ESSENTIAL EXTENSION OF TOPOLOGICAL GROUPS

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Abstract : *Our principle aim is to study an algebraic group properties topologically and specially essential extension of topological groups. Among the results in paper are obtained. We show how you can find extension and essential of topological groups by other given topological group.*
Furthermore, new proposition (as we know) are given at the end of this work.

Key Words: *Injective topological group, essential extension of topological group, topological group, direct sum.*

INTRODUCTION

The aim of this work is to study the properties of topological group [2]. We start with simple cases of algebra and topology, but they are very important, so in our study we concerned with injective, essential extension and extension of topological groups.

In the ending of twentieth century they begin to concern with study of topological groups and at the ending of the 40^{th} they concern with study topological group by the scientist Cabaskee that used quotient group as a basic aim and the type of topological metric and his researcher were continued and in his forth research in 1955, he gave the definition of topological group and the partial topological measurement, the number of research like Dikran Dikranjan, Albertto Tonolo and Nilson touch on the conception of topological group [6], and they concerned with metric space, they did not limit any specific research about topological group [3] and they mentioned that in the researcher that study of topological linear space, topological group and linear group space. They fundamental neighborhood systems of zero is partial groups or contains partial groups.

In this paper we study topological groups especially essential extension of topological group [2]. The important point of this paper is to evaluate the direct sum of injective topological groups, and essential of topological groups. We obtained some results of injective and essential extension of topological groups.

1- Topological Groups

In this section we give the fundamental concepts of this work.

Definition 1.1 ***: [4]***

A topological group is a set G together with two structures:

(1) G is a group.

(2) Topology T on a set G

The two structures are compatible, i.e. the group (binary) operation μ: $G \times G \longrightarrow G$, and the inversion law ν: $G \longrightarrow G$ are both continuous maps.

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***Example 1.2* : [2]**

Every group is a topological group with discrete topology.

***Example 1.3* :** [5]

1. The group form topological group with discrete topology.
2. Every abelian group is topological group with discrete group Z.

Definition (1.4): [5]

Let G and G' be two topological groups f : G $\longrightarrow$ G′ is called topological group homomorphism, if:

1. f is group homomorphism.

2. f is continuous.

2- Essential Extension of Topological Group

Definition (2.1): [6]

Let G be topological group, G is injective if M open subgroup of G and f :M $\longrightarrow$ G be homomorphism topological group then f be extension of homomorphism topological group from M to G.

Remark (2.2):

It is easy to see that ker f is topological subgroup of G, where f is homomorphism topological group from G into G, and image of f is a topological subgroup of G'.

Proposition (2.3): [1]

If G be discrete topological group, G is injective then for any subgroup N from topological group G¢, the homomorphism topological group f :N $\longrightarrow$ G extension of G.

Proposition (2.4): [2]

If $\{G_\alpha\}_{1 \le \alpha \le n}$ finite family of injective discrete topological group then the direct sum $\bigoplus_{1\le\alpha\le n} G_\alpha$ is a discrete injective topological group.

Corollary (2.5): [3]

If $\{G_\alpha\}_{\alpha\in"}$ be a family of injective discrete topological group then the direct sum $\bigoplus_{\alpha\in\Delta} G_\alpha$ is injective topological group.

Definition (2.6): [2]

Let G be topological group and N be subgroup of G, then N is called essential subgroup if N has non zero intersection from all subgroup of G.

Definition (2.7): [1]

Let G be topological group and M be subgroup of G, then G is injective extension if G be injective and M be open of G.

Definition (2.8): [4]

A topological module G is called essential extension of subgroup M if M be open of G and any intersection M of all nonzero submodule from G is a non zero.

Proposition (2.9): [6]

If $M_1 \subset M_2 \subset M_3$ where M_1, M_2, M_3 are subgroups of G and M_2 be essential extension of M_1, M_3 be essential extension of M_2 then M_3 be essential extension of M_1.

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Proposition (2.10):

If $G_1 \oplus G_2$ be injective topological group and $G_1 \oplus G_2$ be open subgroup from F then F be the direct sum of $G_1 \oplus G_2$ and discrete subgroup of F.

