NON – BAYESIAN AND BAYESIAN RELIABILITY ESTIMATION FOR GAMMA STRESS – WEIBULL STRENGTH MODELS

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Abstract : *paper deals with the estimation of reliability function (RF) of Gamma strength- Weibullstress a models with different shape and scale parameters. The maximum likelihood, moments, and three bayes estimators using three different prior distributions of RF, are obtained for different and known shape parameters and different and unknown scale parameters. A simulation based on different sample sizes anddifferent parameter values, is used to study the performance of the reliability estimators.*

INTRODUTION

The estimation of R is very common in the statistical literature, for example, if X is the strength of a system which is subjected to a stress Y, then the reliability (R), is probability of(Y<X), is a measure of system performance and arises in the context of mechanical reliability of a system This system fails if and only if at any time the applied stress is greater than its strength This particular problem was considered by many others ,(see [1], [2] and [3]) .

In the statistical approach to the stress–strength model, most of the considerations depend on the assumption that stress and strength variables are independently but not identically distributed using the same distribution for the two variables with different parameters.

The main aim of this article is to drive a theoriticale expression of reliability function(RF) for stress–strength model when the stress variable y is following weibull distribution with parameters(α, β)and the strength variable X is following gamma distribution with parameters(λ, μ)

As well known from literature that the two distributio ns weibull and gamma are special cases from generalized gamma distribution, with probability density function (pdf) given as:[11]

$$f(x;\beta,\theta,\kappa)=\frac{\beta}{\Gamma(\kappa)\theta^{\kappa}}x^{\kappa\beta-1}\exp\left(-\frac{x^{\beta}}{\theta}\right)$$

;x> 0 $\beta,\theta > 0$ k> 1 .... (1)

When compensation ($\beta = 1$) in the formula (1) it turn to the following formula:

$$f(x;\theta,\kappa)=\frac{1}{\Gamma(\kappa)\theta^{\kappa}}x^{\kappa-1}\exp\left(-\frac{x}{\theta}\right)$$

;x> 0 $\theta > 0$ k > 1 .... (2)

Which represents the pdf of the two parameters gamma distribution. When compensation (k = 1) in the formula (1) it turns to the following formula:

$$f(x;\beta,\theta)=\frac{\beta}{\theta}x^{\beta-1}\exp\left(-\frac{x^{\beta}}{\theta}\right)$$

;x> 0 $\beta > 0$ k> 1 .... (3)

Which represents the pdf of the two parameters weibull distribution.

In this artical we estimate (RF) of gamma strength-weibull stress model ,say R_{Gw} .the maximum likelihood estimator(MLE), moments

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estimator(MOE) and bayes estimators (BE) with three different prior distributions for (RF) assuming that the shape parameters for the two different distributions are known and the scale parameters are unknown and compare between the performance the Non-Bayesian estimators and between the Bayesian estimators using the mean squared error(MSE) values for different sample sizes and different values of the known parameters by simulation.

The Probabilistic Model of Gamma strength-Weibull stress RF(R_{Gw}).

The reliability assessment problem is dealing within the framework of probabilistic stress- strength models [5, 6], i.e. by writing the Reliability function (RF), for a given mission time t, as:

$R=p(Y < X)$ .... (4)

The mathematical model for evaluation of the above, RF, is the object of the proposed estimation procedure, together with other reliability parameters which are of great interest in assessing the performances of the device and possible maintenance strategies.

Let X the strength random variable following Gamma(α, β)with PDF :

$$f(x;\alpha,\beta)=\frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha-1}e^{-\left(\frac{x}{\beta}\right)};x>0,\ \alpha,\beta,>0 \quad ....(5)$$

independent from the stress random variable Y following Weibull (λ, μ),with CDF :

$$F_y(y)=1-e^{-\left(\frac{y^{\lambda}}{\mu}\right)};y>0,\ ,>0 \quad ....(6)$$

Assuming that the shape parameters are known, and the scale parameters are unknown then the probabilistic Expression of Reliability function, R_{Gw} ,can be found as:

$$R_{Gw}=p(Y < X)$$

$$R_{Gw}=\iint_{Y<X} f(x,y)\,dx\,dy$$

assuming the independency between , then by equations (5)and(6), we get:

$$R_{Gw}=\int_{x=0}^{\infty} f_x(x)\,F_y(x)dx$$

$$R_{Gw}=\int_{x=0}^{\infty}\frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha-1}e^{-\left(\frac{x}{\beta}\right)}\left(1-e^{-\left(\frac{x^{\lambda}}{\mu}\right)}\right)dx$$

$$R_{Gw}=1-\frac{1}{\Gamma(\alpha)\beta^{\alpha}}\int_{x=0}^{\infty}x^{\alpha-1}e^{\left(-\frac{x}{\beta}-\frac{x^{\lambda}}{\mu}\right)}dx$$

Using the transformation of $t=\frac{x^{\lambda}}{\mu}$, then :

$$R_{GW}=1-\frac{1}{\Gamma(\alpha)\beta^{\alpha}}\frac{\mu^{\frac{\alpha}{\lambda}}}{\lambda}\int_{0}^{\infty}t^{\frac{\alpha}{\lambda}-1}\exp\left(-\frac{\mu^{\frac{\alpha}{\lambda}}}{\beta}t^{\frac{1}{\lambda}}-t\right)dt \quad ....(7)$$

Now by the Ith degree maclaurin polynomials with degree (I=15) we get:[10]

$$R_{GW}=1-\frac{\mu^{\frac{\alpha}{\lambda}}}{\Gamma(\alpha)\beta^{\alpha}\lambda}\int_{t=0}^{\infty}t^{\frac{\alpha}{\lambda}-1}e^{-t}\sum_{i=0}^{15}\frac{(-1)^i}{i!}\left(\frac{\mu^{\frac{\alpha}{\lambda}}}{\beta}t^{\frac{1}{\lambda}}\right)^i dt$$

$$R_{Gw}=1-\frac{\mu^{\frac{\alpha}{\lambda}}}{\lambda\,\Gamma(\alpha)\beta^{\alpha}}\sum_{i=0}^{15}\frac{(-1)^i\mu^{\frac{i}{\lambda}}}{i!\,\beta^i}\int_{t=0}^{\infty}t^{\frac{\alpha}{\lambda}+\frac{i}{\lambda}-1}e^{-t}dt\ ..8$$

Finally we get R_{GW} as:

$$R_{GW}=1-\frac{1}{\lambda\,\Gamma(\alpha)}\left(\frac{\mu^{\frac{1}{\lambda}}}{\beta}\right)^{\alpha}\sum_{i=0}^{15}\frac{(-1)^{i}}{i!}\left(\frac{\mu^{\frac{1}{\lambda}}}{\beta}\right)^{i}\Gamma\left(\frac{\alpha+i}{\lambda}\right)\text{.........(9)}$$

RELIABILITY ESTIMATION

In this sub section ,Tow non-Bayesian methods of estimation will be used which are Maximum likelihood method and Moments method to estimate the reliability function, also used the Bayesian estimation method with three different priors to derive three Bayesian estimators for RF(R_{GW}). assuming that the shape parameters are known and the scale parameters are unknown.

