A REPLENISHMENT POLICY FOR ITEM WITH PRICE DEPENDENT DEMAND AND DETERIORATING UNDER MARKDOWN POLICY

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ABSTRACT

Many researches on deteriorating inventory model have been developed in recent years. This paper develops deteriorating inventory model with price-dependent demand and uses markdown policy to increase profit. Two examples are used to explain the model and some interesting results are derived. Markdown policy can increase total profit, but the best markdown time and price depend are case dependent.

Keywords:inventory, deteriorating item, markdown, price dependent demand.

1. INTRODUCTION

Some items like vegetables, milks, and fruits have deterioration characteristic. Deterioration is defined as decay, evaporation, obsolescence, loss of quality marginal value of a commodity that results in the decreasing usefulness from the original condition. The longer the items are kept in inventory, the higher the deteriorating cost. Retailer sometime uses markdown strategy to reduce their inventory and increase their profit by assuming that demand will increase with price decrease. In this study we developed a replenishment inventory model for deteriorating items and price-dependent demand. The model can be implemented for single markdown policy with different markdown time period.

Deteriorating inventory has been studied by several researches in recent decades. Goyal and Giri (2001) reviewed many literatures which studied deteriorating inventory since early 1900s. In their review, they studied some variations of deteriorating inventory with deterministic demand such as uniform demand, time-varying demand, stock-dependent demand and price-dependent demand. Kim et al (1995) presented joint price and lot size determination problems with deteriorating products using constant deterioration rate. Wee (1995) developed model for joint pricing and replenishment policy for deteriorating inventory with price elastic demand rate that decline over time. Wee (1997) studied inventory deteriorating model for price-dependent of items that have varying deterioration rate.

You (2005) investigated the problem of jointly determining optimal price and quantity order. He assumed decision maker could adjust price before the end of sales season to increase revenue. Decision variables at this research were quantity order, number of price changes and periodic prices. He assumed equal period price change, for the perishable item, and solved it as a single period problem. At his researched, he just considered price-dependent demand but didn't consider stock dependent demand. The analytical result shows that for any given parameters, the optimal sales prices and order quantities can be found. Padmanabhan and Vret (1998) developed EOQ model for perishable item under stock dependent selling rate. They developed three models with no shortage, permitting shortages and backordering assumption respectively.

Modeling inventory model with markdown price and stock dependent demand is getting the attention of researchers recently. You & Shieh(2007) developed an EOQ model with stock and price sensitive demand. Their objective is to maximize profit by simultaneously determining the order quantity and selling price. They did not allow item deterioration and assumed equal period price changes.

The contribution of this paper is developing replenishment inventory deteriorating model policy when supplier used markdown policy. This paper is presented as follow. The first section presents the paper contribution and reviews the relevant literature. Section two presents calculations and details regarding the deteriorating inventory model. Two examples and experiments are presented in the third section. The fourth section offers conclusions and suggests directions for future research.

2. MODEL DEVELOPMENT

Assumptions:

1. Demand increase as price is reduced. The demand has constant elasticity. The demand at time t is assumed to be $\beta(\alpha p)^{-\varepsilon}$, where β and α is positive constants.
2. A single rate item with a constant rate of deterioration is considered.
3. Shortage is not allowed
4. The on hand stock deteriorates at constant rate
5. Instantaneous replenishment with continuous review order quantity
6. One time markdown price at one planning period is considered
7. Markdown price and time are known.

Notation:

I_t	=	inventory at period t	T^*	=	Optimal replenishment time
θ	=	deterioration rate	TP^*	=	Optimal total profit
α	=	markdown rate	p	=	Initial price
ε	=	increase price rate	r	=	markdown percentage
β	=	constant stock dependent parameter	D	=	demand
Q	=	Ordering quantity	c	=	buying price
Q^*	=	Optimal ordering quantity	RC	=	ordering cost
T_1	=	Markdown offering time	HC	=	holding cost
T	=	Replenishment time	UC	=	cost of purchasing unit