Proof:

Let $G_1 \oplus G_2$ be open subgroup of F, let $G_1 \oplus G_2$ be injective topological group and I_d: $G_1 \oplus GE_2 \longrightarrow G_1 \oplus G_2$ be homomorphism topological group, thus I_d is extension to homomorphism topological group g: $F \longrightarrow G_1 \oplus G_2$.

Now, let x Î F, thus g(x) Î $G_1 \oplus G_2$, we write $g(x) = g_1(x) \oplus g_2(x)$ and

$$I_d(g(x)) = I_d(g_1(x) \oplus g_2(x))$$
$$= ((g_1 \oplus g_2)^o i)(g_1 \oplus g_2)(x)$$
$$= ((g_1 \oplus g_2)^o i)(g_1(x) \oplus g_2(x))$$

That mean $g(x) = g(g(x))$

$$g_1(x) \oplus g_2(x) = (g_1 \oplus g_2)(g_1(x) \oplus g_2(x))$$

but $g_1 \oplus g_2$ is homomorphism topological module, thus

$$(g_1 \oplus g_2)(x - ((g_1 \oplus g_2)(x)) = 0$$
$$(g_1 \oplus g_2)(x - (g_1(x) \oplus g_2(x))) = 0$$
$$x - (g_1(x) \oplus g_2(x)) \in \ker (g_1 \oplus g_2)$$
$$x - ((g_1 \oplus g_2)(x)) \in \ker (g_1 \oplus g_2)$$
$$x \in (G_1 \oplus G_2) + \ker (g_1 \oplus g_2)$$

From the extension

$$(G_1 \oplus G_2) Ç \ker (g_1 \oplus g_2) = 0$$
$$F = (G_1 \oplus G_2) \oplus \ker (g_1 \oplus g_2)$$

$F = (G_1 \oplus G_2) \oplus \ker g$ and $\ker g$ is discrete.

Proposition (2.11):

If $\underset{1\le\alpha\le n}{\oplus} G_\alpha$ be injective topological group and, $\underset{1\le\alpha\le n}{\oplus} G_\alpha$ be open subgroup from F then F be the direct sum of $\underset{1\le\alpha\le n}{\oplus} G_\alpha$ and discrete subgroup of F.

Proof:

Let $\underset{1\le\alpha\le n}{\oplus} G_\alpha$ be open subgroup and injective of F, let I_d: $\underset{1\le\alpha\le n}{\oplus} G_\alpha \longrightarrow \underset{1\le\alpha\le n}{\oplus} G_\alpha$ be homomorphism topological group, thus I_d is an extension to homomorphism topological group g: $F \longrightarrow \underset{1\le\alpha\le n}{\oplus} G_\alpha$.

Now, let $x \in F$, thus $g(x) \in \underset{1\le\alpha\le n}{\oplus} G_\alpha$, we write

$g(x) = \underset{1\le\alpha\le n}{\oplus} g_\alpha(x)$ and

$$I_d(g(x)) = I_d(\underset{1\le\alpha\le n}{\oplus} g_\alpha(x))$$
$$= (\underset{1\le\alpha\le n}{\oplus} g_\alpha \circ i)(\underset{1\le\alpha\le n}{\oplus} g_\alpha(x))$$

That mean $g(x) = g(g(x))$.

$$\underset{1\le\alpha\le n}{\oplus} g_\alpha(x) = (\underset{1\le\alpha\le n}{\oplus} g_\alpha)(\underset{1\le\alpha\le n}{\oplus} g_\alpha(x))$$

But $\underset{1\le\alpha\le n}{\oplus} g_\alpha$ is homomorphism topological group, thus

$$(\underset{1\le\alpha\le n}{\oplus} g_\alpha)(x - (\underset{1\le\alpha\le n}{\oplus} g_\alpha)(x)) = 0$$
$$(\underset{1\le\alpha\le n}{\oplus} g_\alpha)(x - (g_1(x) \oplus g_1(x) \oplus \ldots \oplus g_n(x))) = 0$$
$$x - (g_1(x) \oplus g_1(x) \oplus \ldots \oplus g_n(x)) \in \ker(\underset{1\le\alpha\le n}{\oplus} g_\alpha)$$
$$x - \underset{1\le\alpha\le n}{\oplus} g_\alpha \in \ker(\underset{1\le\alpha\le n}{\oplus} g_\alpha)$$
$$x \in \underset{1\le\alpha\le n}{\oplus} G_\alpha + \ker(\underset{1\le\alpha\le n}{\oplus} g_\alpha)$$

