ON SOMEWHAT #REGULAR GENERALIZED-CONTINUOUS FUNCTIONS AND SOMEWHAT #REGULAR GENERALIZED-IRRESOLUTE FUNCTIONS IN TOPOLOGICAL SPACES

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Abstract: *In this paper, we introduce a new type of somewhat continuous functions called (somewhat #regular generalized-continuous functions and somewhat #regular generalized- irresolute functions) . Also , will be given the relationships of these functions with some other somewhat continuous functions in topological spaces . Furthermore , will be study and proved some of their properties.*

INTRODUCTION

The concepts of somewhat continuous functions was first introduce and investigated by Gentry and Hoyle [14]. After the introduction of somewhat continuous functions, there are many research papers which deal with different types of somewhat continuous functions. Benchalli and Priyanka [8] , Balasubramanian and Chaitahya [3] , Balasubramanian et al [4], [7] , Balasubramanian and Sandya [5] , [6] , Sreeja and Janaki [26]. They introduced and study somewhat b- continuous functions, somewhat ±g- continuous functions, somewhat gpr- continuous functions, somewhat g±- continuous functions, somewhat GS- continuous functions, somewhat rg- continuous functions, somewhat Àgb- continuous functions respectively.

While, the concepts (#rg- closed sets , #rg-open sets , #RG- continuous functions and #RG- irresolute functions)were discussed and introduced by (Syed Ali Fathima and Mariasingam, 2012, in [28],[29]).

In this work, we introduce a new types of somewhat continuous functions which are (somewhat #regular generalized-continuous functions and somewhat #regular generalized- irresolute functions) in topological spaces .Moreover, will be study the characterizations and basic properties of these functions, Also, we give the relation among them.

Throughout this paper (X,t) and (Y,s) (or simply X and Y) represent non-empty topological spaces and the family of all #rg-open (resp . rg-open , gpr-open , gs-open, ±g-open,g±-open , Àg-open , Àgb-open) set of a space (X,Ä) denoted by # RGO(X,Ä) (resp.RGO(X,Ä),GPRO(X,Ä), GSO(X,Ä) , ±GO(X,Ä) , G±O(X,Ä) , ÀGO(X,Ä), ÀGBO(X,Ä) For a sub set A of a space X.**cl (A)**, **int (A)** and **A^c** denoted the closure of A, the interior of A and the complement of A in X respectively.

PRELIMINARIES

Some definitions and basic concepts related to this paper .

Definition 1 :

A subset **A** of a topological space (X,t) is said to a :

1- **semi- open set** [17] if $A \subseteq cl(int(A))$ and **semi- closed set** if int(cL(A)) $\subseteq A$.

2- **±-open se**t [22] if $A \subseteq int(cl(int(A)))$ and ±-**closed set** if $cl(int(cl(A))) \subseteq A$.

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ISSN 0215-9511

3- **Preopen set**[20] if A⊆ int(cl(A)) and **preclosed** if cl(int(A) A.

4- **regular open** [26] if A = int(cl(A)) and re**gular closed** if A = cl(int(A)) .

5- **regular semi open set** [10] if there is a regular open set U in (X,t) such that U ⊆ A ⊆ cl(U).

6- **À-open** set [30] if A is the union of regular open sets .

7- **b-open set** [1] if A ⊆ int(cl(A)) ∪ cl(int(A)) and **b- closed set** if int(cl(A)) ∩ cl(int(A)) ⊆ **A.**

The intersection of all -closed (resp. ±- closed, preclosed and b-closed) subsets of (X,Ä) containing a set A is called semi-closure **(**resp. ±-closure, pre-closure and b- closure) of A and is denoted by **scl(A), (±cl(A), pcl(A) and bcl(A))** respectively .

Definition 2

A subset **A** of a topological space (X,t) is said to a :

1- **generalized closed set** (briefly , g- closed) [16] if cl(A) ⊆ U whenever A⊆ U and U is open set in (X,Ä) .

2- **generalized semi-closed set** (briefly , gs- closed) [2] if scl(A) ⊆ U whenever A and U is open set in (X,Ä) .

3- **generalized ±- closed set** (briefly , g±- closed) [18] if ±cl(A) ⊆ U whenever A⊆ U and U is ±-open set in (X,Ä).

4- **±-generalized closed set** (briefly ,±g- closed) [19] if ±cl(A) ⊆ U whenever A and U is open set in (X,Ä) .

5- **generalized pre regular closed set** (briefly , gpr- closed) [14] if pcl(A) ⊆ U whenever A and U is regular open set in (X,Ä) .

6- **regular generalized closed set** (briefly, rg-closed) [23] if cl(A) ⊆ U whenever A⊆ U and U is regular open set in (X,Ä) .

7- **Àg-closed set** [13] if cl(A) ⊆ U whenever A⊆ U and U is À-open set in (X,Ä) .

8- **Àgb-closed set** [25] if bcl(A) ⊆ U whenever A⊆ U and U is À-open set in (X,Ä)

9- **rw- closed set** [9] if cl(A) ⊆ U whenever A⊆ U and U is regular semi-open set in (X,Ä) .

10- **#rg- closed set** [28] if cl(A) ⊆ U whenever A⊆ U and U is rw- open set in (X, Ä)

The complement of a g-closed (resp. gs-closed , g±-closed ,±g-closed , gpr-closed ,rg-closed, Àg-closed, Àgb-closed , rw –closed and #rg -closed) sets **is called** a g-open (resp. gs-open ,g±-open ,±g-open , gpr-open ,rg-open, Àg-open, Àgb-open , rw –open and #rg-open) sets .

Definition 3 :

A topological spaces (X,Ä) is said to be a :

1- **$T^*_{1/2}$ – space** [23] if every rg- closed sets in (X,Ä) is closed set .

2- **T_b — space** [12] if every gs- closed sets in (X,Ä) is closed set .

3- **αT_b -space** [11] if every ±g- closed sets in (X,Ä) is closed set .

4- **Àgb-space** [25] if every Àgb- closed sets in (X,Ä) is closed set .