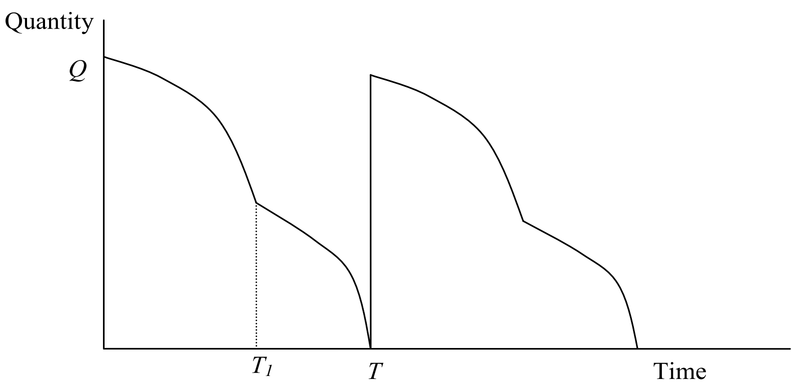

Figure 1. Inventory Level

The inventory level is illustrated in Figure 1 and it can be represented by the following differential equation:

$$\frac{dI}{dt} + \theta I_t = -\beta(\alpha p)^{-\varepsilon}, \quad 0 \le t \le T \tag{1}$$

For $0 \le t \le T_1$, there is no breakdown price, so α =1 and

$$\frac{dI}{dt} + \theta I_t = -\beta p^{-\varepsilon}, \quad 0 \le t \le T_1 \tag{2}$$

Since $I_{(0)}=Q$ when t=0, then

$$I_t = -\frac{\beta p^{-\varepsilon}}{\theta} + (Q + \frac{\beta p^{-\varepsilon}}{\theta})e^{-\theta t}$$

$$I_t = Qe^{-\theta t} + \frac{\beta p^{-\varepsilon}}{\theta}(e^{-\theta t} - 1) \quad \text{for } 0 \le t \le T_1 \tag{3}$$

From equation (3) we know at $t=T_1$, the quantity of inventory is equal to:

$$I_t = Qe^{-\theta T_1} + \frac{\beta p^{-\varepsilon}}{\theta}(e^{-\theta T_1} - 1) \tag{4}$$

And for $T_1 \le t \le T$, the quantity of inventory can be shown as:

$$I_t = (Qe^{-\theta T_1} + \frac{\beta p^{-\varepsilon}}{\theta}(e^{-\theta T_1} - 1))e^{-\theta(t-T_1)} + \frac{\beta(\alpha p)^{-\varepsilon}}{\theta}(e^{-\theta(t-T_1)} - 1) \tag{5}$$

Since $I(t)$ = 0 when $t=T$, one has:

$$(Qe^{-\theta T_1} + \frac{\beta p^{-\varepsilon}}{\theta}(e^{-\theta T_1} - 1))e^{-\theta(T-T_1)} + \frac{\beta(\alpha p)^{-\varepsilon}}{\theta}(e^{-\theta(T-T_1)} - 1) = 0$$

It can be simplified as:

$$Q = -\frac{\beta p^{-\varepsilon}}{\theta}(1 - e^{\theta T_1}) - \frac{\beta(\alpha p)^{-\varepsilon}}{\theta}(e^{\theta T_1} - e^{\theta T}) \tag{6}$$

Before calculate the total cost, we have to define the inventory rate for. $0 \le t \le T$. There are two different inventory rates for $0 \le t \le T_1$ and $T_1 \le t \le T$.

Inventory for $0 \le t \le T_1$ is equal to:

$$\int_0^{T_1} I(t)dt = \int_0^{T_1} Qe^{-\theta t} + \frac{\beta p^{-\varepsilon}}{\theta}(e^{-\theta t} - 1)\,dt$$

$$= \frac{Q(e^{-\theta T_1} - 1)}{\theta} - \frac{\beta(e^{-\theta T_1} + \theta T_1 - 1)p^{-\varepsilon}}{\theta^2} \tag{7}$$

Substitute Q from equation (6), one has:

$$I_t = \frac{-(\beta(p^{-\varepsilon}e^{-\theta T_1} - p^{-\varepsilon} + (\alpha p)^{-\varepsilon}e^{(-T+T_1)\theta} - (\alpha p)^{-\varepsilon})e^{\theta T_1})(e^{-\theta T_1} - 1) - \beta(e^{-\theta T_1} + \theta T_1 - 1)p^{-\varepsilon}}{\theta^2} \tag{8}$$