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Thus, $F = \underset{1\le\alpha\le n}{\oplus} G_\alpha + \ker(\underset{1\le\alpha\le n}{\oplus} g_\alpha)$,

From the extension

$$\underset{1\le\alpha\le n}{\oplus} G_\alpha \cap \ker(\underset{1\le\alpha\le n}{\oplus} g_\alpha) = 0$$

$$F = \underset{1\le\alpha\le n}{\oplus} G_\alpha \oplus \ker(\underset{1\le\alpha\le n}{\oplus} g_\alpha)$$

$$F = \underset{1\le\alpha\le n}{\oplus} G_\alpha \oplus \ker g$$

and ker g is discrete.

<u>Proposition (2.12)</u>:

If $\oplus\{G_a\}_{î„}$ be injective topological group and $\oplus\{G_a\}_{aî„}$ be open subgroup from F then F is the direct sum of $\oplus\{G_a\}_{î„}$ and discrete subgroup of F.

<u>Proof</u>:

Let $\oplus\{G_a\}_{î„}$ be open subgroup and injective topological group of F, let I_d: $\oplus\{G_a\}_{aî„} \longrightarrow \oplus\{G_a\}_{aî„}$ be homomorphism topological group, thus I_d is extension to homomorphism topological group g: F $\longrightarrow \oplus\{G_a\}_{aî„}$.

Now, let x ∈F, thus g(x) ∈ $\oplus\{G_a\}_{aî„}$, we write $g(x) = g_1(x) \oplus g_2(x) \oplus \ldots$ and

$I_d(g(x)) = I_d(g_1(x) \oplus g_2(x) \oplus \ldots) = I_d(g_a(x))_{aî„} = (\oplus\{g_a\}_{aî„})(\oplus\{g_a\}_{aî„})$.

But $\oplus\{g_a\}_{aî„}$ is homomorphism topological group, thus

$(\oplus\{g_a\}_{aî„})(x - (\oplus\{g_a\}_{aî„}(x)) = 0$.

From the extension $\oplus\{G_a\}_{î„}$ Ç ker $(\oplus\{g_a\}_{aî„}) = 0$

$F = \oplus\{G_a\}_{î„} \oplus \ker(\oplus\{g_a\}_{aî„})$

$F = \oplus\{G_a\}_{aî„} \oplus \ker(g)$ and ker(g) is discrete.

<u>Proposition (2.13)</u>:

If $G_1 \oplus G_2$ be open subgroup of F and F be direct sum of $G_1 \oplus G_2$ iff F is an essential extension of $G_1 \oplus G_2$ and $G_1 \oplus G_2 = F$.

<u>Proof</u>:

⇒) Let $G_1 \oplus G_2 \in F$, and F be essential extension of topological group $G_1 \oplus G_2$, let $G_1 \oplus G_2$ be open subgroup of F, F be direct sum of $G_1 \oplus G_2$ and discrete subgroup X of F, thus

$(G_1 \oplus G_2) \oplus X = F$

$(G_1 \oplus G_2) + X = F$

and $(G_1 \oplus G_2) \cap X = 0$

but F be essential extension of $G_1 \oplus G_2$ that mean X = 0 and $F = G_1 \oplus E_2$.

⇐) Let $F = G_1 \oplus G_2$ and F be essential extension of $G_1 \oplus G_2$, thus there exist discrete subgroup X = 0 such that

$G_1 \oplus G_2 \cap X = 0$

$G_1 \oplus G_2 + X = F$

$(G_1 \oplus G_2) \oplus X = F$ where X = ker(g) by proposition (2.10), thus F is discrete sum of $G_1 \oplus G_2$.

<u>Proposition (2.14)</u>:

If $\underset{1\le\alpha\le n}{\oplus} G_\alpha$ be open subgroup of F, F be direct sum of $\underset{1\le\alpha\le n}{\oplus} G_\alpha$ and discrete subgroup of F iff F is an essential extension of $\underset{1\le\alpha\le n}{\oplus} G_\alpha$ and $\underset{1\le\alpha\le n}{\oplus} G_\alpha = F$.