5- **$T_{\#rg}$-space** [29] if every #rg - closed sets in (X,Ä) is closed set .

6- **submaximal space**[21] if every preopen (preclosed) set in (X,Ä) is open (closed)set.

7- **pre-regular $T_{1/2}$ - space** [15] if every gpr-closed sets in (X,Ä) is preclosed set.

Definition 4:

A function *f : (X ,Ä) '! (Y ,Ã)* is said to be a :

1. **Regular- continuous** [24] if the inverse image every open (closed) set in(Y,Ã) is a regular open (regular closed) set in (X,Ä) .
2. **#rg- continuous** [29] if the inverse image of every open (closed) set in(Y,Ã) is a #rg -open (#rg-closed) set in (X,Ä) .
3. **#rg- irresolute** [29] if the inverse image of every #rg- open (#rg-closed) set in(Y,Ã) is a #rg -open (#rg-closed) set in (X,Ä) .

Definition 5:

A function *f : (X ,Ä) '! (Y ,Ã)* is said to be a :

1. **Somewhat-continuous** [14] if for U ∈ Ã and f^{-1}(U) ≠ ∅ , there exists a non empty open set Vin (X,Ä) such that V ⊆ f^{-1}(U).
2. **Somewhat b-continuous** [8] if for U ∈ Ã and f^{-1}(U) ≠ ∅, there exists a non empty b-open set Vin (X,Ä) such that V ⊆ (U).
3. **Somewhat rg-continuous** [6] if for U ∈ Ã and f^{-1}(U) ≠ ∅ , there exists a non empty rg-open set Vin (X,Ä) such that V ⊆ f^{-1}(U).
4. **Somewhat gpr-continuous** [4] if for U ∈ Ã and f^{-1}(U) ≠ ∅ , there exists a non empty gpr-open set Vin (X,Ä) such that V ⊆ (U).
5. **Somewhat Àgb-continuous** [26] if for U ∈ Ã and f^{-1}(U) ≠ ∅ , there exists a non empty Àgb-open set Vin (X,Ä) such that V ⊆ (U).
6. **Somewhat gs-continuous** [5] if for U ∈ Ã and f^{-1}(U) ≠ ∅ , there exists a non empty gs-open set Vin (X,Ä) such that V ⊆ (U).
7. **Somewhat ±g-continuous** [3] if for U ∈ Ã and f^{-1}(U) ≠ ∅ , there exists a non empty ±g-open set Vin (X,Ä) such that V ⊆ (U)) .
8. **Somewhat g±-continuous** [7] if for U ∈ Ã and f^{-1}(U) ≠ ∅, there exists a non empty g±-open set Vin (X,Ä) such that V ⊆ f^{-1}(U) .

On Somewhat #Regular Generalized- Continuous Functions

In this section ,we introduce a new type of somewhat –continuous which is somewhat #regular generalized-continuous functions , and will be find the relation between these functions with some other somewhat continuous- functions and we study some of their properties.

Definition 1:

A function *f* :(X,Ä)'!(Y,Ã) is said to be **Somewhat #regular generalized –continuous** (briefly , **somewhat #rg -continuous**) if for U ∈ Ã and f^{-1}(U) ≠ ∅ , there exists a non empty #rg- open set V in (X,Ä) such that V ⊆ f^{-1}(U) .

Proposition 1:

If *f* : (X ,Ä) --> (Y ,Ã) is somewhat -continuous function , then *f* is a somewhat #rg –continuous.

Proof:

Let U be an open set in (Y,Ã) such that f^{-1}(U) ≠ ∅. Since *f* is a somewhat -continuous function , then there exists a non empty open set V in (X,Ä) such that V⊆ f^{-1}(U). Since (Every open set is #rg-open , [28]) . Thus , Vis a #rg-open set in (X,Ä) such that V⊆ f^{-1}(U). Hence, *f* is a somewhat #rg – continuous.

Corollary1 :

(i) Every continuous function is somewhat #rg – continuous.

(ii) Every #rg-continuous function is somewhat #rg –continuous.

ISSN 0215-9511

Proof:

(i) It follows from the fact (Every continuous function is somewhat – continuous , [14]) and proposition 1).

(ii) It is clear from definition 1.

The following example shows the converse of above proposition and corollary need not be true in general**.**

Example 1 :

(i) Let X={a, b ,c ,d},Ä={X,Ø,{a},{b},{a ,b}},and #RGO(X, Ä)= {X, Ø,{a},{b},{c},{d},{a ,b},{a,c},{a ,d},{b ,c},{b ,d}}. The map *f* : (X ,Ä) '! (X ,Ä) is defined as *f* (a)= d , *f*(b) =b, *f* (c) =a and *f* (d)=c. Then clearly *f* is somewhat #rg –continuous , but *f* is not somewhat –continuous. Since for open set U={a} in (X ,Ä), f^{-1}(U)= f^{-1}({a})={c} . It is observe that there is no non empty open set V in (X ,Ä) such that V ⊆ f^{-1}(U)={ c}.

(ii) Let X = Y = {a, b, c}, Ä={X,$,{a}}, Ã={Y,Ø,{a,c}} and RGO (X, Ä)={X, $,{a}}. Define function *f* : (X , Ä) --> (Y , Ã) by *f* (a) = a, *f* (b) = c and, *f* (c) = b. Then clearly *f* is somewhat #rg-continuous function, but *f* is not #rg-continuous function, Since for open set U={a,b} in (Y,Ã). f^{-1}(U)= f^{-1}({a,,b})={a,c}is not #rg-open set V in (X , Ä).

Proposition 2 :

If *f* : (X ,Ä) --> (Y ,Ã) is somewhat #rg –continuous function , then *f* is a

(i) somewhat gs –continuous function

(ii) somewhat Àgb–continuous function

(iii) somewhat ±g–continuous function

(iv) somewhat rg –continuous function

(v) somewhat gpr –continuous function

Proof:

(i) Let U be an open set in (Y ,Ã) such that f^{-1}(U) ≠ Ø. Since *f* is a somewhat #rg-continuous function , then there exists a non empty #rg- open set V in (X,Ä) such that V⊆ f^{-1}(U). Since (Every #rg-open set is g-open, [28] and every g-open set is gs-open, [2]). Thus ,Vis a non empty gs-open set in (X, Ä) such that V⊆ f^{-1}(U). Hence, *f* is a somewhat gs – continuous.