Inventory for $T_1 \le t \le T$ is:

$$\int_{T_1}^{T} I(t)dt = \int_{T_1}^{T} Qe^{-\theta T_1} + \frac{\beta p^{-\varepsilon}}{\theta}(e^{-\theta T_1} - 1) + \frac{\beta(\alpha p)^{-\varepsilon} a}{\theta}(e^{-\theta(t-T_1)} - 1)\,dt$$

Substitute Q from (6) and simplify the equation, one has:

$$I_t = \left(\frac{\beta(\alpha p)^{-\varepsilon} e^{-T\theta}(e^{T\theta}(\theta T_1 - \theta T - 1) + e^{-\theta(T_1 - 2T)}}{\theta^2}\right) \tag{9}$$

The total revenue consists of the revenue before the markdown price is implemented and the revenue when the markdown is realized. The revenue equation can be obtained as:

$$TR_1 = \beta p^{1-\varepsilon}, \quad 0 \le t \le T_1 \tag{10}$$

where TR_1 is the total revenue for $0 \le t \le T_1$, and

$$TR_2 = \beta(\alpha p)^{1-\varepsilon}, \quad T_1 \le t \le T \tag{11}$$

Where TR_2 is the total revenue for $T_1 \le t \le T$,

Adding equation (10) and (11), and divide by the total time, the revenue rate is:

$$TR = \frac{\beta(p^{1-\varepsilon})T_1 + \beta(\alpha p)^{1-\varepsilon}(T - T_1)}{T} \tag{12}$$

The total relevant cost per unit time consists of:

a. Cost of placing orders = $\dfrac{RC}{T}$ (13)

b. Cost of purchasing units = $\dfrac{cQ}{T}$

Substitute Q from equation (6) to equation (14), the purchasing rate is:

$$UC = \frac{c}{T}\left(-\frac{\beta p^{-\varepsilon}}{\theta}(1 - e^{\theta T_1}) - \frac{\beta(\alpha p)^{-\varepsilon}}{\theta}(e^{\theta T_1} - e^{\theta T})\right) \tag{14}$$

c. Cost of carrying inventory = $\dfrac{I_t HC}{T} = \dfrac{HC\left(\int_0^{T_1} I(t)dt + \int_{T_1}^{T} I(t)dt\right)}{T}$ (15)

Total profit is equal to total revenue less total cost. Subtracting equation (13)-(15) to equation (12), the total profit is:

$$\begin{aligned} TP = {} & \frac{\beta(p^{1-\varepsilon})T_1 + \beta(\alpha p)^{1-\varepsilon}(T - T_1)}{T} - \frac{RC}{T} \\ & - c\left(-\frac{\beta p^{-\varepsilon}(e^{-\theta T_1} - 1 + \alpha e^{(-T+T_1)\theta} - \alpha)e^{\theta T_1}}{\theta T}\right) \\ & - \frac{HC}{T}\left(\left(\frac{-(\beta(p^{-\varepsilon}e^{-\theta T_1} - p^{-\varepsilon} + (\alpha p)^{-\varepsilon}e^{(-T+T_1)\theta} - (\alpha p)^{-\varepsilon})e^{\theta T_1})(e^{-\theta T_1} - 1) - \beta(e^{-\theta T_1} + \theta T_1 - 1)p^{-\varepsilon}}{\theta^2}\right)\right. \\ & \left. + \left(\frac{\beta(\alpha p)^{-\varepsilon} e^{-T\theta}(e^{T\theta}(\theta T_1 - \theta T - 1) + e^{-\theta(T_1 - 2T)}}{\theta^2}\right)\right) \end{aligned} \tag{16}$$

The objective of this case is to maximize the net profit. It is done by differentiating *TP* with respect to *T*. Differentiating *TP* with respect to *T* and substitute T_1 with $r*T$, one has equation in appendix A. For the function to be maximized, the second derivative must be less than zero. The analysis showed that the equation is failed to identify the convexity, with all parameters is positive and parameters r, α, θ are between 0 and 1 (see Appendix 2). Since the second derivation is failed to identify the convexity, so we have to check the convexity for each case in feasible solution area. The calculation steps are explained in Section 3.