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Proof:

$\Rightarrow$) Let $\underset{1\le\alpha\le n}{\oplus} G_\alpha \subset F$, and F be essential extension of topological group $\underset{1\le\alpha\le n}{\oplus} G_\alpha$, let be open subgroup of F, F be direct sum of $\underset{1\le\alpha\le n}{\oplus} G_\alpha$ and discrete subgroup X of F, thus

$$\underset{1\le\alpha\le n}{\oplus} G_\alpha \oplus X = F$$

$$\underset{1\le\alpha\le n}{\oplus} G_\alpha + X = F$$

and $\underset{1\le\alpha\le n}{\oplus} G_\alpha \cap X = 0$.

But F be essential extension of $\underset{1\le\alpha\le n}{\oplus} G_\alpha$ that mean X = 0 and F = $\underset{1\le\alpha\le n}{\oplus} G_\alpha$.

$\Leftarrow$) Let F = $\underset{1\le\alpha\le n}{\oplus} G_\alpha$ and F be essential extension of $\underset{1\le\alpha\le n}{\oplus} G_\alpha$, thus there exist discrete subgroup X = 0 such that

$$\underset{1\le\alpha\le n}{\oplus} G_\alpha \cap X = 0$$

$\underset{1\le\alpha\le n}{\oplus} G_\alpha \oplus X = F$, where X = ker(g) by proposition (2.11), thus F is discrete sum of $\underset{1\le\alpha\le n}{\oplus} G_\alpha$.

Proposition (2.15):

If $\oplus\{G_a\}_{î„}$ be open subgroup of F, F be direct sum of $\oplus\{G_a\}_{aî„}$ and discrete subgroup of F iff F is an essential extension of $\oplus\{G_a\}_{aî„}$ and $\oplus\{G_a\}_{î„} = F$.

Proof:

$\Rightarrow$) Let $\oplus\{G_a\}_{aî„} \subset F$, and F be essential extension of topological group $\oplus\{G_a\}_{aî„}$, let $\oplus\{G_a\}_{aî„}$ be open subgroup of F, F be direct sum of $\oplus\{G_a\}_{aî„}$ and discrete subgroup X of F, thus

$\oplus\{G_a\}_{aî„} \oplus X = F$

$\oplus\{G_a\}_{aî„} + X = F$

and $\oplus\{G_a\}_{î„} \cap X = 0$

but F be essential extension of $\oplus\{G_a\}_{aî„}$ that mean X = 0 and F = $\oplus\{G_a\}_{a„}$.

$\Leftarrow$) Let F = $\oplus\{G_a\}_{aî„}$ and F be essential extension of $\oplus\{G_a\}_{aî„}$, thus there exist discrete subgroup X = 0 such that

$\oplus\{G_a\}_{aî„} \cap X = 0$

$\oplus\{G_a\}_{aî„} \oplus X = F$, where X = ker(g) by proposition (2.12), thus F is discrete sum of $\oplus\{G_a\}_{aî„}$.

Proposition (2.16):

If $G_1 \oplus G_2$ be topological group and F be essential extension of $G_1 \oplus G_2$ then $G_1 \oplus G_2$ is injective.

Proof:

Let $G_1 \oplus G_2 \neq F$, $G_1 \oplus G_2$ is not essential extension, let F' be injective topological group contains $G_1 \oplus G_2$ where $G_1 \oplus G_2$ be open subgroup of F' and let M be another subgroup of F satisfy $M \cap (G_1 \oplus G_2) = 0$.

Clear, $M + (G_1 \oplus G_2)/M \subseteq F'/M$ special case $M + (G_1 \oplus G_2)/M$ be equivalent topological group of $G_1 \oplus G_2$.

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Claim F'/M be essential extension of M + ($G_1 \oplus G_2$)/M. Let K be subgroup of F' contains $G_1 \oplus G_2$, thus K/M be subgroup of F'/M.

Let (M + ($G_1 \oplus G_2$)/M) $\cap$ K/M = 0

K $\cap$ (($G_1 \oplus G_2$) + M) $\subset$ ($G_1 \oplus G_2$)

K $\cap$ ($G_1 \oplus G_2$) $\subset$ M' $\cap$ ($G_1 \oplus G_2$) = 0.