1. Maximum likelihood Estimation(MLE).

Using the invariance property of ML method, the ML estimator of R_{GW} ,say $R_{GW(MLE)}$, based on ML estimators of the parameters(β and μ), is from eq.(9):

$$R_{GW(MLE)}=1-\frac{1}{\lambda\,\Gamma(\alpha)}\left(\frac{\mu_{(MLE)}^{\frac{1}{\lambda}}}{\beta_{(MLE)}}\right)^{\alpha}\sum_{i=0}^{15}\frac{(-1)^{i}}{i!}\left(\frac{\mu_{(MLE)}^{\frac{1}{\lambda}}}{\beta_{(MLE)}}\right)^{i}\Gamma\left(\frac{\alpha+i}{\lambda}\right)\text{.........(10)}$$

Where $\beta_{(MLE)}$ and $\mu_{(MLE)}$ are ML estimators of scale parameters β and μ , respectively. Suppose that X_1..........X_n is a random sample from Gam (α,β) and Y_1..........Y_m is a random sample from Wei(λ,μ),then the likelihood functions of the two random samples are:

$$L_x(\alpha,\beta;\ x_1\text{.......}x_n)=\left(\frac{1}{\Gamma(\alpha)\,\beta^{\alpha}}\right)^{n}\prod_{i=1}^{n}x_i^{\alpha-1}\,e^{-\left(\frac{\sum_{i=1}^{n}x_i}{\beta}\right)}\text{........(11)}$$

$$L_y(\lambda,\mu;y_1\text{.........}y_m)=\left(\frac{\lambda}{\mu}\right)^{m}\prod_{i=1}^{m}y_i^{\lambda-1}\,e^{-\left(\frac{\sum_{i=1}^{m}y_i^{\lambda}}{\mu}\right)}\text{.........(12)}$$

And the log-likelihood functions are:

$$\ell_x(\alpha,\beta)=-n\ln\Gamma\alpha-n\alpha\ln\beta+(\alpha-1)\sum_{i=1}^{n}\ln x_i-\frac{\sum_{i=1}^{n}x_i}{\beta}$$

$$\ell_y(\lambda,\mu)=m\ln\lambda-m\ln\mu+(\lambda-1)\sum_{i=1}^{m}\ln y_i-\frac{\sum_{i=1}^{m}y_i^{\lambda}}{\mu}$$

The fi[illegible]lhood function with respect to β and μ for each function, are given by:

$$\frac{\partial\ell_x}{\partial\beta}=-\frac{n\alpha}{\beta}+\frac{\sum_{i=1}^{n}x_i}{\beta^2}=0$$

$$\frac{\partial\ell_y}{\partial\mu}=-\frac{m}{\mu}+\frac{\sum_{i=1}^{m}y_i^{\lambda}}{\mu^2}=0$$

ISSN 0215-9511

Then the ML estimators for the unknown parameter β and μ ,say $\beta_{(MLE)}$ and can be obtained as the solution of above equations, as:[7]

$$\beta_{(MLE)}=\frac{\bar{x}}{\alpha} \qquad \text{and} \qquad \mu_{(MLE)}=\frac{\sum_{i=1}^{m} y_i^{\lambda}}{m}$$

2. **Moments Estimation (ME).**

When the method of estimation of unknown parameter is changed from ML to any other traditional method, the invariance principle does not hold good to estimate the parametric function. However, such an adoption is attempted in different situations by different authors. [8]and[4].In this direction we propose the Moments method to estimate the RF, by considering the estimators of model parameters. Since the strength X is Gamma random variable with(α,β), and the stress Y is weibull random variable with(λ,μ), then their population means are given by:

$E(X)=\alpha\beta$ and $E(Y)=\mu^{\frac{1}{\lambda}}\Gamma\left(1+\frac{1}{\lambda}\right)$

According to the method of moments, equating the samples means with the corresponding populations means. then the ME's of β and μ denoted $\beta_{(ME)}$ and $\mu_{(ME)}$, respectively, are:

$$\beta_{(ME)}=\frac{\bar{X}}{\alpha} \qquad \text{and} \qquad \mu_{(ME)}=\left(\frac{\bar{Y}}{\Gamma\left(1+\frac{1}{\lambda}\right)}\right)^{\lambda}$$

The ME of R_{GW} ,say $R_{GW(ME)}$, is obtained by substitute $\beta_{(ME)}$ and $\mu_{(ME)}$ in eq. (9) as:

$$R_{GW(ME)}=1-\frac{1}{\lambda\,\Gamma(\alpha)}\left(\frac{\mu_{(ME)}^{\frac{1}{\lambda}}}{\beta_{(ME)}}\right)^{\alpha}\sum_{i=0}^{15}\frac{(-1)^i}{i!}\left(\frac{\mu_{(ME)}^{\frac{1}{\lambda}}}{\beta_{(ME)}}\right)^{i}\Gamma\left(\frac{\alpha+i}{\lambda}\right)\ldots\ldots\ldots(13)$$

3. **Bayesian Estimation Method (BE).**

In the following subsections the threebayesian estimators (BE's) of R_{GW} are obtained under squared error loss function ,by using three different prior distributions

3.1. Noninformative prior distribution.

For any parameter θ, as a random variable following noninformative type of priors with:[9]

$$f(\theta)=\frac{1}{\theta}\,\theta>0 \qquad \ldots\ldots\ldots\ldots(14)$$

then the prior densities of the two parametersβ and μ ,assuming they are independent random variables following the noninformative distribution, are

$$f(\beta)=\frac{1}{\beta}\,\beta>0 \quad , \quad f(\mu)=\frac{1}{\mu}\,\mu>0$$

combining the prior densities of β and μ, and the likelihood functions given in equations (10)and(11), to obtain the joint posterior density of (β, μ) as:

$$\pi_1(\beta, \mu / \underline{x}, \underline{y}) = \frac{\pi(\beta,\mu)}{\int_0^\infty \int_0^\infty \pi(\beta,\mu)\, d\beta\, d\mu} \quad ....(15)$$

Where

$$\pi(\beta, \mu) = f(\alpha, \beta/\underline{x}) f(\lambda, \mu/\underline{y}) \quad ....(16)$$

$$\pi(\beta, \mu) = \frac{T_x\, e^{-\left(\frac{U_x}{\beta}\right)}}{(\Gamma\alpha)^n\, \beta^{\alpha n+1}} \frac{\lambda^m T_y\, e^{-\left(\frac{U_y}{\mu}\right)}}{\mu^{m+1}}$$