(ii) Let U be an open set in (Y ,Ã) such that (U). Since *f* is a somewhat #rg-continuous function, then there exists a non empty #rg- open set V in (X,Ä) such that VSince (Every #rg-open set is Àg-open, [28] and every Àg-open set is Àgb-open,[25]).Thus ,V is a non empty Àgb-open set in (X,Ä) such that VHence, *f* is a somewhat Àgb –continuous.

The proof of step –iii-,-iv-and-v- are similar to step-i-and –ii-.

The following examples shows the converse of proposition(3-2) need not be true in general.

Example2:

Let X=Y={a,b,c}, Ä={ X, $,{a},{a,c}}, Ã={ Y, Ø,{b}} , RGO(X,Ä)={ X, $,{a},{a,c}} and GSO(X,Ä) = ±GO(X,Ä) = { X,$,{a},{c},{a,b},{a,c}} . Define function *f* : (X ,Ä) '! (Y ,Ã) by *f* (a)= a, *f* (b) =c and, *f* (c) =b. Then clearly *f* is (somewhat gs-continuous function and somewhat ±g-continuous function) , but f is not somewhat #rg-continuous function, Since for open set U={b} in (Y ,Ã). f^{-1}(U)= f^{-1}({b})={c}. It is observe that there is no non empty #rg- open set V in (X ,Ä) such that V⊆ (U)={c} .

Example3:

Let X={a,b,c}, Ä={ X, $,{a}}, RGO(X,Ä)={ X, $,{a}} and RGO(X,Ä) =GPRO(X , Ä) ={ X,$,{a} ,{b},{c},{a, b},{a, c},{b ,c}} . Define function *f*: (X ,Ä) '! (Y ,Ã) by *f* (a)= c, *f* (b) =a and *f* (c) =b. Then clearly *f* is (somewhat rg-continuous and somewhat gpr-continuous), but *f* is not somewhat #rg-continuous function, Since for open set U={a} in (Y ,Ã). (U)=({a})={b}. It is observe that there is no non empty # rg- open set V in (X ,Ä) such that V (U)={b}.

Example 4:

Let X=Y={a,b,c}, Ä={X,$,{a}}, Ã={Y,∅,{b}}, #RGO(X,Ä) = {X,$,{a}} and ÀGBO(X,Ä) = {X,$, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}}. Define function *f*: (X ,Ä) --> (Y ,Ã) by *f* (a)= c, *f* (b) = b and, *f* (c) = a. Then clearly *f* is somewhat Àgb-continuous function, but *f* is not somewhat #rg-continuous function, Since for open set U={b} in (Y ,Ã). f^{-1}(U) = f^{-1}({b}) ={b}. It is observe that there is no non empty #rg- open set V in (X , Ä) such that V⊆ (U) = {b}.

Remark 1:

The concepts of somewhat b- continuous function and somewhat g±-continuous function are independent to somewhat #rg- continuous function. As shows in the following examples.

Example5:

(i) Let X = {a ,b ,c ,d} , Ä = { X, $,{a},{b},{a ,b}}# RGO (X,Ä) = { X, $, {a}, {b}, {c}, {d}, {a ,b}, {a, c}, {a , d},{b,c}, {b,d}} and the b-open sets in (X,Ä) are {X,$, {a}, {b}, {a,b}, {a,c}, {a,d}, {b,c}, {b,d}, {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}}. Define function *f* : (X ,Ä) --> (X ,Ä) by *f* (a) = c, *f* (b) = d , *f* (c) = a and *f* (d)=b. Then clearly *f* is somewhat #rg-continuous function, but *f* is not somewhat b-continuous function . Since for open set U={a} in (X ,Ä). f^{-1}(U)= f^{-1}({a})={c}. It is observe that there is no non empty b- open set V in (X ,Ä) such that V⊆ (U)={c} .

(ii) Let X=Y={a ,b ,c ,d}, Ä={X,$}, Ã = {Y, $,{a},{b},{a ,b}}, #RGO(X ,Ä) = { X, $} and b-open sets in (X,Ä) are {{a} ,{b},{c} ,{d} ,{a ,b} ,{a ,c} ,{a ,d} ,{b ,c},{b, d}, {c, d}, {a , b ,c},{a ,b ,d},{a ,c ,d},{b ,c ,d}} . Define function *f* : (X ,Ä) '! (Y ,Ã) by *f* (a)= b, *f* (b) =c , *f* (c) =d and *f* (d)=a.. Then clearly *f* is somewhat b- continuous ,but f is not somewhat #rg -continuous function, Since for open set U={b} in (Y ,Ã). (U)=({b})={a}. It is observe that there is no non empty #rg- open set V in (X, Ä) such that V (U)={a}.

Example 6:

(i) Let X={a ,b ,c ,d},Ä={X,$,{a},{b},{a ,b},{a ,b ,c}}, #RGO(X ,Ä)={X,$,{a},{b},{c},{a ,b},{a ,c},{b ,c},{a ,b ,c}} and G±O(X ,Ä)={X,$,{a} ,{b},{a ,b},{a ,b ,c},{a ,b ,d}} . Define function *f*: (X ,Ä) '! (X ,Ä) by *f* (a)= c, *f* (b) =d and *f*(c) =a and *f* (d)=b.. Then clearly *f* is somewhat #rg-continuous function, but *f* is not somewhat g±- continuous function .Since for open set U={a} in (X ,Ä). f^{-1}(U)= f^{-1}({a})={c}. It is observe that there is no non empty g± -open set V in (X ,Ä) such that V⊆ (U)={c} .