Thus K $\cap$ ($G_1 \oplus G_2$) = 0, but M is maximal, thus K = M and K/M = 0. So F'/M be essential extension of ($G_1 \oplus G_2$) $\cong$ M + ($G_1 \oplus G_2$)/M.

By hypothesis $G_1 \oplus G_2$ is not essential extension and M + ($G_1 \oplus G_2$)/M, thus F'/M $\cong G_1 \oplus G_2$ $\Rightarrow$ F' = M + ($G_1 \oplus G_2$). Thus $G_1 \oplus G_2$ direct summand of F' and $G_1 \oplus G_2$ is an injective by proposition (2.10).

Proposition (2.17):

If $G_1 \oplus G_2$ be injective topological group, M be open topological subgroup of $G_1 \oplus G_2$ be not contain of another subgroup and $G_1 \oplus G_2$ be essential extension of M then M is an injective topological group.

Proof:

Let F be essential extension of M and $G_1 \oplus G_2$ be injective, there exists injective map f: F $\longrightarrow$ $G_1 \oplus G_2$ but f(F) be subgroup of G contains M and essential extension of M, thus f(F) = M but f is one-to-one, so F = M.

Proposition (2.18):

If $\bigoplus_{1\le\alpha\le n} G_\alpha$ be left topological group, F be essential extension of $\bigoplus_{1\le\alpha\le n} G_\alpha$ then $\bigoplus_{1\le\alpha\le n} G_\alpha$ then $\bigoplus_{1\le\alpha\le n} G_\alpha$ is an injective topological group.

Proof:

Let $\bigoplus_{1\le\alpha\le n} G_\alpha \neq F$, is not essential extension, let F' be injective topological group contains $\bigoplus_{1\le\alpha\le n} G_\alpha$ where $\bigoplus_{1\le\alpha\le n} G_\alpha$ be open subgroup of F$\not\subset$' and let M be another subgroup of F satisfy M $\cap$ $\bigoplus_{1\le\alpha\le n} G_\alpha = 0$.

Clear, M + ($\bigoplus_{1\le\alpha\le n} G_\alpha$)/M $\subseteq$ F′/M special case M + + ($\bigoplus_{1\le\alpha\le n} G_\alpha$)/M be equivalent topological module of $\bigoplus_{1\le\alpha\le n} G_\alpha$.

Claim F'/M be essential extension of M + ($\bigoplus_{1\le\alpha\le n} G_\alpha$)/M. Let K be submodule of F'contains′ $\bigoplus_{1\le\alpha\le n} G_\alpha$, thus K/M be subgroup of F'/M.

Let M + ($\bigoplus_{1\le\alpha\le n} G_\alpha$)/M)$\cap$K/M = 0

K $\cap$ ($\bigoplus_{1\le\alpha\le n} G_\alpha$) + M) $\subset$ $\bigoplus_{1\le\alpha\le n} G_\alpha$

K $\cap$ ($\bigoplus_{1\le\alpha\le n} G_\alpha$) $\subset$ M' $\cap$ ($\bigoplus_{1\le\alpha\le n} G_\alpha$) = 0

Thus K $\cap$ ($\bigoplus_{1\le\alpha\le n} G_\alpha$) = 0, but M is maximal, thus K = M and K/M = 0. So F'/M be essential extension of ($\bigoplus_{1\le\alpha\le n} G_\alpha$) $\cong$ M + ($\bigoplus_{1\le\alpha\le n} G_\alpha$)/M.

By hypothesis $\bigoplus_{1\le\alpha\le n} G_\alpha$ is not essential extension and M + ($\bigoplus_{1\le\alpha\le n} G_\alpha$)/M, thus F'/M$\cong$ $\bigoplus_{1\le\alpha\le n} G_\alpha$ $\Rightarrow$ F' = M + $\bigoplus_{1\le\alpha\le n} G_\alpha$. Thus $\bigoplus_{1\le\alpha\le n} G_\alpha$ direct summand of F' and $\bigoplus_{1\le\alpha\le n} G_\alpha$ be injective by proposition (2.11).