Where $T_x = \prod_{i=1}^n x_i^{\alpha-1}$, $T_y = \prod_{i=1}^m y_i^{\lambda-1}$, $U_x = \sum_{i=1}^n x_i$

and $U_y = \sum_{i=1}^m y_i^\lambda$, then:

$$\int_0^\infty \int_0^\infty \pi(\beta, \mu)\, d\beta\, d\mu =$$

$$\frac{T_x T_y \lambda^m}{(\Gamma\alpha)^n} \int_0^\infty \int_0^\infty \beta^{-\alpha n-1} e^{-\left(\frac{U_x}{\beta}\right)} \mu^{-m-1} e^{-\left(\frac{U_y}{\mu}\right)} d\beta\, d\mu$$

Now let $k = \frac{U_x}{\beta}$ and $t = \frac{U_y}{\mu}$ using the transformation technique we get:

$$\int_0^\infty \int_0^\infty \pi(\beta, \mu)\, d\beta\, d\mu = \frac{T_x\, T_y\, \lambda^m}{(\Gamma\alpha)^n\, U_x^{\alpha n}\, U_y^m} \int_0^\infty \int_0^\infty k^{\alpha n-1} e^{-k} t^{m-1} e^{-t} dt\, dk$$

$$= \frac{T_x\, T_y\, \lambda^m}{(\Gamma\alpha)^n\, U_x^{\alpha n}\, U_y^m} \Gamma(\alpha n)\Gamma(m)$$

Then the joint posterior density will be:

$$\pi_1(\beta, \mu / \underline{x}, \underline{y}) = \frac{U_x^{\alpha n}\, U_y^m}{\Gamma(\alpha n)\, \Gamma(m)} \beta^{-\alpha n-1} \mu^{-m-1} e^{-\left(\frac{U_x}{\beta}\right)} e^{-\left(\frac{U_y}{\beta}\right)} \beta > 0, \mu > 0$$

Now under squared error loss function the Bayes estimator of R_{GW},denoted by $R_{GW(B1)}$,defined as:

$$R_{GW(B1)} = E(R_{GW} / \underline{x}, \underline{y})$$

$$R_{GW(B1)} = \int_0^\infty \int_0^\infty R_{GW}\, \pi_1(\beta, \mu / \underline{x}, \underline{y})\, d\beta\, d\mu$$

$$R_{GW(B1)} = \int_0^\infty \int_0^\infty \left[1 - \frac{1}{\lambda\, \Gamma(\alpha)} \left(\frac{\mu^{\frac{1}{\lambda}}}{\beta}\right)^\alpha \sum_{i=0}^{15} \frac{(-1)^i}{i!} \left(\frac{\mu^{\frac{1}{\lambda}}}{\beta}\right)^i \Gamma\left(\frac{\alpha+i}{\lambda}\right)\right] \pi_1(\beta, \mu / \underline{x}, \underline{y})\, d\beta\, d\mu$$

ISSN 0215-9511

$R_{GW(B1)}=$

$$\int_0^\infty \int_0^\infty \pi_1(\beta, \mu/\underline{x}, \underline{y})\ d\beta\, d\mu \quad -$$

$$\int_0^\infty \int_0^\infty \frac{1}{\lambda\,\Gamma(\alpha)} \left(\frac{\mu^{\frac{1}{\lambda}}}{\beta}\right)^{\alpha} \sum_{i=0}^{15} \frac{(-1)^i}{i!} \left(\frac{\mu^{\frac{1}{\lambda}}}{\beta}\right)^{i} \Gamma\left(\frac{\alpha+i}{\lambda}\right) \pi_1\left(\beta, \mu/\underline{x}, \underline{y}\right)\ d\beta\, d\mu \qquad \text{......................} (17)$$

$R_{GW(B1)}=R_{11} - R_{12}$

Where

$$R_{11} = \int_0^\infty \int_0^\infty \pi_1(\beta, \mu/\underline{x}, \underline{y})\ d\beta\, d\mu$$

$$R_{11}=\frac{U_x^{\alpha n}\ U_y^{m}}{\Gamma(\alpha n)\ \Gamma(m)} \int_0^\infty \int_0^\infty \beta^{-\alpha n-1} \mu^{-m-1} e^{-\left(\frac{U_x}{\beta}\right)} e^{-\left(\frac{U_y}{\mu}\right)}\ d\beta\, d\mu$$

Using the same transformation of $k = \frac{U_x}{\beta}$, $t = \frac{U_y}{\mu}$ · we get:

$$R_{11}=\frac{1}{\Gamma(\alpha n)\ \Gamma(m)} \int_0^\infty \int_0^\infty k^{\alpha n-1} e^{-k} t^{m-1} e^{-t} dt\, dk$$

$$R_{11}=\frac{\Gamma(\alpha n)\ \Gamma(m)}{\Gamma(\alpha n)\ \Gamma(m)}=1$$

And

$$R_{12} = \frac{U_x^{\alpha n}\ U_y^{m}}{\lambda\,\Gamma(\alpha)\Gamma(\alpha n)\ \Gamma(m)} \sum_{i=0}^{15} \frac{(-1)^i}{i!}\ \Gamma\left(\frac{\alpha+i}{\lambda}\right)$$

$$\int_0^\infty \int_0^\infty \beta^{-\alpha-i-\alpha n-1}\ e^{-\left(\frac{u_x}{\beta}\right)} \mu^{\frac{\alpha+i}{\lambda}-m-1} e^{-\left(\frac{u_y}{\mu}\right)}\ d\beta\, d\mu$$

By the k and t transformation above we get:

$$R_{12} = \frac{1}{\lambda\,\Gamma(\alpha)\Gamma(\alpha n)\ \Gamma(m)} \sum_{i=0}^{15} \frac{(-1)^i}{i!}\ \Gamma\left(\frac{\alpha+i}{\lambda}\right) \frac{U_y^{\frac{\alpha+i}{\lambda}}}{U_x^{\alpha+i}}$$

$$\int_0^\infty \int_0^\infty k^{\alpha+i+\alpha n-1} e^{-k}\ t^{m-\frac{\alpha+i}{\lambda}-1} e^{-t}\ dt\, dk$$

ISSN 0215-9511

$$R_{12} = \frac{1}{\lambda\,\Gamma(\alpha)\Gamma(\alpha n)\ \Gamma(m)} \sum_{i=0}^{15} \frac{(-1)^i}{i!}\ \Gamma\left(\frac{\alpha+i}{\lambda}\right) \frac{U_y^{\frac{\alpha+i}{\lambda}}}{U_x^{\alpha+i}}\ \Gamma(\alpha(1+n)+i)\ \Gamma\left(m-\frac{\alpha+i}{\lambda}\right)$$

So then

$$R_{GW(B1)} = 1 - \frac{1}{\lambda\,\Gamma(\alpha)\Gamma(\alpha n)\ \Gamma(m)} \sum_{i=0}^{15} \frac{(-1)^i}{i!}\ \Gamma\left(\frac{\alpha+i}{\lambda}\right) \frac{U_y^{\frac{\alpha+i}{\lambda}}}{U_x^{\alpha+i}}$$

$$\Gamma(\alpha(n+1)+i)\ \Gamma\left(m-\frac{\alpha+i}{\lambda}\right)$$

3.2. Jeffery prior Information.

The Jeffery prior information for any parameter θ is given as:[12]

$f(\theta) = \frac{a}{\theta^{\,b}}$ where a is a constant, $b \in R^{+}$ .... (18)

The prior densities of the two independent parameters β and μ is given as:

$f(\beta)=\frac{a}{\beta^b}\ \beta>0$, $f(\mu)=\frac{a}{\mu^b}\ \mu>0$.

