\sqrt{2pr}} f_{ij}(q) \quad (1)$$

where σ_{ij} is the stress acting on an element dx dy at a distance r from the crack tip and an angle of θ from the crack plane and $f_{ij}(\theta)$ is a function of θ which is known. K_I is the stress intensity factor for mode I fracture mode, subscript I stands for Mode I fracture mode.

Mode I Stress Intensity Factor (K_I) Calculation Formula (CT-Specimen)

According to [7], Mode I stress intensity factor for compact tension specimen (CT-Specimen, Figure 3) is calculated based on the following formula :

$$K_I = \frac{P}{B\sqrt{W}} f\left(\frac{a}{W}\right) \quad (2)$$

where:

$$f\left(\frac{a}{W}\right) = \frac{2 + \frac{a}{W}}{(1 - \frac{a}{W})^{3/2}}[0.886 + 4.64\left(\frac{a}{W}\right) - 13.32\left(\frac{a}{W}\right)^2 + 14.72\left(\frac{a}{W}\right)^3 - 5.60\left(\frac{a}{W}\right)^4] \quad (3)$$

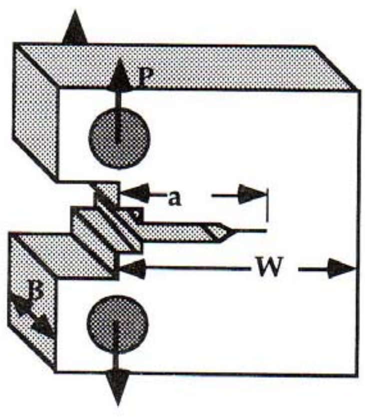

Figure 3. Mode I Fracture Testing Specimen Geometry (CT-Specimen) [7]

Mode I Stress Intensity Factor (K_I) Calculation Formula for three-point bend specimen

The analytical formula [7] for three point bend specimen crack problem (Figure 4) is :

$$K_I = \frac{P}{B\sqrt{W}} f\left(\frac{a}{W}\right) \quad (4)$$

where:

$$f\left(\frac{a}{W}\right)=\frac{3\frac{S}{W}\sqrt{\frac{a}{W}}}{2\left\{1+2\left(\frac{a}{W}\right)\right\}\left\{1-\left(\frac{a}{W}\right)\right\}^{3/2}}\left[1.99-\frac{a}{W}\left(1-\frac{a}{W}\right)\left\{2.15-3.93\left(\frac{a}{W}\right)+2.7\left(\frac{a}{W}\right)^{2}\right\}\right] \quad (5)$$

W is the height of the beam and B is the width of the beam analyzed.

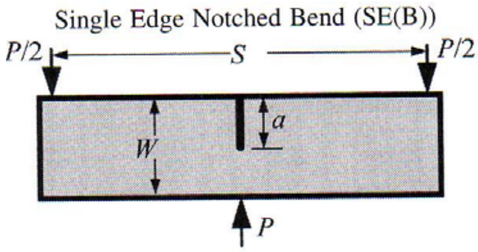

Figure 4. Typical Geometry of Mode I Fracture Testing of Beam Specimen [7]

Crack extension will occur when K reaches a certain value, which is called as fracture toughness (K_c). Fracture toughness can be obtained by undertaking the standardized testing under a particular condition of fracture mode.

Quarter Point Element

Modelling the singularity at the crack tip is an essential part in predicting crack propagation using fracture mechanics, but being able to model the singularity is not sufficient, since stress intensity factor is the most important parameter in fracture mechanics to check if the crack propagates, the proper method should be available for this task to be done. Figure 5 illustrates the quadratic isoparametric element which is used in this study to satisfy the stress singularity at the crack tip for stress intensity factor calculation [8]. Eight-node isoparametric plane strain elements were chosen with the mid-side nodes of elements surrounding the crack tips moved to the quarter point of each element side, so that a square root singular deformation field at the crack tip could be achieved.