3. NUMERICAL EXAMPLE

The preceding theory can be illustrated using the numerical example. We use two examples with varying markdown time and price. The parameters are as follows:

Example 1: Suppose $RC = 1000$, $c = 10$, $p = 30$, $HC = 0.05$, $\beta = 100000$, $\theta = 0.3$, $\varepsilon = 1.8$. Value of r is varying from 0.5 to 0.9 and α is varying from 0.7 to 0.9. We solve the problem using Maple 8. The first step to solve the problem is to check the convexity and the feasibility of the solution. The second derivative is used to check this convexity. For this problem with $r = 0.5$ and $\alpha = 0.7$, we check the second derivative equation in the feasible area between T=0.5 to T=20. The plot can be shown in Figure 2. Figure 2 shows that all second derivative values are negative at T, so the maximum solution is verified for this case at T=0.5 to T=20.

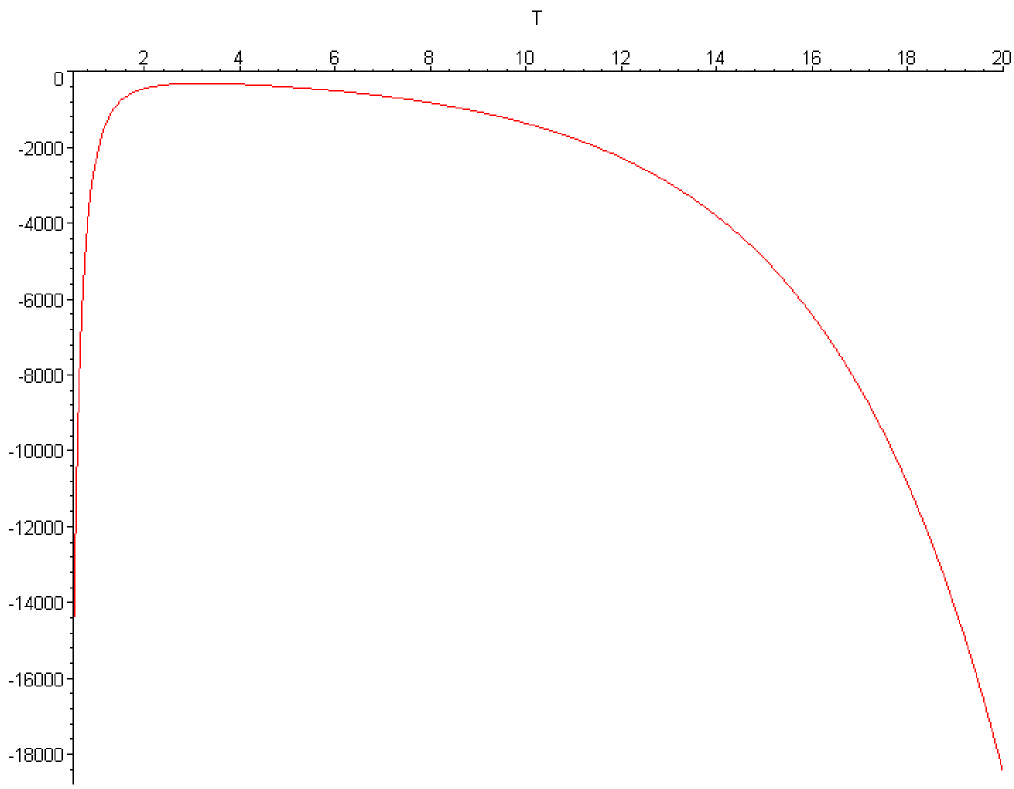

Figure 2. Second derivative function

Find the value of T^* using the first derivative equation, one has $T^* = 1.18$. This T value is in the range of maximum area. The value of Q^* can be calculated using equation (6) and one has Q^* = 461.1 units. Substitute the value of T^* to equation (16), and one obtained the optimal profit is 2886.3.

Table 1. Experimental result for Example 1

α	$r = 0.5$			$r = 0.7$			$r = 0.9$		
	T^*	Q^*	TP^*	T^*	Q^*	TP^*	T^*	Q^*	TP^*
0.7	1.18	461.1	2886.3	1.25	433.4	2944.6	1.38	414.3	3053.3
0.8	1.29	438.5	3033.3	1.34	422.6	3047.6	1.42	412.4	3098.1
0.9	1.39	422.4	3105.3	1.42	415.5	3108.0	1.46	411.4	3122.3

Example 2: Suppose RC = 1000, c = 10, p = 30, HC = 0.05, β = 100000, θ = 0.05, ε = 1.8. In the second example, we decrease the deteriorating rate parameter (θ) to 0.05 instead of 0.3 in Example 1. Using the same steps as Example 1, the result of this example can be seen in Table 2.