(ii) Let X=Y={a ,b, c}, Ä={X,$,{a},{b ,c}}, Ã={Y,$,{a}}, #RGO(X,Ä)= {X,$,{a},{b ,c}} , and G±O(X ,Ä)={X,$,{a},{b},{c},{a, b},{a ,c},{b ,c}}. Define function *f*: (X ,Ä) '! (Y ,Ã) by *f*(a)= b, *f* (b) =a and *f* (c) =c. Then clearly *f* is somewhat g±-continuous function, but *f* is not somewhat #rg-continuous function, Since for open set U={a}in (Y,Ã) , (U)=({a})={b}. It is observe that there is no non empty #rg- open set V in (X, Ä) such that V (U)={b}.

Next, we introduce some new results, that shall needed in this work:

Proposition 3:

If a space (X,Ä) is a

(i) $T^*_{1/2}$ – space , then every rg-open set in (X,Ä) is an open set.

(ii) T_b – space , then every gs-open set in (X,Ä) is an open set.

(iii) αT_b -space , then every ±g-open set in (X,Ä) is an open set.

(iv) πgb – space , then every Àgb-open set in (X,Ä) is an open set.

(v) $T_{\#rg}$-space , then every #rg-open set in (X,Ä) is an open set.

Proof:

Let U be a rg-open set in (X,Ä) . Then U^c is a rg-closed set in (X,Ä) . Since (X,Ä) is – space and by using definition (2-3) step-1- we have is a closed set in (X,Ä) . Thus , U is an open set in (X,Ä).

The proof of step-ii- , -iii-, -iv- and step-v- are similar to step-i- .

Corollary 2:

If a space (X ,Ä) is a αT_b -space , then every g±-open set in (X ,Ä) is an open set.

Proof:

Let U is a g±-open set in (X ,Ä). Since (Every g±-open set (X ,Ä) is ±g-open set ,[11]) . Then U is a ±g-open set in(X ,Ä). Since (X ,Ä) is – space and by using proposition(3-3)step-iii- we get U is an open set in (X ,Ä) .

Proposition 4:

Let (X ,Ä) be a submaximal and pre-regular-space . Then every gpr-open set in (X, Ä) is an open set.

Proof:

Let U be a gpr-open set in (X, Ä) . Then is a gpr-closed set in (X ,Ä) . Since (X ,Ä) is pre-regular-space and by using definition (2-3) step-7- we have is a preclosed set in (X ,Ä) . Also , since (X ,Ä) is a submaximal space and by using definition (2-3) step-6- we get is a closed set in (X ,Ä) . Thus, U is an open set in (X ,Ä) .

The following propositions give the condition to make the propositions (3-1),(3-2) and Remark(3-1) are true :

Proposition 5:

If *f* : (X ,Ä) '! (Y ,Ã) is somewhat #rg – continuous function and X is a $T_{\#rg}$-space . Then *f* is a somewhat –continuous .

Proof:

Let U be an open set in (Y,Ã) such that f^{-1}(U) $\neq \emptyset$. Since *f* is a somewhat #rg-continuous function , then there exists a non empty #rg- open set V in (X ,Ä) such that V$\subseteq f^{-1}(U)$. Since X is a -space . Then , by using proposition (3-3) step-v- we have Vis an open set in (X,Ä) such that VHence, *f* is a somewhat -continuous .

ISSN 0215-9511

Proposition 6:

Let f : (X ,Ä) '! (Y ,Ã) be any function , then f is somewhat #rg –continuous function if (X ,Ä) is a

(i) T_b-space and f is a somewhat gs –continuous.

(ii) $\pi gb-space$. and f is a somewhat Àgb – continuous.

(iii) αT_b -space and f is a somewhat ±g –continuous

(iv) $T^*_{1/2}$-space and f is a somewhat rg –continuous

Proof:

(i) Let f : (X ,Ä) --> (Y ,Ã) be a somewhat gs –continuous function and U be an open set in (Y , Ã) such that f^{-1}(U) $\neq \emptyset$. Since f is a somewhat gs-continuous function , then there exists a non empty gs- open set V in (X,Ä) such that V$\subseteq f^{-1}(U)$. Since X is T_b-space,by using proposition(3-3)step-ii- we get V is an open set in (X,Ä). Also , since (Every open set is #rg-open set ,[28]).Thus, V is a #rg-open set in (X , Ä), such that V $\subseteq f^{-1}(U)$. Hence, f is a somewhat #rg-continuous.

The proof of step-ii- , -iii-, and step-iv- are similar to step-i- .

Proposition 7:

Let f : (X ,Ä) '! (Y ,Ã) be a somewhat gpr-continuous function and a space (X ,Ä) be a submaximal and pre-regular-$T_{1/2}$ space, then f is somewhat #rg –continuous function .

Proof:

Let U be an open set in (Y ,Ã) such that f^{-1}(U) $\neq \emptyset$. Since f is a somewhat gpr-continuous function , then there exists a non empty gpr- open set V in (X ,Ä) such that V$\subseteq f^{-1}(U)$. Since X is a submaximal and pre-regular- space, then by proposition(3-4) we get Vis an open set in (X ,Ä) . Also ,since (Every open set is a #rg-open ,[28]) .Thus, V is a #rg-open set such that VHence, f is a somewhat #rg -continuous.

Proposition 8:

Let (X,Ä) be $T_{\#rg}$-space and Àgb-space .Then a function f : (X ,Ä) ---> (Y ,Ã) is somewhat #rg–continuous function if and only if f is somewhat b –continuous function.

Proof:

Suppose that f is a somewhat #rg –continuous function and let U be an open set in (Y ,Ã) such that f^{-1}(U) $\neq \emptyset$. Since f is a somewhat #rg-continuous function , then there exists a non empty #rg-open set V in (X ,Ä) such that V$\subseteq f^{-1}(U)$. By hypotheses X is a -space, and by proposition(3-3) step-v- we get Vis an open set in (X , Ä) and since (Every open set is a b-open , [1]).Thus, V is a b-open set such that VHence, f is a somewhat b -continuous function . **Conversely,** assume that f is a somewhat b -continuous function , then there exists a non empty b-open set V in (X ,Ä) such that Vand also, since (Every b-open set is a Àgb-open, [25]) Thus ,V is a Àgb-open set in (X,Ä). By hypotheses X is a Àgb-space and by proposition(3-3) step-iv- we get Vis an open set in (X ,Ä) , and since (Every open set is a #rg-open [28]).Thus , V is a #rg-open set such that VHence, f is a somewhat #rg -continuous function.