Then the combain between the prior densities and the likelihood function in eq. (11) and (12) from eq. (16) is given as:

$$\pi(\beta,\mu)=\frac{a}{\beta^b}\frac{T_x}{(\Gamma\alpha)^n\,\beta^{\alpha n}}\ e^{-\left(\frac{U_x}{\beta}\right)}\frac{a}{\mu^b}\frac{\lambda^m T_y}{\mu^b}e^{-\left(\frac{U_y}{\mu}\right)}$$

Then

$$\int_0^\infty\int_0^\infty \pi(\beta,\mu)\,d\beta\,d\mu = \frac{a^2\ T_x\,T_y\ \lambda^m}{(\Gamma\alpha)^n}$$

$$\cdot\int_0^\infty\int_0^\infty \beta^{-b-\alpha n}e^{-\left(\frac{U_x}{\beta}\right)}\mu^{-b-m}e^{-\left(\frac{U_y}{\mu}\right)}\ d\beta\,d\mu$$

Using the $k=\frac{U_x}{\beta}$ and $t=\frac{U_y}{\mu}$ transformation we get:

$$\int_0^\infty\int_0^\infty \pi(\beta,\mu)\,d\beta\,d\mu = \frac{a^2\ T_x\,T_y\ \lambda^m}{(\Gamma\alpha)^n}\Gamma(b+\alpha n-1)\ \Gamma(b+m-1)$$

The joint posterior density from eq.(15), will be:

$$\pi_2(\beta,\mu/\underline{x},\underline{y}) = \frac{U_x^{b+\alpha n-1}\ U_y^{b+m-1}\ e^{-\left(\frac{U_x}{\beta}\right)}e^{-\left(\frac{U_y}{\mu}\right)}}{\Gamma(b+\alpha n-1)\,\Gamma(b+m-1)\ \beta^{b+\alpha n}\mu^{b+m}}\ \beta>0,\mu>0$$

ISSN 0215-9511

Then under squared error loss function the second Bayes estimator of R_{GW},denoted by $R_{GW(B2)}$, from eq.(17) can be found as:

$$R_{GW(B2)}=\int_0^{\infty}\int_0^{\infty} R_{GW}\,\pi_2(\beta,\mu/\underline{x},\underline{y})\; d\beta\, d\mu \quad\text{..........(19)}$$

$$R_{GW(B2)}=R_{11}-R_{12}$$

Where :

$$R_{11}=\frac{U_x^{b+\alpha n-1}\; U_y^{b+m-1}}{\Gamma(b+\alpha n-1)\,\Gamma(b+m-1)}\int_0^{\infty}\int_0^{\infty}\beta^{-b-\alpha n}e^{-\left(\frac{U_x}{\beta}\right)}\mu^{-b-m}e^{-\left(\frac{U_y}{\mu}\right)}\; d\beta\, d\mu$$

$$R_{11}=\frac{\Gamma(b+\alpha n-1)\,\Gamma(b+m-1)}{\Gamma(b+\alpha n-1)\,\Gamma(b+m-1)}=1$$

So then :

$$R_{12}=\frac{U_x^{b+\alpha n-1}\; U_y^{b+m-1}}{\lambda\,\Gamma(b+\alpha n-1)\,\Gamma(b+m-1)\Gamma(\alpha)}\sum_{i=0}^{15}\frac{(-1)^i}{i!}\,\Gamma\left(\frac{\alpha+i}{\lambda}\right)$$

$$\cdot\int_0^{\infty}\int_0^{\infty}\beta^{-\alpha-i-b-\alpha n}\mu^{\frac{\alpha+i}{\lambda}-b-m}e^{-\left(\frac{U_x}{\beta}\right)}e^{-\left(\frac{U_y}{\mu}\right)}\; d\beta\, d\mu$$

By transformation we get:

$$R_{12}=\frac{U_x^{-\alpha}\; U_y^{\frac{\alpha}{\lambda}}}{\lambda\,\Gamma(b+\alpha n-1)\,\Gamma(b+m-1)\Gamma(\alpha)}\sum_{i=0}^{15}\frac{(-1)^i}{i!}\,\Gamma\left(\frac{\alpha+i}{\lambda}\right)\frac{U_y^{\frac{i}{\lambda}}}{U_x^{i}}$$

$$\Gamma(b+\alpha(n+1)+i-1)\;\Gamma\left(b+m-\frac{\alpha+i}{\lambda}-1\right)$$

So then:

$$R_{GW(B2)}=1-\frac{1}{\lambda\,\Gamma(b+\alpha n-1)\,\Gamma(b+m-1)\Gamma(\alpha)}\sum_{i=0}^{15}\frac{(-1)^i}{i!}\,\Gamma\left(\frac{\alpha+i}{\lambda}\right)\frac{U_y^{\frac{\alpha+i}{\lambda}}}{U_x^{\alpha+i}}$$

$$.\Gamma(b+\alpha(n+1)+i-1)\;\Gamma\left(b+m-\frac{\alpha+i}{\lambda}-1\right)$$

ISSN 0215-9511

3.3. Inverse Gamma Distribution

Finally using the Inverse Gamma distribution for the unknown parameters as a prior distribution for any parameter θ :[13]

$$f(\theta) = \frac{a^b}{\Gamma(b)} \theta^{-b-1} e^{-\left(\frac{a}{\theta}\right)} \theta > 0 \quad ,a,b > 0 \quad ....(20)$$

Then the prior distributions for β and μ will given as:

$$f(\beta) = \frac{a^b}{\Gamma(b)} \beta^{-b-1} e^{-\left(\frac{a}{\beta}\right)} a,b > 0 \quad , \beta > 0$$

$$f(\mu) = \frac{a^b}{\Gamma(b)} \mu^{-b-1} e^{-\left(\frac{a}{\mu}\right)} a,b,$$

Assuming the independency between β and μ ,the combin function as in eq.(16) will given as:

$$\pi(\beta,\mu) = \frac{a^{2b} \lambda^m T_x T_y e^{-\left(\frac{A_x}{\beta}\right)} e^{-\left(\frac{A_y}{\mu}\right)}}{(\Gamma b)^2 (\Gamma\alpha)^n \beta^{b+\alpha n+1} \mu^{b+m+1}}$$