Stress Intensity Factor Calculation Methods

The Displacement Correlation Technique (DCT) [8] uses all four nodes on the crack faces of quarter point element (QPE), as can be seen in Figure 6. The stress intensity factors for Mode I :

$$K_I = \frac{2G}{(k+1)}\sqrt{\frac{p}{2L}}\left[4\left(v_2 - v_1\right) - \left(v_4 - v_3\right)\right] \quad (6)$$

where:

$\kappa = 3\text{-}4\mu$, μ = poison's ratio, G = shear modulus = $E/[2(1+\mu)]$ for plane strain, L=length of quarter point element and u = quarter point node displacement toward u direction, v = quarter point node displacement toward v direction.

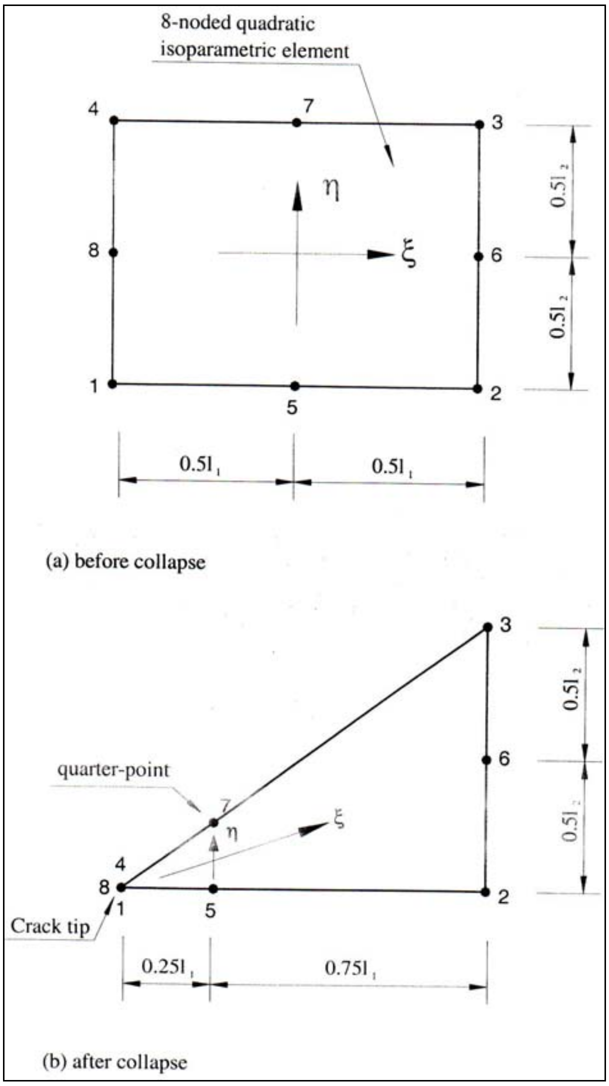

Figure 5. Formation of Triangular Crack-Tip Element From 8-Noded Quadratic Quadrilateral Element

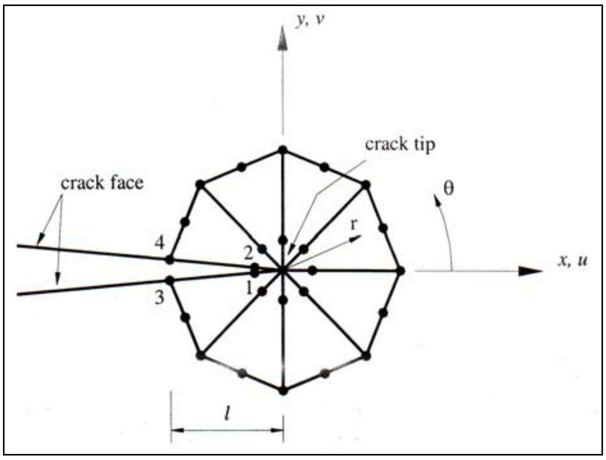

Figure 6. Quarter-Point Elements Modeled Around Crack Tip [8]

The Quarter Point Displacement Technique (QPDT) is slightly different to the Displacement Correlation Technique, the Quarter Point Displacement Technique only uses two adjacent nodes at the crack surface to calculate the stress intensity factors. According to Quarter Point Displacement Technique the stress intensity factors are as follows:

$$K_I = \frac{2G}{k+1}\sqrt{\frac{2p}{L}}\left(v_2 - v_1\right) \quad (6)$$

where:

$\kappa = 3\text{-}4\mu$, μ = poison's ratio, G = shear modulus = $E/[2(1+\mu)]$ for plane strain, L=length of quarter point element, v = quarter point node displacement toward v direction.