Similarly, we proof the following proposition**:**

Proposition 9:

Let (X ,Ä)be a $T_{\#rg}$-space and αT_b –space .Then a function f : (X ,Ä) '! (Y ,Ã) is somewhat #rg –continuous function if and only if f is somewhat g± –continuous function .

Next, will be define #rg-dense set in a space (X,Ä) , that will be needed in this paper.

ISSN 0215-9511

Definition 2:

A subset M of a space(X,Ä) is called #rg-dense set if there is no proper #rg-closed set C in (X,Ä) such that M ⊂ C ⊂ X .

Proposition 10:

For surjective function *f* : (X ,Ä) '! (Y ,Ã) , the following are equivalent :

(i) *f* is somewhat#rg –continuous function .

(ii) If C is a closed subset of (Y ,Ã) such that f^{-1}(C) $\neq X$ .Then there is a proper #rg-closed subset D of (X ,Ä) such that $f^{-1}(C) \subseteq D$.

(iii) If M is a #rg-dense subset in (X ,Ä) , then *f* (M) is a dense subset in (Y,Ã) .

Proof:

(i)'!(ii):Let C is a closed subset of (Y ,Ã) such that f^{-1}(C) $\neq X$ .Then C^c is an open subset in (Y,Ã) such that (=(=X-(C) . By step-i- there exists a non empty #rg-open set V in (X ,Ä) such that V= (=X-(C) . This implies that *(*C) ⊂ X-C and let D=X-C , then D is a proper #rg-closed subset of (X,Ä).

(ii)'!(i):Let U ∈ σ and $f^{-1}(U) \neq \emptyset$. Then U^c is a closed subset in (Y,Ã) such that $f^{-1}(U^c)$ = $f^{-1}(Y-U)$=X-(U) . By step-ii- there is a proper subset #rg-closed subset D in (X ,Ä) such that = (=X-(U) ⊂ D. Thus, X-D ⊂ (D) and X-D is a #rg-open subset of (X,Ä) such that X-D . therefore, *f* is a somewhat #rg –continuous function .

(ii)-->(iii):Let M be a #rg-dense subset in (X ,Ä). Suppose that *f* (M) is not dense in(Y,Ã) there exists a proper closed subset C in (Y ,Ã) such that *f* (M) ⊂ C ⊂ Y .Clearly f^{-1}*(*C) $\neq X$. By step-ii- there exists a proper #rg-closed subset D in (X,Ä), such that M ⊂ (C) ⊂ D ⊂ X (which is contradiction).Since M is a #rg-dense subset in (X ,Ä) .

(iii)'!(ii):Suppose that step-ii- is not true , then there is a closed subset C in (Y,Ã) such that f^{-1}(C) $\neq X$, but there is no proper #rg-closed set D in (X,Ä) , such that (C) ⊂ D . this means that *(*C) is a #rg-dense in (X,Ä) . But by step-ii-we have *f((*C))=C must be dense in(Y,Ã) , which is contradiction to the choice of C .

Next, we give some propositions about the composition of Somewhat #rg –continuous functions:

Proposition 11 :

If *f* : (X ,Ä) '!(Y ,Ã) is a somewhat #rg- continuous function and *g:* (Y , Ã) '!(Z,μ) is continuous function. Then *g¿f* : (X ,Ä) '!(Z ,μ) is somewhat #rg- continuous function.

Proof:

Let U be an open set in (Z ,μ) such that $(gof)^{-1}$*(*U) $\neq \emptyset$ .Since *g* is a continuous function . Then g^{-1}(U) is an open set (Y ,Ã) . By hypotheses *f* is a somewhat #rg- continuous function , then there exists a non-empty #rg-open set V in (X ,Ä) such that V ⊆ f^{-1}*((*U)) . But *((U)*) = (U) . Hence , V (U) . Therefore , *g¿f* is somewhat #rg- continuous function.

Similarly, we proof the following corollary **:**

Corollary 3:

If *f* : (X ,Ä) '!(Y ,Ã) is a somewhat #rg- continuous function and *g*: (Y , Ã) '!(Z,μ) is regular-continuous function. Then *g¿f* : (X ,Ä) '!(Z ,μ) is somewhat #rg- continuous function .

Remark 2:

In the above proposition _(i)if *f* is continuous (or #rg-continuous) function and *g* is a somewhat #rg-continuous function , then is not necessarily *g¿f* is somewhat #rg- continuous . (ii)if *f* and *g* are two somewhat #rg- continuous function , then is not necessarily *g¿f* is somewhat #rg- continuous.

The following examples serves this purpose :

Example 7:

Let X=Y= Z={a ,b ,c ,d}, Ä={X, $,{a},{b},{a ,b},{ a ,b ,c}} ,Ã= {Y, $,{a},{b},{a, b}, µ= {Z, $,{a},{b},{a,b}}. Define a function *f* : (X ,Ä) --> (Y ,Ã) by *f* (a)=a, *f* (b)=b , *f* (c)=c, *f*(d)=d. and define *g*: (Y , Ã) --> (Z,µ) by *g*(a)=d ,*g*(b)=b ,*g*(c)=c and *g*(d)=a. Then, clearly *f* is continuous(and somewhat #rg- continuous) function ,and *g* is a somewhat #rg- continuous , but *g¿f* is not somewhat #rg- continuous. Since for #rg-open set U={a} in (Z ,µ). $(gof)^{-1}$(U)= f^{-1}(g^{-1}(U)) = (({a})) = ({d})={d} . It is observe that there is no non empty #rg-open set V in (X ,Ä) such that V (U)= ((U))={d}.