Where $A_x = \sum_{i=1}^{n} x_i + a$ and $A_y = \sum_{i=1}^{m} y_i^{\lambda} + a$, then:

$$\int_0^\infty \int_0^\infty \pi(\beta,\mu)\, d\beta\, d\mu = \frac{a^{2b} \lambda^m T_x T_y}{(\Gamma b)^2 (\Gamma\alpha)^n}$$

$$\cdot \int_0^\infty \int_0^\infty \beta^{-b-\alpha n-1} e^{-\left(\frac{A_x}{\beta}\right)} \mu^{-b-m-1} \quad e^{-\left(\frac{A_y}{\mu}\right)} d\beta\, d\mu$$

$$\int_0^\infty \int_0^\infty \pi(\beta,\mu)\, d\beta\, d\mu = \frac{a^{2b} \lambda^m T_x T_y}{(\Gamma b)^2 (\Gamma\alpha)^n A_x^{b+\alpha n} A_y^{b+m}} \Gamma(b+\alpha n)\Gamma(b+m)$$

And the Joint posterior density from eq.(15), will be:

$$\pi_3(\beta,\mu/\underline{x},\underline{y}) = \frac{A_x^{b+\alpha n} A_y^{b+m} e^{-\left(\frac{A_x}{\beta}\right)} e^{-\left(\frac{A_y}{\mu}\right)}}{\Gamma(b+\alpha n)\, \Gamma(b+m) \beta^{b+\alpha n+1} \mu^{b+m+1}} \beta > 0, \mu > 0$$

Under squared error loss function the third Bayes estimator of R_{GW},denoted by $R_{GW(B3)}$, from eq.(17), is:

$$R_{GW(B3)} = \int_0^\infty \int_0^\infty R_{GW}\, \pi_3(\beta,\mu/\underline{x},\underline{y})\ d\beta\, d\mu ....(21)$$

$$R_{GW(B3)} = R_{11} - R_{12}$$

Where :

$$R_{11}=\frac{A_x^{b+\alpha n}A_y^{b+m}}{\Gamma(b+\alpha n)\,\Gamma(b+m)}\int_0^\infty\int_0^\infty \beta^{-b-\alpha n-1}e^{-\left(\frac{A_x}{\beta}\right)}\mu^{-b-m-1}e^{-\left(\frac{A_y}{\mu}\right)}\,d\beta\,d\mu$$

$$R_{11}=\frac{\Gamma(b+\alpha n)\,\Gamma(b+m)}{\Gamma(b+\alpha n)\,\Gamma(b+m)}=1$$

$$R_{12}=\frac{A_x^{b+\alpha n}A_y^{b+m}}{\Gamma(b+\alpha n)\,\Gamma(b+m)\lambda\,\Gamma(\alpha)}\sum_{i=0}^{15}\frac{(-1)^i}{i!}\,\Gamma\left(\frac{\alpha+i}{\lambda}\right)$$

$$\cdot\int_0^\infty\int_0^\infty \beta^{-\alpha-i-b-\alpha n-1}e^{-\left(\frac{A_x}{\beta}\right)}\mu^{\frac{\alpha+i}{\lambda}-b-m-1}e^{-\left(\frac{A_y}{\mu}\right)}\,d\beta\,d\mu$$

$$R_{12}=\frac{1}{\lambda_1\Gamma(\alpha)\,\Gamma(b+\alpha n)\,\Gamma(b+m)}\sum_{i=0}^{15}\frac{(-1)^i}{i!}\frac{A_y^{\frac{\alpha+i}{\lambda}}}{A_x^{\alpha+i}}\Gamma\left(\frac{\alpha+i}{\lambda}\right)$$

$$.\Gamma(b+i+\alpha+\alpha n)\,\Gamma\left(b+m-\frac{\alpha+i}{\lambda}\right)$$

Finally the third Bayesian estimator for R_{GW} ,will be in form:

$$R_{GW(B3)}=1-\frac{1}{\lambda\,\Gamma(\alpha)\,\Gamma(b+\alpha n)\,\Gamma(b+m)}\sum_{i=0}^{15}\frac{(-1)^i}{i!}\frac{A_y^{\frac{\alpha+i}{\lambda}}}{A_x^{\alpha+i}}\Gamma\left(\frac{\alpha+i}{\lambda}\right)$$

$$.\Gamma(b+\alpha(n+1)+i)\,\Gamma\left(b+m-\frac{\alpha+i}{\lambda}\right)$$

After five estimators was derived for RF , We will use simulations to compare the estimation methods, by generating random values of the variables X with Gam$(\alpha,\beta)$and Y with wei(λ,μ) for different sample sizes varied between (10,15) for small samples and (25) sample medium and (50) for large sample sizes.

Also for different parameter values as in the following models:

Model1:($\alpha=1.2,\ \beta=1.5,\ \lambda=1.9,2\ ,\ \mu=1.3,2.2$)

Model2:($\alpha=0.5\ ,\ \beta=1.5\ ,\ \lambda=1.5,1.9\ ,\ \mu=1.3,2.2$)

Model 3:($\alpha=2.5\ ,\ \beta=2\ ,\ \lambda=1.9,2.4\ ,\ \mu=2.2,1.3$)

Model 4:($\alpha=1.8\ ,\ \beta=2\ ,\ \lambda=1.5,1.9\ ,\ \mu=2.2,1.3$)

Model 5:($\alpha=2\ ,\ \beta=1.5\ ,\ \lambda=2.4,2.8\ ,\ \mu=1.3,3.6$)

The comparisons are made between the non- Bayesian estimators from equations (10) for ML and (13) for Mo estimators, and made between the Bayesian estimators by equations(17),(19) and (21), by the MSE values for each one. The results are recorded in the tables from (2)to (6), under(1000) replications obtained by using matlab(2012) .

Table(1) Models Selected Parameters for The Simulation

model	α	β	λ	μ	R_{GW}
1	1.2	1.5	1.9	1.3	0.6218
	1.2	1.5	2	2.2	0.5379
2	0.5	1.5	1.5	1.3	0.2978
	0.5	1.5	1.9	2.2	0.2297
3	2.5	2	1.9	2.2	0.9149
	2.5	2	2.4	1.3	0.9560
4	1.8	2	1.5	2.2	0.7706
	1.8	2	1.9	1.3	0.8638
5	2	1.5	2.4	1.3	0.8509
	2	1.5	2..8	3.6	0.7562

The conclusions from these tables are summarized as follows:

1- According to the volumes of sample and proven parameters appeared the best way through the tables of (2-6) ,(B2).

2- In the case of the different parameters and small sample size(10,15) appeared the best way through the same tables(B2).

3- Depending on the parameters and size of the sample medium (25) appeared the best way through the same tables(B2).

4- When the different Parameters and large sample size (50) was the advantage for the (B2) through the same tables.

5- Equal volumes of samples n = m and different Parameters(B2) was the best way .

6- Between non-Bayesian appeared the best way through the tables of (2-6) Moment estimations (MOM)

7- Also Between the Bayesian appeared the best way through the same tables of (B2).

8- Through comparison between the total appeared the best way through the tables of (2-6),(B2).