RESEARCH METHODOLOGY

Finite Element Analysis

Two types of standardized specimen were analyzed in this present study. Compact tension and three-point bend specimen were used for the analysis of Mode I stress intensity factors (K_I) for obtaining the fracture toughness. The Illustration about the positions of quarter point nodes is depicted in Figure 7. The mesh configurations for Mode I stress intensity factors analysis are depicted in Figure 8 and 9. The compact specimen and beam specimen are modeled using plane strain in two dimensional finite element analysis. Figure 8a utilizes triangular quadrilateral elements around the crack tip. Figure 8b describes the usage of triangular quadrilateral elements around the crack tip, it has the same number of elements as Figure 8a, but with the smaller triangle size. The finer mesh is used in Figure 8c around the crack tip while the coarser one is in Figure 8d. Lastly, orthogonal mesh is used with the mid nodes moved to the quarter-point of the element in Figure 8e.

Figure 9a is a finite element mesh arrangement around the crack tip using orthogonal mesh whose mid nodes moved to the quarter point of the element. Figure 9b and Figure 9c are representing the use of triangular elements around the crack tip but with different sizes of quarter point elements. Figure 9d and 9e are explaining the use of triangular elements but have different number of elements around the crack tip.

In the finite element analyses that carried out in this study, the specimen was loaded by a unit load to get the displacements of the quarter point nodes around the crack tip, once these displacements obtained, they will be used for calculating stress intensity factors using either Quarter Point Displacement Technique or Displacement Correlation Technique. Fracture toughness then calculated by multiplying the value of stress intensity factors with the fracture loads (P_f), which obtained from the experiment.