Table 2. Experimental result for Example 2

α	$r = 0.5$			$r = 0.7$			$r = 0.9$		
	T^*	Q^*	TP^*	T^*	Q^*	TP^*	T^*	Q^*	TP^*
0.7	2.99	1035.0	3833.2	3.18	971.9	3835.6	3.53	930.1	3858.3
0.8	3.28	984.1	3896.1	3.42	948.1	3881.0	3.65	926.3	3876.9
0.9	3.56	948.4	3902.7	3.64	932.9	3890.1	3.75	924.2	3882.0

Table 1 and 2 show that if the value of markdown price is different at a given markdown time, the variance of the replenishment time is small but give totally different profits. The variance of total profit is bigger when markdown is applied earlier.

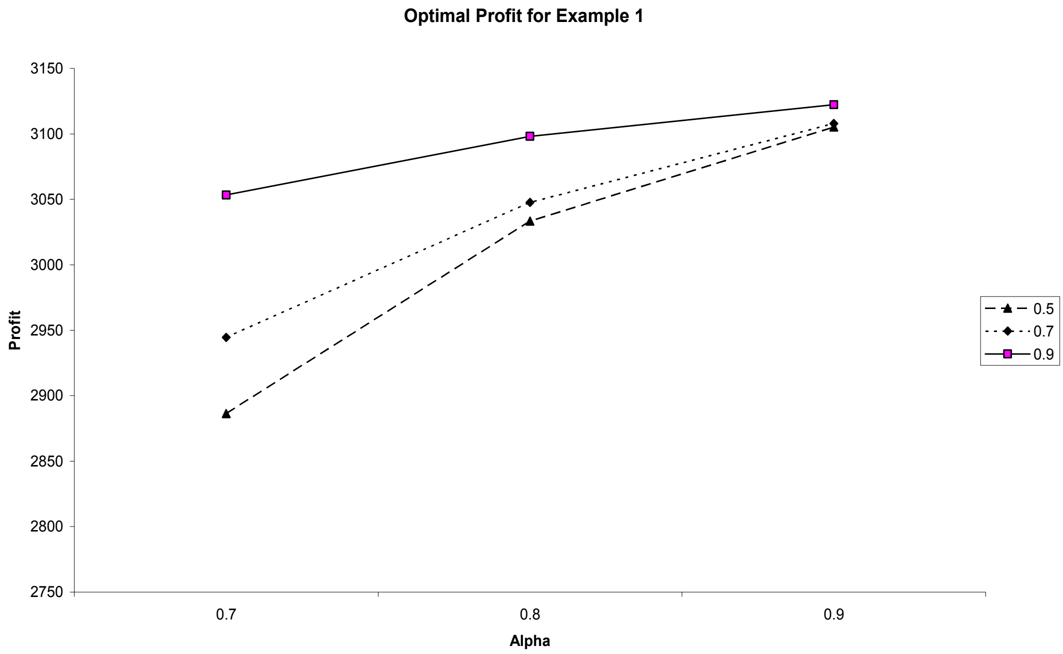

Figure 3. Optimal Profit for Example 1

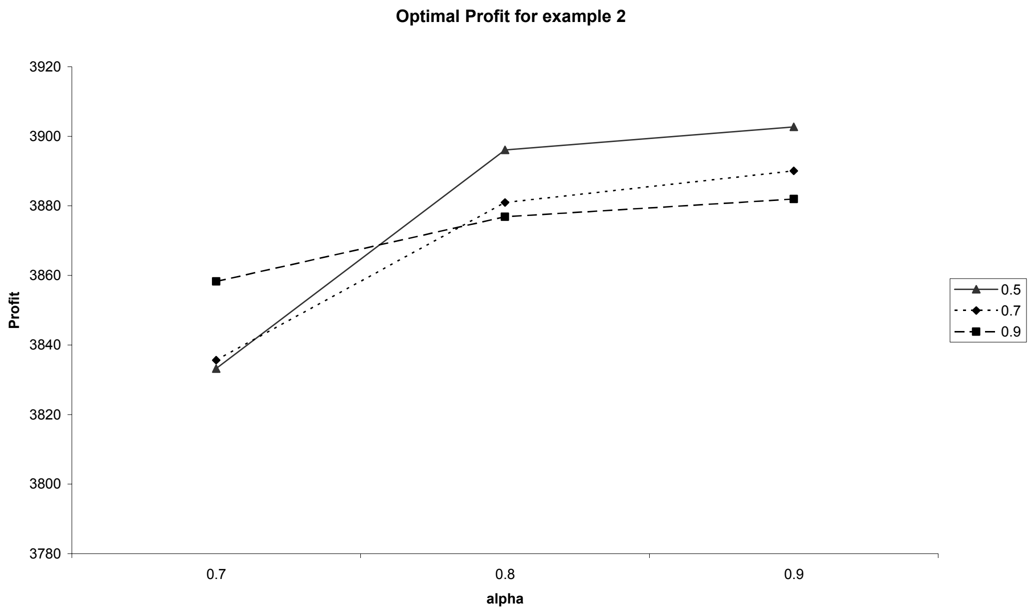

Figure 4. Optimal Profit for Example 2

The effect of markdown price and time to optimize total profit are case dependent. In Example 1, when the price is reduced to 90% of the initial price, it dominants others markdown price at any markdown time. This condition can be seen in Figure 3. This circumstance does not happen in Example 2. We can see from Figure 4, that when markdown price strategy start at alpha equal to 0.8 and 0.9, $r = 0.5$ give the best profit but, when alpha equal to 0.7, then $r = 0.9$ give the best profit. These two examples also show when deteriorating rate is bigger, bigger markdown price and markdown time tends to derive optimal profit.

4. CONCLUSION

In this study, a deteriorating inventory model with price dependent demand under markdown policy has been developed. The optimal replenishment time and optimal ordering time were derived and two examples were shown to illustrate the model. The result demonstrates that markdown time and price give significant contribution to optimize the total profit and a policy maker must be careful to set markdown time and price because optimum policy is different for different case.