9- In the first Model value R_{GW} near to 0.5, and Model two less than 0.5 in the case of proven gamma and weibull change.

10- value $R_{GW}$was near to one and is best in the third model in same cases of proven gamma and change weibull,but in Model fourth and fifth the value ofis greater than 0.7 .

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Table 2: the MSE values for estimation methods of R_{GW} from model(1)

		$\alpha = 1.2$, $\beta = 1.5$, $\lambda = 1.9$, $\mu = 1.3$ R_{GW} =0.6218				
N	M	MLE	MOM	B1	B2	B3
10	10	9.8816 E-06	1.5095 E-05	9.9009 E-06	7.9425 E-06	8.9375 E-06
	15	1.8908 E-05	2.2023 E-05	2.0307 E-05	1.6609 E-05	1.8197 E-05
	25	1.9370 E-05	1.8920 E-05	2.2078 E-05	1.7713 E-05	1.9238 E-05
	50	1.5231 E-05	1.9502 E-05	1.8765 E-05	1.4467 E-05	1.5602 E-05
15	10	7.5944 E-05	7.6311 E-05	7.3391 E-05	6.8892 E-05	7.3493 E-05
	15	1.6457 E-05	1.5867 E-05	1.6363 E-05	1.4052 E-05	1.5567 E-05
	25	2.4641 E-06	4.0621 E-06	3.1776 E-06	2.1327 E-06	2.4582 E-06
	50	1.1774 E-05	1.3946 E-05	1.3684 E-05	1.1166 E-05	1.1923 E-05
25	10	7.8803 E-06	1.3908 E-05	5.9278 E-06	6.0259 E-06	6.6305 E-06
	15	2.3458 E-06	6.8693 E-06	1.9101 E-06	1.6077 E-06	1.9705 E-06
	25	2.1216 E-04	2.2332 E-04	2.1183 E-04	2.0150 E-04	2.0875 E-04
	50	1.8849 E-05	2.3544 E-05	1.9751 E-05	1.8011 E-05	1.8710 E-05
50	10	2.1667 E-05	3.1018 E-05	1.7679 E-05	1.7705 E-05	1.9472 E-05
	15	2.1414 E-05	2.8962 E-05	1.9097 E-05	1.9312 E-05	2.0007 E-05
	25	1.8817 E-05	2.3567 E-05	1.7858 E-05	1.7577 E-05	1.8077 E-05
	50	1.2035 E-05	1.7155 E-05	1.2041 E-05	1.1365 E-05	1.1815 E-05
		$\alpha = 1.2$, $\beta = 1.5$, $\lambda = 2$, $\mu = 2.2$ R_{GW} =0.5379				
N	M	MLE	MOM	B1	B2	B3
10	10	4.3335E-05	4.4722E-05	4.3449E-05	3.8628E-05	4.1342E-05
	15	5.6969E-05	5.2533E-05	5.9319E-05	5.2834E-05	5.5728E-05
	25	3.5167E-05	3.5137E-05	3.9644E-05	3.3241E-05	3.5317E-05
	50	2.7936E-09	9.2970E-09	5.8950E-07	5.6734E-08	4.5776E-08
15	10	7.3204E-05	7.4192E-05	7.0206E-05	6.6913E-05	7.0118E-05
	15	4.7525E-05	4.9573E-05	4.7570E-05	4.3790E-05	4.6097E-05
	25	6.6416E-05	6.3578E-05	6.8192E-05	6.3406E-05	6.5611E-05
	50	5.0157E-05	5.1066E-05	5.3559E-05	4.8633E-05	5.0268E-05
25	10	6.7750E-05	6.8532E-05	6.2917E-05	6.2590E-05	6.4479E-05
	15	5.4367E-05	5.0162E-05	5.2357E-05	5.0868E-05	5.2489E-05
	25	4.3387E-05	4.4342E-05	4.3475E-05	4.1398E-05	4.2584E-05
	50	4.6524E-05	4.6234E-05	4.7997E-05	4.5313E-05	4.6351E-05
50	10	4.4381E-05	4.6669E-05	3.8689E-05	4.0041E-05	4.1290E-05
	15	3.5893E-05	3.0804E-05	3.2872E-05	3.2704E-05	3.4088E-05
	25	3.0008E-05	2.7433E-05	2.8876E-05	2.8604E-05	2.9124E-05
	50	7.5466E-05	7.4747E-05	7.5373E-05	7.3910E-05	7.4902E-05

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Table 3: the MSE values for estimation methods of R_{GW} from model(2)

		$\alpha = 0.5$, $\beta = 1.5$, $\lambda = 1.5$, $\mu = 1.3$ R_{GW} =0.2978				
N	M	MLE	MOM	B1	B2	B3
10	10	1.2968 E-05	1.1937 E-05	0.28 E-02	4.2540 E-05	5.8044 E-07
	15	8.4039 E-08	2.8532 E-07	4.0507 E-05	1.3602 E-05	3.8705 E-05
	25	6.7547 E-06	6.1316 E-06	5.02 E-01	0.410 E-01	2.025 E-01
	50	1.5512 E-05	1.6239 E-05	3.0123 E-05	1.7063 E-05	2.0223 E-05
15	10	7.0488 E-05	7.1198 E-05	7.6441 E-05	6.4493 E-05	7.1363 E-05
	15	2.3666 E-05	2.8723 E-05	3.0703 E-05	2.3123 E-05	2.5893 E-05
	25	2.8315 E-05	3.1415 E-05	3.7546 E-05	2.8111 E-05	3.1295 E-05
	50	9.8131 E-06	1.0612 E-05	1.7113 E-05	1.0842 E-05	1.2360 E-05
25	10	5.4204 E-05	6.1877 E-05	5.4192 E-05	4.8124 E-05	5.3469 E-05
	15	3.5137 E-06	2.7230 E-06	4.7905 E-06	3.5789 E-06	4.0368 E-06
	25	1.2543 E-06	2.1884 E-06	2.5476 E-06	1.4914 E-06	1.7606 E-06
	50	3.4950 E-05	3.8717 E-05	4.0858 E-05	3.4764 E-05	3.6914 E-05
50	10	1.2936 E-05	1.0568 E-05	1.1285 E-05	1.1495 E-05	1.2176 E-05
	15	1.1444 E-06	7.5252 E-07	1.1068 E-06	1.0494 E-06	1.1606 E-06
	25	4.2759 E-09	3.1780 E-08	4.1711 E-08	1.2169 E-08	1.9775 E-08
	50	4.8660 E-06	7.8724 E-06	5.9031 E-06	4.9950 E-06	5.2858 E-06
		$\alpha = 0.5$, $\beta = 1.5$, $\lambda = 1.9$, $\mu = 2.2$ R_{GW} =0.2297				
N	M	MLE	MOM	B1	B2	B3
10	10	6.0998E-05	5.6269E-05	7.9705E-05	5.7916E-05	6.6422E-05
	15	1.0811E-04	1.0263E-04	1.3169E-04	1.0373E-04	1.1449E-04
	25	8.1340E-06	8.4651E-06	2.0642E-05	1.0795E-05	1.2407E-05
	50	4.0489E-05	3.8150E-05	6.3010E-05	4.2309E-05	4.7820E-05
15	10	7.8458E-05	7.0550E-05	8.8081E-05	7.3829E-05	8.0894E-05
	15	6.1608E-05	5.0847E-05	7.3487E-05	5.9546E-05	6.5219E-05
	25	2.8171E-05	2.7233E-05	3.9200E-05	2.9093E-05	3.2035E-05
	50	1.7350E-05	1.7636E-05	2.7577E-05	1.9118E-05	2.1059E-05
25	10	2.6471E-05	2.2990E-05	2.8634E-05	2.4663E-05	2.7216E-05
	15	2.1993E-05	2.1807E-05	2.567E-05	2.1895E-05	2.3369E-05
	25	1.9597E-05	1.7631E-05	2.4347E-05	2.0022E-05	2.1402E-05
	50	1.3229E-05	1.1973E-05	1.8071E-05	1.4231E-05	1.5117E-05
50	10	2.6841E-05	2.3402E-05	2.5429E-05	2.5125E-05	2.6225E-05
	15	2.9710E-05	2.5715E-05	2.9937E-05	2.8101E-05	2.9720E-05
	25	2.2246E-05	2.0864E-05	2.3731E-05	2.1799E-05	2.2812E-
	50	2.9296E-05	2.5724E-05	3.1938E-05	2.9349E-05	053.0285E-05