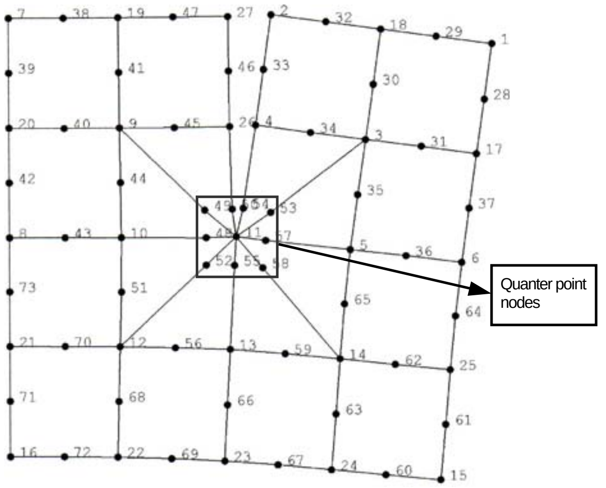

Figure 7. Illustration of Quarter Point Nodes in Finite Element Mesh

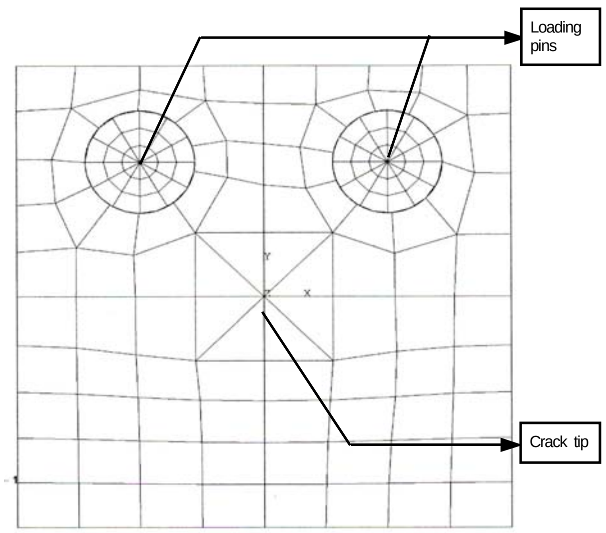

Figure 8. (a) QPE –T10

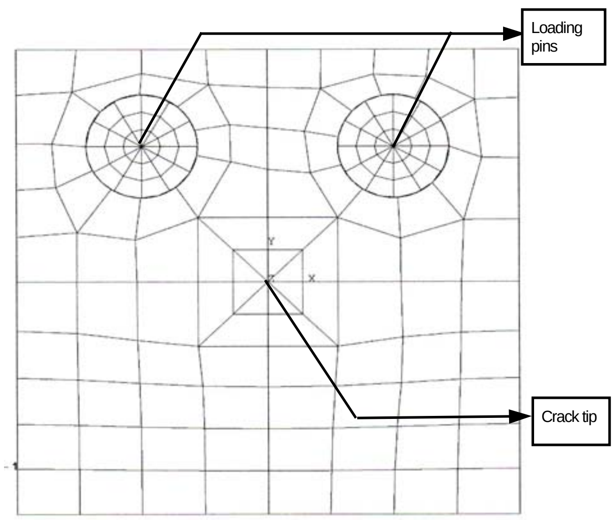

Figure 8.(b) QPE-T5

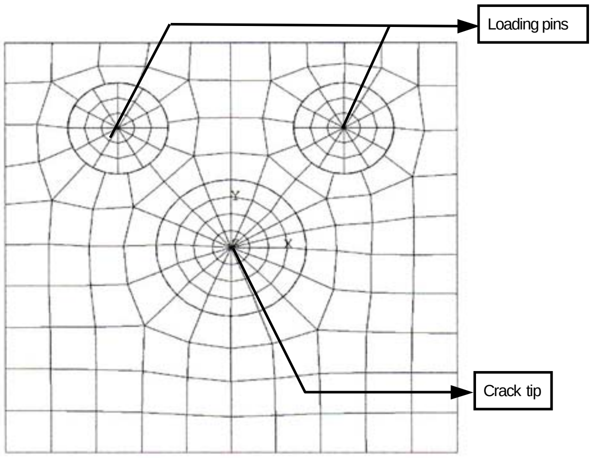

Figure 8.(c) QPE-T2

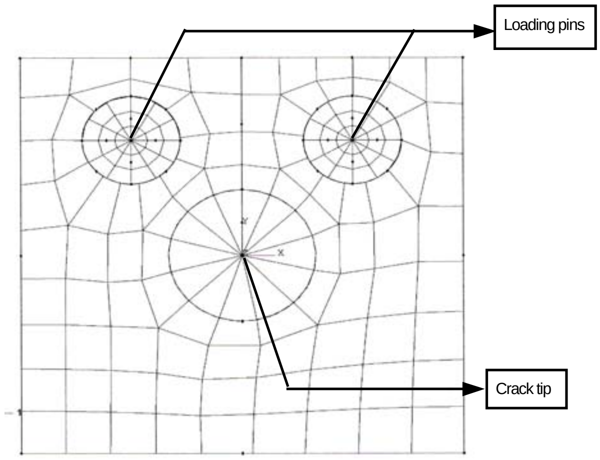

Figure 8. (d) QPE-T12

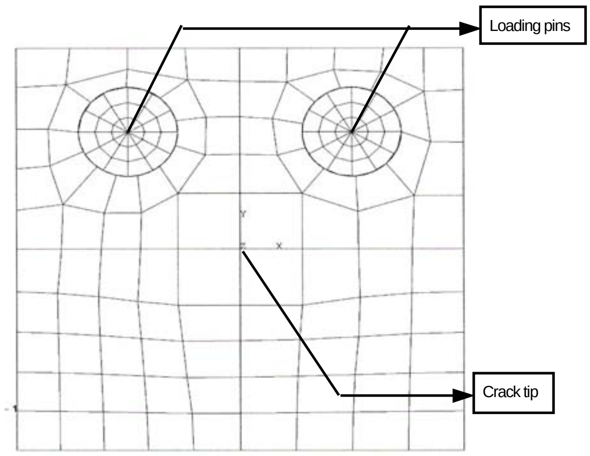

Figure 8. (e) QPE-Sq10

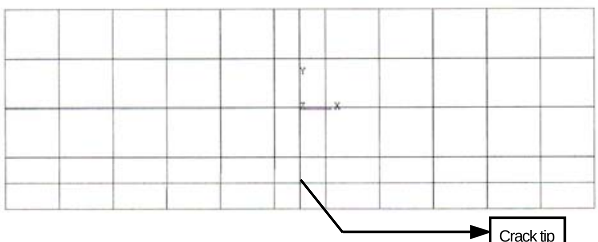

Figure 9. (a) QPE –Sq4.36B

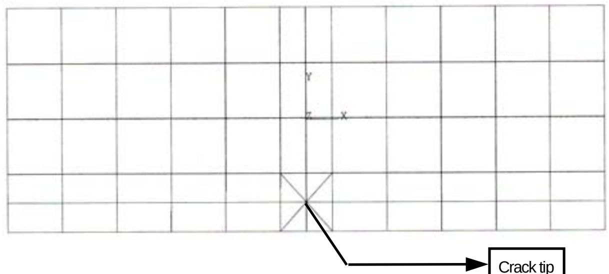

Figure 9. (b) QPE-T4.36B

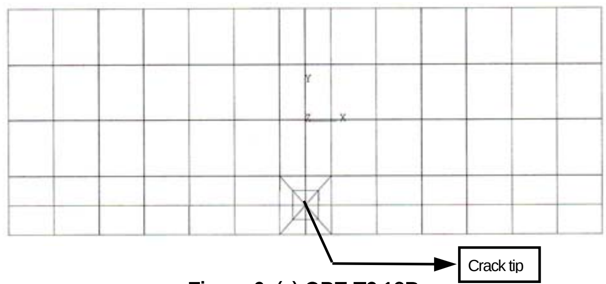

Figure 9. (c) QPE-T2.18B

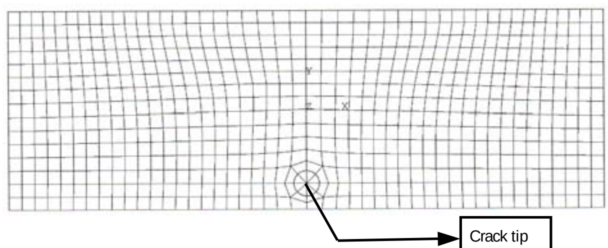

Figure 9. (d) QPE-T2.18BR

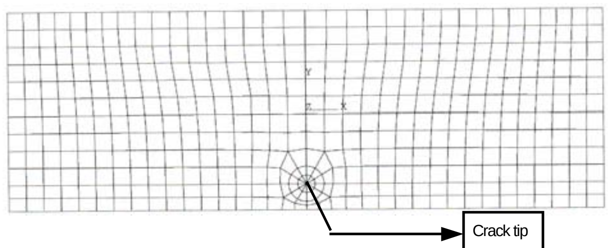

Figure 9. (e) QPE-T1.5B

Experimental Method

For compact tension specimen, the specimen used is cemented carbonate soils which made by mixing an amount of water with the mixture of carbonate sand and cement. The carbonate sand obtained from Coogee beach (Western Australia) and as cementing agent, ordinary Portland Cement Type II is used. The degree of cementation in this present study is 50%; the weight of cement is calculated based on the weight of carbonate soil, the mixture that is applied in this research is adopted from [9]. The amount of water that added to all specimens was 20% of total weight of soil and cement. All specimens are cured within 14 days before testing. The specimens types are, compact tension specimens (5 specimens) for fracture toughness determination (K_{IC}), cylindrical specimens (3 specimens) for Young's modulus and Poisson's ratio test (E and μ) and three point bend specimens, which adopted from [10].