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Proposition (2.19):

If $\underset{1\leq\alpha\leq n}{\oplus} G_\alpha$ be injective topological group and let M be open topological subgroup of $\underset{1\leq\alpha\leq n}{\oplus} G_\alpha$ not contain another subgroup and $\underset{1\leq\alpha\leq n}{\oplus} G_\alpha$ be essential extension of M then M be injective topological group.

Proof:

Let F be essential extension of M and $\underset{1\leq\alpha\leq n}{\oplus} G_\alpha$ be injective, there exists injective map f: F $\longrightarrow$ $\underset{1\leq\alpha\leq n}{\oplus} G_\alpha$ but f (F) be subgroup of $\underset{1\leq\alpha\leq n}{\oplus} G_\alpha$ contains M and essential extension of M, thus f (F) = M, but f is an one-to-one, so F = M.

Proposition (2.20):

If $\oplus\{G_\alpha\}_{\alpha\in"}$ be left topological group and F be essential extension of $\oplus\{G_\alpha\}_{\alpha\in"}$ then $\oplus\{G_\alpha\}_{\alpha\in"}$ be injective topological group.

Proof:

Let $\oplus\{G_\alpha\}_{\alpha\in"} \neq$ F, $\oplus\{G_\alpha\}_{\alpha\in"}$ is not an essential extension, let F' be injective topological group contains $\oplus\{G_\alpha\}_{\alpha\in"}$ where $\oplus\{G_\alpha\}_{\alpha\in"}$ be open subgroup of F' and let M be another subgroup of F satisfy M $\cap \oplus\{G_\alpha\}_{\alpha\in"}$ = 0.

Clear, M + $(\oplus\{G_\alpha\}_{\alpha\in"})$/M $\subseteq$ F'/M special case M + $(\oplus\{G_\alpha\}_{\alpha\in"})$/M be$(\oplus\{G_\alpha\}_{\alpha\in"})$/ equivalent topological group of $\oplus\{G_\alpha\}_{\alpha\in"}$

Claim F'/M be essential extension of M + $(\oplus\{G_a\}_{\alpha\in"})$/M. Let K be submodule of F' contains $\oplus\{G_a\}_{a\hat{\imath}",\alpha\in"}$ thus K/M be subgroup of F'/M.

Let (M + $(\oplus\{G_\alpha\}_{\alpha\in"})$/M)$\cap$K/M = 0

K $\cap$ $(\oplus\{G_a\}_{\alpha\in"})$ + M) $\subset \oplus\{G_a\}_{\alpha\in"}$

K $\cap$ $(\oplus\{G_a\}_{\alpha\in"})$ $\subset$ M' Ç $(\oplus\{G_a\}_{\alpha\in"})$ = 0

Thus K $\cap$ $(\oplus\{G_a\}_{\alpha\in"})$= 0, but M is maximal, thus K = M and K/M = 0. So F¢/M be essential extension of $(\oplus\{G_a\}_{\alpha\in"}) \cong$ M + $(\oplus\{G_a\}_{\alpha\in"})$/M.

By hypothesis $\oplus\{G_a\}_{\alpha\in"}$ is not essential extension and M + $(\oplus\{G_a\}_{\alpha\in"})$/M, thus F'/M $\cong \oplus\{G_a\}_{\alpha\in"} \Rightarrow$ F' = M + $\oplus\{G_a\}_{\alpha\in"}$. Thus $\oplus\{G_a\}_{\alpha\in""}$ direct summand of F' and $\oplus\{G_a\}_{\alpha\in""}$ is an injective topological group by proposition (2.12).

Proposition (2.21):

If $\oplus\{G_a\}_{\alpha\in"}$ be injective topological group, M be open topological subgroup of $\oplus\{G_a\}_{a"}$ not contain another subgroup and $\oplus\{_{Ga}\}_{\hat{\imath}"}$ is essential extension of M then M be injective topological group.

Proof:

Let F be essential extension of M and $\oplus\{G_\alpha\}_{\alpha\in"}$ be injective, there exists injective map f: F $\longrightarrow$ $\oplus\{G_\alpha\}_{\in"}$ but f(F) be subgroup of $\oplus\{G_a\}_{\in"}$ contains M and essential extension of M, thus f (F) = M, but f is one-to-one, so F = M.

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