Example(3-8):

Let X=Y= Z={a ,b ,c ,d}, Ä={X, $,{a},{b},{a ,b},{ a ,b,c}} ,Ã= {Y, $,{a},{b},{a, b}, µ= {Z, $,{a}}. Let *f* : (X ,Ä) '! (Y ,Ã) be the identity function , define *g*: (Y , Ã) '!(Z,µ) by *g*(a)=b*, g*(b)=d ,*g*(c)=c and *g*(d)=a. Then, clearly *f* and g are a somewhat #rg- continuous, but *g¿f* is not somewhat #rg- continuous . Since for #rg-open set U={a} in (Z ,µ). (U)= ((U)) = (({a})) = ({d})={d}. It is observe that there is no non empty #rg-open set V in (X ,Ä) such that V (U)= ((U))={d}.

4- Somewhat #Regular Generalized - Irresolute Functions:

In this section , will be given other type of somewhat #rg –continuous functions called somewhat #rg – irresolute functions.

Definition 1:

A function *f* : (X ,Ä) '! (Y ,Ã) is said to be **Somewhat #regular generalized –irresolute** (briefly , **somewhat #rg- irresolute**) if for U #RGO(Y,Ã) and (U) , there exists a non empty #rg- open set Vin (X,Ä) such that V (U) .

Proposition 1:

If *f* : (X ,Ä) '! (Y ,Ã) is #rg-irresolute function .Then *f* is a somewhat #rg-irresolute.

Proof:

It is clear from definition(4-1).

The following examples shows the converse of proposition(4-1) need not be true in general .

Example1:

Let X=Y={a ,b ,c}, Ä={ X,$,{a}}, Ã={Y, Ø ,{a ,b}},#RGO(X ,Ä)={ X, $,{a}}and #RGO(Y,Ã)={Y, $,{a ,b}}. Then the identity function *f* : (X ,Ä) '! (Y ,Ã) is somewhat #rg-irresolute function , but *f* is not #rg-irresolute function . Since for #rg-open set U={a,b} in (Y,Ã), f^{-1}(U)= f^{-1}({a,b})={a,b}is not #rg-open set in (X,Ä).

Proposition 2:

Every somewhat #rg-irresolute function is somewhat #rg continuous-function .

Proof:

Let *f* : (X ,Ä) '! (Y ,Ã) is somewhat #rg-irresolute function and let U a be an open set in (Y,Ã) such that (U) . Since (Every open set is an #rg-open , [28]) Thus , U is a #rg-open set in(Y ,Ã) . Since *f* is a somewhat #rg-irresolute function . Then there exists a non empty #rg-open set V in (X ,Ä) such that Vence, *f* is a somewhat #rg continuous function.

Corollary 1:

If *f* : (X ,Ä) '!(Y ,Ã) is somewhat #rg –irresolute function ,then *f* is a

(i) somewhat gs –continuous function.

(ii) somewhat Àgb–continuous function.

(iii) somewhat ±g–continuous function.

(iv) somewhat rg –continuous function .

(v) somewhat gpr –continuous function.

Proof:

It follows from proposition(4-2) and proposition(3-2). The following examples show that the converse of Proposition(4-2) and corollary(4-1) need not be true in general :

Example 2:

Let X=Y={a ,b ,c ,d}, Ä={X, $,{a},{b},{a ,b},{a ,b ,c}} ,Ã={Y, $,{a},{b},{a, b}}, #RGO(X ,Ä)={X,$,{a},{b},{c},{a ,b},{a ,c},{b ,c},{a ,b ,c}} and #RGO(X ,Ã) ={X,$,{a} ,{b}, {c},{d},{a ,b}, {a ,c},{a ,d},{b ,c},{b ,d}} . Then the identity function *f* : (X ,Ä) '! (Y ,Ã) is a somewhat #rg-continuous function, but *f* is not somewhat #rg- irresolute function .Since for #rg-open set U={d} in (Y ,Ã). f^{-1}(U)= f^{-1}({d})={d}. It is observe that there is no non empty #rg-open set V in (X ,Ä) such that V⊆ (U)={d}.

Example 3:

Let X=Y={a ,b ,c}, Ä={X,$,{a},{b ,c}}, Ã ={Y,Ø,{b}}, #RGO(X,Ä) ={X, $, {a},{b ,c}}, #RGO(Y ,Ã) = {Y,$,{b}} , and GSO(X ,Ä) = ±GO (X,Ä) = {X,$, {a},{b},{c},{a ,b},{a ,c},{b ,c}}. Define a function *f* :(X,Ä) '!(Y,Ã) by *f* (a)=c *f(b)=b* and *f* (c)=a Then clearly *f* is somewhat ±g- continuous function and somewhat gs- continuous function. but *f* is not somewhat #rg- irresolute function. Since for #rg-open set U={b} in (Y ,Ã). f^{-1}(U)= f^{-1}({b})={b}. It is observe that there is no non empty #rg-open set V in (X ,Ä) such that V⊆ (U)={b}.

Example 4:

Let X=Y={a ,b ,c}, Ä= {X,$,{a},{b},{a ,b}}, Ã={Y,Ø,{a}}, #RGO(X ,Ä)={X, $,{a},{b},{a ,b}},#RGO(Y ,Ã)={Y,$,{a}}, and RGO(X, Ä)= GPRO (X ,Ä)={X,$,{a},{b},{c},{a ,b}} . Define a function *f* : (X ,Ä) '! (Y ,Ã) by *f*(a)=c , *f*(b)=b and *f*(c)=a Then clearly *f* is somewhat rg- continuous function and somewhat gpr- continuous function , but *f* is not somewhat #rg- irresolute function . Since for #rg-open set U={a} in (Y ,Ã). f^{-1}(U)= f^{-1}({a})={c}. It is observe that there is no non empty #rg-open set V in (X ,Ä) such that V⊆ (U)={c}.