Fracture testing for opening mode (Mode I) adopted from [11] (Compact Tension Specimen). Figure 10 shows the picture of test specimen which measures 72 x 72 x 36 mm with the pre-crack length of 36 mm from the top to the middle of the specimen. The apparatus that used for fracture testing consisted of two clevises holding two steel pins which slot in the specimen holes as depicted in Figure 11. The load was applied using Wykeham Farrance loading machine to cause tensile force to the crack tip. The load was measured by load cell and an LVDT was used for displacement measurement. All the data from fracture testing were recorded automatically using data logger.

For the purpose of fracture toughness determination, the beam specimen was formed to have dimension of 36.7 mm width, 33.5 mm height, 4.36 mm pre-crack length and 100 mm span width (L_o) (Figure 12).

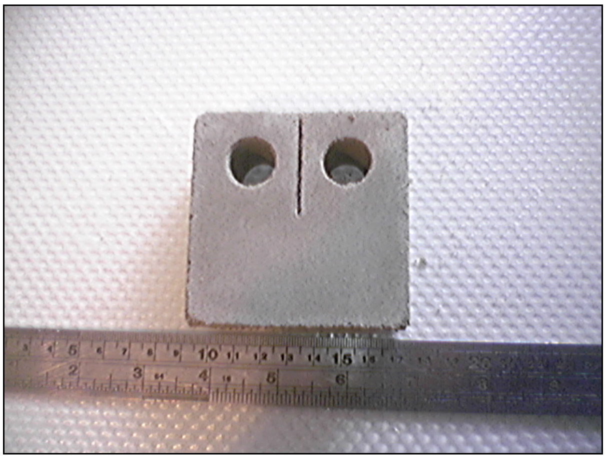

Figure 10. Actual Size of the Mode I Fracture Testing of CT-Specimen

Figure 11. Set up of Mode I Fracture Testing of CT-Specimen

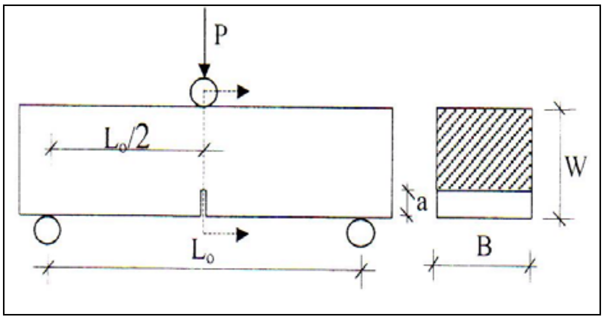

Figure 12. Geometry of Beam Specimen Tested [10]

RESULTS AND DISCUSSION

Mode I fracture of CT Specimen

Based on the mode I fracture testing over CT Specimen made of cemented carbonate soil, the fracture load (P_f) was 500 N for the specimen with 50% cement content. The elastic parameters were obtained from standard test for concrete cylinder, which resulted Young's Modulus (E) of 28 GPa and Poisson's ratio (μ) of 0.3.

Figure 13. Typical Failure Pattern of Mode I Fracture Testing (CT-Specimen)

The fracture toughness resulted from analytical formula (equation 2) is 12 MPa$\sqrt{mm}$, with the fracture load of P_f = 500 N. It can be seen from Table 1 that in general the results calculated using Quarter Point Displacement Technique are reasonably in good agreement compared to the result from analytical solution [7]. While for some particular mesh arrangements, the Displacement Correlation Technique results are quite far from [7]. The largest differences that occurred were for mesh QPE-T10, where Quarter Point Displacement Technique had 7% difference, while Displacement Correlation Technique had 27.91% difference, relative to the analytical solution. The best results for both Quarter Point Displacement Technique and

Displacement Correlation Technique were obtained by arranging the mesh as in QPE-T2, triangular quarter point element and relatively short quarter point element side (L = 2 mm), in this case, the mode I fracture toughness differences were 2.5% and 5% respectively for Quarter Point Displacement Technique and Displacement Correlation Technique, relative to the analytical solution. The most precise numerical analysis obtained was from rectangular quarter point element, which was calculated using Quarter Point Displacement Technique (0.41% difference) but it was not quite accurate for Displacement Correlation Technique (9.58% difference).