In this paper, we have not investigated the effect of another parameters of different deteriorating rate to find replenishment time and optimal profit. Different types of deteriorating function can be considered as future research. The model can also be extended to consider stochastic demand dependent rate instead of deterministic rate as in this paper.

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Appendix A : The derivation of *equation* (16)

$$
\begin{aligned}
Topt :=\ & \frac{RC}{T^2} + \frac{c\,\beta\,(-p^{(-\varepsilon)} + p^{(-\varepsilon)}\,e^{(r\,T\,\theta)} - (\alpha\,p)^{(-\varepsilon)}\,e^{(r\,T\,\theta)} + (\alpha\,p)^{(-\varepsilon)}\,e^{(T\,\theta)})}{T^2\,\theta} \\
& - \frac{c\,\beta\,(p^{(-\varepsilon)}\,r\,\theta\,e^{(r\,T\,\theta)} - (\alpha\,p)^{(-\varepsilon)}\,r\,\theta\,e^{(r\,T\,\theta)} + (\alpha\,p)^{(-\varepsilon)}\,\theta\,e^{(T\,\theta)})}{T\,\theta} + hc\Bigg(\\
& - \frac{\beta\,(-p^{(-\varepsilon)} + p^{(-\varepsilon)}\,e^{(r\,T\,\theta)} - (\alpha\,p)^{(-\varepsilon)}\,e^{(r\,T\,\theta)} + (\alpha\,p)^{(-\varepsilon)}\,e^{(T\,\theta)})\,(e^{(-r\,T\,\theta)} - 1)}{\theta^2} \\
& - \frac{\beta\,(e^{(-r\,T\,\theta)} + r\,T\,\theta - 1)\,p^{(-\varepsilon)}}{\theta^2}\Bigg) \Big/ T^2 - hc\Bigg(\\
& - \frac{\beta\,(p^{(-\varepsilon)}\,r\,\theta\,e^{(r\,T\,\theta)} - (\alpha\,p)^{(-\varepsilon)}\,r\,\theta\,e^{(r\,T\,\theta)} + (\alpha\,p)^{(-\varepsilon)}\,\theta\,e^{(T\,\theta)})\,(e^{(-r\,T\,\theta)} - 1)}{\theta^2} \\
& + \frac{\beta\,(-p^{(-\varepsilon)} + p^{(-\varepsilon)}\,e^{(r\,T\,\theta)} - (\alpha\,p)^{(-\varepsilon)}\,e^{(r\,T\,\theta)} + (\alpha\,p)^{(-\varepsilon)}\,e^{(T\,\theta)})\,r\,e^{(-r\,T\,\theta)}}{\theta} \\
& - \frac{\beta\,(-r\,\theta\,e^{(-r\,T\,\theta)} + r\,\theta)\,p^{(-\varepsilon)}}{\theta^2}\Bigg) / T \\
& + \frac{hc\,\beta\,e^{(-T\,\theta)}\,p^{(-\varepsilon)}\,\alpha^{(-\varepsilon)}\,(-e^{(T\,\theta)} + e^{(-\theta\,(r\,T - 2\,T))} - T\,\theta\,e^{(T\,\theta)} + \theta\,r\,T\,e^{(T\,\theta)})}{T^2\,\theta^2}
\end{aligned}
$$