Example 5:

Let X={a ,b ,c ,d},Y={a ,b, c}, Ä ={X,$},Ã ={Y,Ø,{a}},#RGO(X,Ä) ={X,$}, #RGO(Y ,Ã)={Y,$,{a}}, and ÀGBO(X ,Ä) = {X,$, {a},{b},{c},{d},{a ,b},{a ,c},{a ,d},{b ,c},{b ,d},{c ,d},{a ,b ,c},{a ,b ,d},{a ,c, d},{b ,c ,d}} . Define a function *f* : (X ,Ä) --> (Y ,Ã) by *f*(a)=a , *f*(b)= *f*(c)=b and *f*(d)=c . Then clearly *f* is somewhat Àgb-continuous function , but *f* is not somewhat #rg-irresolute function . Since for #rg-open set U={a} in (Y ,Ã). f^{-1}(U)= f^{-1}({a})={a}. It is observe that there is no non empty #rg-open set V in (X ,Ä) such that V⊆ (U)={a}.

The following proposition give the condition to make proposition 2 and corollary 1 true.

ISSN 0215-9511

Proposition 3:

If *f* : (X ,Ä) '! (Y ,Ã) is somewhat #rg –continuous function and a space (Y ,Ã) is $T_{\#rg}$-space .Then a function *f* is a somewhat #rg –irresolute function.

Proof:

Let U be a #rg-open set in (Y ,Ã) such that f^{-1}(U) $\neq \emptyset$.Since (Y ,Ã) is $T_{\#rg}$-space and by using proposition(3-3)step-v- we have U is an open set in (Y ,Ã) .Also. Since *f* is a somewhat #rg-continuous function , then there exists a non empty #rg-open set V in (X ,Ä) such that V$\subseteq$ $f^{-1}(U)$. This implies that *f* is a somewhat #rg -irresolute function .

Proposition 4:

Let *f* : (X ,Ä) '! (Y ,Ã) be any function ,and (Y ,Ã) be a $T_{\#rg}$-space , then *f* is somewhat #rg –irresolute function if (X ,Ä) is a

(i) T_b-space and *f* is a somewhat gs –continuous .

(ii) $\pi gb - space$ and *f* is a somewhat Àgb –continuous.

(iii) αT_b -space and *f* is a somewhat ±g –continuous.

(iv) $T^*{}_{1/2}$-space and *f* is a somewhat rg –continuous.

Proof:

(i) Let U be a #rg-open set in (Y ,Ã) such that f^{-1}(U) $\neq \emptyset$.Since (Y,Ã) is $T_{\#rg}$-space and by using proposition(3-3)step-v- we have U is an open set in (Y,Ã) Also ,since *f* is a somewhat gs-continuous function , then there exists a non empty gs-open set V in (X ,Ä) such that V$\subseteq$ $f^{-1}(U)$. By hypotheses (X ,Ä) is a T_b-space . Thus , Vis an open set in (X ,Ä) and since (Every open set is a #rg-open [28]).Hence , V is a #rg-open set in (X ,Ä) such that V$\subseteq$ $f^{-1}(U)$. This implies that *f* is a somewhat #rg -irresolute function .

The proof of step-ii- , -iii- , and –iv- are similar to step-i- .

Proposition 5:

Let *f* : (X ,Ä) '! (Y ,Ã) be any function , and space (X,Ä) be a submaximal and pre-regular-$T_{1/2}$ space , (Y ,Ã) is $T_{\#rg}$- space , then *f* is somewhat gpr –continuous function if and only if *f* is somewhat #rg –irresolute function .

Proof:

Suppose that *f* is a somewhat gpr –continuous and let U be #rg- open set in (Y ,Ã) such that f^{-1}(U) $\neq \emptyset$. Since (Y ,Ã) is $T_{\#rg}$-space and by using proposition(3-3)step-v- we have U is an open set in (Y ,Ã). Also , since *f* is a somewhat gpr-continuous function , then there exists a non empty gpr- open set V in (X ,Ä) such that V$\subseteq$ $f^{-1}(U)$. By hypothese , X is a submaximal and pre-regular-$T_{1/2}$ space, then by proposition(3-4) we get Vis an open set in (X ,Ä) and since (Every open set is a #rg-open [28]).Thus , V is a #rg-open set such that V$\subseteq$ $f^{-1}(U)$. Hence, *f* is a somewhat #rg -irresolute .**Conversely** ,assume that *f* is #rg –irresolute function . By corollary (4-1) step-v- we get *f* is somewhat gpr –continuous function .

Remark 1:

The concepts of somewhat - continuous , somewhat b- continuous function and somewhat g±-continuous function are independent to somewhat #rg- irresolute function . As shows in the following examples.

Example 6:

(i) Let X={a ,b ,c ,d} with the topology Ä={ X, $,{a},{d},{a ,d},{c , d },{a ,c, d}},# RGO(X ,Ä)={ X, $,{a},{c},{d},{a ,d},{c ,d},{a ,c ,d}} . Define a function *f* : (X ,Ä) '! (X ,Ä) by *f*(a)= c, *f*(b) =b , *f*(c)

=a and *f* (d)=d.. Then clearly *f* is somewhat #rg-irresolute function, but *f* is not somewhat -continuous function . Since for open set U={a} in (X ,Ä). f^{-1}(U)= f^{-1}({a})={c}. It is observe that there is no non empty b- open set V in (X ,Ä) such that V⊆ (U)={c}.

(ii) Let X=Y={a ,b ,c ,d}, Ä={ X, $,{a},{d},{a ,d},{c ,d},{a ,c .d}},Ã = {Y, $,{a},{b},{a ,b}}, # RGO(X ,Ä)={ X, $,{a},{c},{d},{a, d},{c ,d},{a, c, d}} and #RGO(Y, Ã) ={Y ,$, ,{a},{b},{c},{d},{a ,b},{a ,c},{a ,d},{b ,c},{b ,d}} . Define function *f* : (X ,Ä) ’! (Y ,Ã) by *f* (a)= a, *f* (b) =c , *f* (c) =d and *f* (d)=b.. Then clearly *f* is somewhat - continuous ,but *f* is not somewhat #rg -irresolute function, Since for open set U={c} in (X ,Ä). f^{-1}(U)= f^{-1}({c})={b}. It is observe that there is no non empty #rg- open set V in (X ,Ä) such that V⊆ (U)={b}.