Mode I fracture three-point bend specimen

Mode I fracture testing over the beam specimen resulted the fracture toughness of 31.9 MPa$\sqrt{mm}$, calculated using analytical formula (eq. 4), where the fracture load is P_f = 2450 N. The beam specimen (three-point bend specimen) has E = 26.34 GPa and Poisson's ratio (μ) of 0.2.

In general, the fracture toughness calculations using stress intensity factor derived from finite element analysis and analytical formula are reasonably in good agreement except the result which revealed from finite element analysis which used the mesh arrangement as depicted in Figure 9 (d), the elements are relatively uniform in size, which caused difficulty to reach converged stress intensity factor calculation. While the use of fine elements around the crack tip as in Figure 9 (e), which converged at the crack tip, resulted the closest fracture toughness calculation to the analytical formula result, either for Displacement Correlation Technique or Quarter Point Displacement Technique. It seemed that the use of Displacement Correlation Technique and Quarter Point Displacement Technique for cracked-beam analysis were reasonably acceptable as long as combined with the appropriate mesh arrangement around the crack tip.

CONCLUSION AND RECOMMENDATION

Conclusion

It may be concluded from this present study that:

1. There is a tendency that Displacement Correlation Technique and Quarter Point Displacement Technique are acceptable for cracked-beam analysis as far as coupled with the appropriate mesh arrangement. While Quarter Point Displacement Technique was more suitable for compact tension specimen.
2. The finer elements around the crack tip are recommended (converged from large to small element) for the accuracy of stress intensity factor calculation.
3. The use of rectangular elements around the crack tip is not recommended due to inconsistent result that might be resulted either calculated by Displacement Correlation Technique or Quarter Point Displacement Technique.

Recommendation

In this present study, the analysis is limited to the standardized specimens, further study is needed to simulate the crack in the actual structural member like beam, column and plate.

Table 1. Comparison of Mode I Fracture Toughness Obtained From Finite Element vs Analytical Formula (CT-Specimen)

Mesh	QPE type	Number of QPE around crack tip	L (mm)	$K_{IC(DCT)}$ (MPa $\sqrt{mm}$)	% diff	$K_{IC(QPDT)}$ (MPa $\sqrt{mm}$)	% diff	$K_{IC} = K_{I(eq. 2)} \times P_f$ (MPa $\sqrt{mm}$) (Analytical Formula)
QPE –T10	Triangle	8	10	8.65	-27.91	11.15	-7.08	12
QPE-T2	Triangle	8	2	11.4	-5	11.7	-2.5	12
QPE-T5	Triangle	8	5	10.5	-12.5	11.65	-2.91	12
QPE-T12	Triangle	8	12	9.9	-17.5	11.75	-2.08	12
QPE-Sq10	Rectangular	4	10	10.85	-9.58	12.05	+0.41	12

Note: % diff = percentage of difference between finite element analysis and analytical formula results

Table 2. Comparison of Mode I Fracture toughness obtained from Finite Element vs Analytical Formula (Beam Specimen)

Mesh	QPE type	Number of QPE around crack tip	L (mm)	$K_{IC(DCT)}$ (MPa $\sqrt{mm}$)	% diff	$K_{IC(QPDT)}$ (MPa $\sqrt{mm}$)	% diff	$K_{IC} = K_{I\,(eq.4)} \times P_f$ (MPa $\sqrt{mm}$) (Analytical Formula)
QPE –T4.36B	Triangle	8	4.36	30.9	-3.13	31.1	-2.5	31.9
QPE-T2.18B	Triangle	8	2.18	31.1	-2.5	31.1	-2.5	31.9
QPE-T2.18BR	Triangle	8	2.18	23.1	-27.58	26.2	-17.86	31.9
QPE-T1.5B	Triangle	12	1.5	31.5	-1.25	31.3	-1.88	31.9
QPE-Sq4.36B	Rectangular	4	4.36	31.3	-1.88	31.11	+2.47	31.9

Acknowledgement

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