$$+\frac{hc\,\beta\,e^{(-T\theta)}\,p^{(-\varepsilon)}\,\alpha^{(-\varepsilon)}\,(-e^{(T\theta)}+e^{(-\theta(rT-2T))}-T\theta\,e^{(T\theta)}+\theta\,r\,T\,e^{(T\theta)})}{T\theta}-hc\,\beta\,e^{(-T\theta)}\,p^{(-\varepsilon)}\,\alpha^{(-\varepsilon)}(-2\theta\,e^{(T\theta)}-\theta(r-2)\,e^{(-\theta(rT-2T))}-T\theta^2\,e^{(T\theta)}+\theta\,r\,e^{(T\theta)}+\theta^2\,r\,T\,e^{(T\theta)})\Big/(T\theta^2)$$

Appendix B: The proof of the total profit function's convexity

Hessian matrix of equation (16) is

$$H:=\Bigg[-\frac{2\,RC}{T^3}-\frac{2\,c\,\beta\,(-p^{(-\varepsilon)}+p^{(-\varepsilon)}\,e^{(rT\theta)}-(\alpha p)^{(-\varepsilon)}\,e^{(rT\theta)}+(\alpha p)^{(-\varepsilon)}\,e^{(T\theta)})}{T^3\,\theta}$$
$$+\frac{2\,c\,\beta\,(p^{(-\varepsilon)}\,r\,\theta\,e^{(rT\theta)}-(\alpha p)^{(-\varepsilon)}\,r\,\theta\,e^{(rT\theta)}+(\alpha p)^{(-\varepsilon)}\,\theta\,e^{(T\theta)})}{T^2\,\theta}$$
$$-\frac{c\,\beta\,(p^{(-\varepsilon)}\,r^2\,\theta^2\,e^{(rT\theta)}-(\alpha p)^{(-\varepsilon)}\,r^2\,\theta^2\,e^{(rT\theta)}+(\alpha p)^{(-\varepsilon)}\,\theta^2\,e^{(T\theta)})}{T\theta}+2\,hc\,\beta\,($$
$$p^{(-\varepsilon)}\,e^{(-rT)}-2\,p^{(-\varepsilon)}-p^{(-\varepsilon)}\,e^{(rT\theta-rT)}+p^{(-\varepsilon)}\,e^{(rT\theta)}+(\alpha p)^{(-\varepsilon)}\,e^{(rT\theta-rT)}$$
$$-(\alpha p)^{(-\varepsilon)}\,e^{(rT\theta)}-(\alpha p)^{(-\varepsilon)}\,e^{(T\theta-rT)}+(\alpha p)^{(-\varepsilon)}\,e^{(T\theta)}+p^{(-\varepsilon)}\,e^{(-rT\theta)}+p^{(-\varepsilon)}\,\theta\,r\,T)$$
$$\Big/(T^3\,\theta^2)-2\,hc\,\beta\,(-p^{(-\varepsilon)}\,r\,e^{(-rT)}-p^{(-\varepsilon)}\,(r\theta-r)\,e^{(rT\theta-rT)}+p^{(-\varepsilon)}\,r\,\theta\,e^{(rT\theta)}$$
$$+(\alpha p)^{(-\varepsilon)}\,(r\theta-r)\,e^{(rT\theta-rT)}-(\alpha p)^{(-\varepsilon)}\,r\,\theta\,e^{(rT\theta)}-(\alpha p)^{(-\varepsilon)}\,(\theta-r)\,e^{(T\theta-rT)}$$
$$+(\alpha p)^{(-\varepsilon)}\,\theta\,e^{(T\theta)}-p^{(-\varepsilon)}\,r\,\theta\,e^{(-rT\theta)}+p^{(-\varepsilon)}\,\theta\,r)\Big/(T^2\,\theta^2)+hc\,\beta\,(p^{(-\varepsilon)}\,r^2\,e^{(-rT)}$$