Table 4: the MSE values for estimation methods of $R_{GW}$from model(3)

		$\alpha = 2.5$, $\beta = 2$, $\lambda = 1.9$, $\mu = 2.2$ R_{GW} =0.9149				
N	M	MLE	MOM	B1	B2	B3
10	10	5.5286 E-06	5.2733 E-06	5.1675 E-06	5.1566 E-06	5.2283 E-06
	15	3.1790 E-06	3.0633 E-06	2.8225 E-06	2.6906 E-06	2.8158 E-06
	25	4.9436 E-06	4.7346 E-06	4.8029 E-06	4.6547 E-06	4.7447 E-06
	50	4.6678 E-06	4.5382 E-06	4.6146 E-06	4.4380 E-06	4.5117 E-06
15	10	5.7977 E-06	5.5528 E-06	5.4727 E-06	5.4334 E-06	5.5585 E-06
	15	3.0386 E-06	2.9327 E-06	2.6524 E-06	2.6287 E-06	2.7106 E-06
	25	5.4792 E-06	5.4005 E-06	5.3521 E-06	5.2901 E-06	5.3407 E-06
	50	2.7164 E-06	2.5339 E-06	2.6405 E-06	2.5010 E-06	2.5623 E-06
25	10	4.6264 E-06	4.4706 E-06	4.1114 E-06	4.1361 E-06	4.2867 E-06
	15	3.8701 E-06	3.8487 E-06	3.4976 E-06	3.5265 E-06	3.5979 E-06
	25	3.0858 E-06	3.0686 E-06	2.8605 E-06	2.8366 E-06	2.8909 E-06
	50	3.0010 E-06	2.9186 E-06	2.9045 E-06	2.8405 E-06	2.8795 E-06
50	10	3.8675 E-06	3.4718 E-06	3.2568 E-06	3.4514 E-06	3.4971 E-06
	15	5.2937 E-06	5.2667 E-06	5.0279 E-06	5.0428 E-06	5.1205 E-06
	25	5.0026 E-06	4.7809 E-06	4.8356 E-06	4.8361 E-06	4.8822 E-06
	50	3.8429 E-06	3.9495 E-06	3.7416 E-06	3.7250 E-06	3.7538 E-06
		$\alpha = 2.5$, $\beta = 2$, $\lambda = 2.4$, $\mu = 1.3$ R_{GW} =0.9560				
N	M	MLE	MOM	B1	B2	B3
10	10	8.8489E-07	1.0339E-06	7.5542E-07	6.9689E-07	7.6642E-07
	15	1.6483E-06	1.7843E-06	1.6172E-06	1.5675E-06	1.6127E-06
	25	1.2947E-06	1.3459E-06	1.2631E-06	1.1992E-06	1.2420E-06
	50	7.0319E-08	2.4838E-07	6.6655E-08	3.1442E-08	4.2469E-08
15	10	7.8344E-07	1.2225E-06	6.4459E-07	6.1349E-07	6.7472E-07
	15	1.1580E-06	1.3019E-06	1.0875E-06	9.7166E-07	1.0919E-06
	25	1.1584E-06	1.4443E-06	1.1198E-06	1.0739E-06	1.1099E-06
	50	9.7832E-07	1.2602E-06	9.6069E-07	9.0633E-07	9.3810E-07
25	10	5.9735E-07	1.1158E-06	4.5125E-07	4.7449E-07	4.9719E-07
	15	9.7155E-07	1.3232E-06	8.8830E-07	8.6538E-07	9.0701E-07
	25	4.3324E-07	8.3494E-07	3.8286E-07	3.6323E-07	3.8512E-07
	50	4.7551E-07	7.9360E-07	4.5365E-07	4.2935E-07	4.4290E-07
50	10	1.2635E-06	1.4916E-06	1.1567E-06	1.1196E-06	1.1983E-06
	15	9.5752E-07	1.1264E-06	8.7120E-07	8.7379E-07	9.0048E-07
	25	1.0218E-06	1.3847E-06	9.7442E-07	9.6068E-07	9.8626E-07
	50	1.1682E-06	1.4119E-06	1.1484E-06	1.1333E-06	1.1490E-06

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Table : 5 the MSE values for estimation methods of R_{GW} from model(4)

		$\alpha = 1.8$, $\beta = 2$, $\lambda = 1.5$, $\mu = 2.2$		R_{GW} =0.7706		
N	M	MLE	MOM	B1	B2	B3
10	10	2.2868 E-05	2.0990 E-05	1.9704 E-05	2.0178 E-05	2.0588 E-05
	15	2.0090 E-05	1.9948 E-05	1.8630 E-05	1.7653 E-05	1.8432 E-05
	25	8.9219 E-06	7.6996 E-06	8.7887 E-06	7.7211 E-06	8.1311 E-06
	50	2.8652 E-05	2.6886 E-05	2.8889 E-05	2.7370 E-05	2.7982 E-05
15	10	1.1313 E-05	9.5191 E-06	4.3480 E-06	1.2494 E-05	1.2583 E-05
	15	2.1702 E-05	2.0153 E-05	1.9826 E-05	1.9561 E-05	2.0099 E-05
	25	2.0943 E-05	1.9577 E-05	2.0158 E-05	1.9470 E-05	1.9928 E-05
	50	2.1219 E-05	1.9346 E-05	2.1218 E-05	2.0224 E-05	2.0637 E-05
25	10	2.3256 E-05	2.1170 E-05	1.9617 E-05	2.0936 E-05	2.0975 E-05
	15	2.9634 E-05	2.7781 E-05	2.7638 E-05	2.7885 E-05	2.8208 E-05
	25	2.4973 E-05	2.3088 E-05	2.3891 E-05	2.3718 E-05	2.4027 E-05
	50	2.5148 E-05	2.4192 E-05	2.4827 E-05	2.4374 E-05	2.4625 E-05
50	10	1.7209 E-05	1.6718 E-05	1.3205 E-05	1.4875 E-05	1.4985 E-05
	15	1.2586 E-05	1.1021 E-05	1.0263 E-05	1.1055 E-05	1.1168 E-05
	25	2.1617 E-05	1.9988 E-05	2.0281 E-05	2.0522 E-05	2.0700 E-05
	50	2.1199 E-05	1.9442 E-05	2.0657 E-05	2.0554 E-05	2.0720 E-05
		$\alpha = 1.8$, $\beta = 2$, $\lambda = 1.9$, $\mu = 1.3$		R_{GW} =0.8638		
N	M	MLE	MOM	B1	B2	B3
10	10	4.0965E-06	5.8320E-06	3.3142E-06	3.0232E-06	3.3464E-06
	15	1.1681E-05	1.2335E-05	1.1332E-05	1.0590E-05	1.1205E-05
	25	1.2255E-05	1.2686E-05	1.2155E-05	1.1590E-05	1.1932E-05
	50	6.8033E-06	8.1937E-06	6.9566E-06	6.2430E-06	6.5227E-06
15	10	4.8732E-07	5.2529E-08	1.1783E-06	9.7765E-07	9.6391E-07
	15	2.6264E-06	3.7487E-06	2.1802E-06	2.0632E-06	2.1864E-06
	25	5.5257E-06	6.6165E-06	5.3195E-06	4.8885E-06	5.1706E-06
	50	3.6640E-06	5.0705E-06	3.7142E-06	3.2687E-06	3.4614E-06
25	10	5.1977E-06	5.9442E-06	4.1199E-06	4.1071E-06	4.4839E-06
	15	2.9176E-07	1.4324E-06	9.9795E-08	1.2229E-07	1.4119E-07
	25	3.3616E-06	5.4557E-06	3.0757E-06	2.9244E-06	3.0737E-06
	50	5.9722E-06	7.7867E-06	5.8959E-06	5.6490E-06	5.7828E-06
50	10	6.4060E-07	1.5103E-06	1.5201E-07	3.6516E-07	3.2455E-07
	15	7.5510E-06	9.3908E-06	6.8328E-06	6.8955E-06	7.0919E-06
	25	2.8556E-06	4.4472E-06	2.5035E-06	2.5015E-06	2.6014E-06
	50	3.6408E-06	5.2244E-06	3.4965E-06	3.4017E-06	3.4932E-06

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Table 6: the MSE values for estimation methods of $R_{GW}$from model(5)

		$\alpha = 2$, $\beta = 1.5$, $\lambda = 2.4$, $\mu = 1.3$ R_{GW} =0.8509				
N	M	MLE	MOM	B1	B2	B3
10	10	1.5011 E-05	1.7769 E-05	1.4391 E-05	1.2917 E-05	1.4367 E-05
	15	1.1695 E-05	1.3582 E-05	1.1316 E-05	1.0490 E-05	1.1095 E-05
	25	1.1788 E-05	1.4098 E-05	1.1694 E-05	1.0857 E-05	1.1340 E-05
	50	1.4660 E-05	1.6142 E-05	1.4724 E-05	1.4088 E-05	1.4369 E-05
15	10	9.9707 E-06	1.1370 E-05	9.0412 E-06	7.6419 E-06	9.2072 E-06
	15	6.1497 E-06	1.0247 E-05	5.6376 E-06	4.9714 E-06	5.5943 E-06
	25	9.0585 E-06	1.2179 E-05	8.8558 E-06	8.3755 E-06	8.6599 E-06
	50	9.7677 E-06	1.2069 E-05	9.7893 E-06	9.3261 E-06	9.4926 E-06
25	10	5.0814 E-07	1.3272 E-06	1.4183 E-07	1.9662 E-07	2.4570 E-07
	15	8.4727 E-06	1.3532 E-05	7.8381 E-06	7.2982 E-06	7.9645 E-06
	25	9.5682 E-06	1.1300 E-05	9.2642 E-06	8.9500 E-06	9.2333 E-06
	50	1.1040 E-05	1.2885 E-05	1.0963 E-05	1.0601 E-05	1.0833 E-05
50	10	1.3383 E-05	1.3125 E-05	1.2451 E-05	1.1885 E-05	1.2828 E-05
	15	9.2560 E-06	1.0545 E-05	8.5513 E-06	8.3627 E-06	8.7997 E-06
	25	8.1926 E-06	1.1255 E-05	7.8034 E-06	7.7241 E-06	7.8977 E-06
	50	1.0395 E-05	1.3701 E-05	1.0248 E-05	1.0080 E-05	1.0231 E-05
		$\alpha = 2$, $\beta = 1.5$, $\lambda = 2.8$, $\mu = 3.6$ R_{GW} =0.7562				
N	M	MLE	MOM	B1	B2	B3
10	10	1.9058E-05	1.7432E-05	1.7755E-05	1.6615E-05	1.7420E-05
	15	2.1717E-05	2.0845E-05	2.1252E-05	1.9665E-05	2.0466E-05
	25	2.8333E-05	2.6488E-05	2.8427E-05	2.6510E-05	2.7388E-05
	50	3.7102E-05	3.5906E-05	3.7448E-05	3.5839E-05	3.6432E-05
15	10	2.9328E-05	2.7121E-05	2.7633E-05	2.6404E-05	2.7830E-05
	15	2.8018E-05	2.7163E-05	2.7141E-05	2.6203E-05	2.6909E-05
	25	2.1318E-05	2.0638E-05	2.1131E-05	1.9927E-05	2.0538E-05
	50	3.5259E-05	3.3721E-05	3.5387E-05	3.4291E-05	3.4730E-05
25	10	1.2618E-05	1.0443E-05	1.0704E-05	1.0881E-05	1.1313E-05
	15	2.8258E-05	2.5211E-05	2.7108E-05	2.6612E-05	2.7277E-05
		1.8174E-05	1.7135E-05	1.7688E-05	1.7102E-05	1.7521E-05
	50	2.4985E-05	2.2984E-05	2.4951E-05	2.4220E-05	2.4556E-05
50	10	3.2987E-05	3.1663E-05	3.0949E-05	3.1005E-05	3.1784E-05
	15	3.2672E-05	2.9997E-05	3.1409E-05	3.1209E-05	3.1842E-05
	25	2.4784E-05	2.2494E-05	2.4049E-05	2.3753E-05	2.4203E-05
	50	3.0445E-05	2.9040E-05	3.0197E-05	2.9864E-05	3.0120E-05

ISSN 0215-9511

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