$$-p^{(-\varepsilon)}\,(r\theta-r)^2\,e^{(rT\theta-rT)}+p^{(-\varepsilon)}\,r^2\,\theta^2\,e^{(rT\theta)}+(\alpha p)^{(-\varepsilon)}\,(r\theta-r)^2\,e^{(rT\theta-rT)}$$
$$-(\alpha p)^{(-\varepsilon)}\,r^2\,\theta^2\,e^{(rT\theta)}-(\alpha p)^{(-\varepsilon)}\,(\theta-r)^2\,e^{(T\theta-rT)}+(\alpha p)^{(-\varepsilon)}\,\theta^2\,e^{(T\theta)}$$
$$+p^{(-\varepsilon)}\,r^2\,\theta^2\,e^{(-rT\theta)})\Big/(T\theta^2)$$
$$-\frac{2\,hc\,\beta\,e^{(-T\theta)}\,p^{(-\varepsilon)}\,\alpha^{(-\varepsilon)}\,(-e^{(T\theta)}+e^{(-\theta(rT-2T))}-T\theta\,e^{(T\theta)}+\theta\,r\,T\,e^{(T\theta)})}{T^3\,\theta^2}$$
$$-\frac{2\,hc\,\beta\,e^{(-T\theta)}\,p^{(-\varepsilon)}\,\alpha^{(-\varepsilon)}\,(-e^{(T\theta)}+e^{(-\theta(rT-2T))}-T\theta\,e^{(T\theta)}+\theta\,r\,T\,e^{(T\theta)})}{T^2\,\theta}+2\,hc\,\beta$$
$$e^{(-T\theta)}\,p^{(-\varepsilon)}\,\alpha^{(-\varepsilon)}$$
$$(-2\theta\,e^{(T\theta)}-\theta(r-2)\,e^{(-\theta(rT-2T))}-T\theta^2\,e^{(T\theta)}+\theta\,r\,e^{(T\theta)}+\theta^2\,r\,T\,e^{(T\theta)})\Big/(T^2\,\theta^2)$$

$$- \frac{hc\, \beta\, e^{(-T\theta)}\, p^{(-\varepsilon)}\, \alpha^{(-\varepsilon)} (-e^{(T\theta)} + e^{(-\theta(rT-2T))} - T\theta\, e^{(T\theta)} + \theta\, r\, T\, e^{(T\theta)})}{T} + 2\, hc\, \beta$$

$$e^{(-T\theta)}\, p^{(-\varepsilon)}\, \alpha^{(-\varepsilon)}$$

$$(-2\,\theta\, e^{(T\theta)} - \theta\,(r-2)\, e^{(-\theta(rT-2T))} - T\,\theta^2\, e^{(T\theta)} + \theta\, r\, e^{(T\theta)} + \theta^2\, r\, T\, e^{(T\theta)})/(T\,\theta) -$$

$$hc\, \beta\, e^{(-T\theta)}\, p^{(-\varepsilon)}\, \alpha^{(-\varepsilon)}$$

$$(-3\,\theta^2\, e^{(T\theta)} + \theta^2\,(r-2)^2\, e^{(-\theta(rT-2T))} - T\,\theta^3\, e^{(T\theta)} + 2\,\theta^2\, r\, e^{(T\theta)} + \theta^3\, r\, T\, e^{(T\theta)}) \Big/ ($$

$$T\,\theta^2) \Big]$$

with assumptions on RC, T, c, β, p, ε, r, θ, α and hc

The equation was calculated using maple 8. The Hessian fail to proof negative eigenvalues with assumption on RC, T, c, p, ε and hc are positive and r, θ, α are between 0 and 1.