Example 7:

Let X={a ,b,c}, Ä={ X,$,{a},{b ,c}}, #RGO(X ,Ä)={X,$,{a},{b ,c}}, and G±O(X, Ä) ={X,$,{a},{b},{c},{a ,b},{a ,c},{b ,c}}. Define a function *f* : (X ,Ä) ’! (X ,Ä) by *f* (a)=b , *f* (b)=c and *f* (c)=a . Then clearly *f* is somewhat g±- continuous function, but *f* is not somewhat #rg- irresolute function . Since for #rg-open set U={a} in (X ,Ä). f^{-1}(U)= f^{-1}({a}) ={c}. It is observe that there is no non empty #rg-open set V in (X ,Ä) such that V⊆ (U)={c} . Also , in example (3-6) step-i- , it is observe that *f* is somewhat #rg- irresolute, but *f* is not somewhat g±-continuous.

Example 8:

(i) Let X=Y={a ,b,c}, Ä={ X,$,{a},{b ,c}}, Ã= {Y ,$, {c}} , #RGO(X,Ä)={X,$,{a},{b,c}},#RGO(Y,Ã) = {Y,$, {c}} and b-open sets in (X ,Ä)are {X ,$,{a},{b}, {c},{b ,c}} . Define a function *f* : (X ,Ä) ’! (Y ,Ã) by *f*(a)=a , *f* (b)=c and *f* (c)=b . Then clearly *f* is somewhat b- continuous function, but *f* is not somewhat #rg- irresolute. Since for #rg-open set U={c} in (Y ,Ã). f^{-1}(U)= f^{-1}({c})={b}. It is observe that there is no non empty #rg-open set V in (X, Ä) such that V⊆ (U)={b}.

(ii) Let X=Y={a ,b ,c ,d}, Ä={X, $,{a},{b},{a ,b}} ,Ã={Y, $,{a},{b},{a, b},{a ,b ,c} }, #RGO(X ,Ä)={X,$,{a},{b},{c}, {d},{a ,b},{a ,c}, {a ,d},{b ,c},{b ,d}, {b ,c}}, #RGO(Y ,Ã) ={X,$,{a} ,{b}, {c},{a ,b}, {a ,c},}, {b ,c},{a ,b ,c}} , and b- open sets in (X ,Ä)={X,$,{a},{b},{a ,b},{a ,c}, {a ,d},{b ,c},{b ,d}, {a, b, c},{a ,b, d},{a, c, d},{b ,c, d}}. Define a function *f* : (X ,Ä) ’! (Y ,Ã) by *f* (a)=d , *f* (b)=b , *f* (c)=a, *f*(d)=c. Then *f* is a somewhat #rg-irresolute function, but *f* is not somewhat b- continuous function . Since for b-open set U={a} in (Y ,Ã). f^{-1}(U)= f^{-1}({a})={c}. It is observe that there is no non empty #rg-open set V in (X ,Ä) such that V⊆ (U)={c}.

Proposition 6:

For surjective function *f* : (X ,Ä) ’! (Y ,Ã) , the following are equivalent :

(i) *f* is somewhat #rg –irresolute function .

(ii)If C is a closed subset of (Y ,Ã) such that f^{-1}(C) $\neq X$ .Then there is a proper #rg-closed subset D of (X ,Ä) such that $f^{-1}(C) \subseteq$ D.

(iii) If M is a #rg-dense subset in (X ,Ä) , then *f* (M) is a #rg- dense subset in (Y,Ã) .

Proof:

The proof is similar to that proposition(3-10) . Thus , it is omitted.

Proposition 7:

If *f* : (X ,Ä) ’!(Y ,Ã) is a somewhat #rg- irresolute function and *g*: (Y , Ã) ’!(Z,µ) is #rg-irresolute function. Then *g¿f* : (X ,Ä) ’!(Z ,µ) is somewhat #rg-irresolute function.

ISSN 0215-9511

Proof:

Let U be #rg- open set in (Z ,µ) such that $(gof)^{-1}$(U) $\neq \emptyset$ .Since g is #rg-irresolute function .Then g^{-1}(U) is #rg- open set (Y,Ã) . By hypotheses *f* is a somewhat #rg- irresolute function , then there exists a non-empty #rg-open set V in (X ,Ä) such that V $\subseteq f^{-1}$((U)) . Bu ((U)) = (U) . Hence ,V (U) . Therefore , *g¿f* is somewhat #rg-irresolute function.

Similarly, we proof the following corollary **:**

Corollary 2:

Let *f*: (X ,Ä) '!(Y ,Ã) and g: (Y , Ã) '!(Z,µ) be any tow functions .

Then *g¿f*: (X ,Ä) '!(Z ,µ) is somewhat #rg- irresolute function , if *f* is a somewhat #rg- irresolute function and *g* is :

(i) #rg-continuous function

(ii) continuous function

(iii) regular continuous function

Remark 2:

In the above proposition **:**

(i) if *f* is continuous (or #rg-continuous) function and *g* is a somewhat #rg- irresolute function, then is not necessarily *g¿f* is somewhat #rg-irresolute.

(ii) if *f* and *g* are two somewhat #rg- irresolute function, then is not necessarily *g¿f* is somewhat #rg-irresolute continuous.

The following examples serves this purpose :

Example 9:

Let X=Y= {a ,b ,c ,d}, Z= {a ,b ,c} Ä={X, $,{a},{b},{a ,b}},Ã= {Y, $,{a},{b},{a, b},and µ= {Z, $,{c}}. Let *f* : (X ,Ä) '! (Y ,Ã) be the identity function and define *g*: (Y , Ã) '!(Z,µ) by *g*(a)= *g*(b)=a ,*g*(c)=c and *g*(d)=d . Then, clearly *f* is continuous (and somewhat #rg- continuous) function ,and *g* is a somewhat #rg- irresolute , but *g¿f* is not somewhat #rg- irresolute . Since for #rg-open set U={c} in (Z ,µ). $(gof)^{-1}$(U)= f^{-1}(g^{-1}(U)) = (({c})) = ({c})={d} . It is observe that there is no non empty #rg-open set V in (X ,Ä) such that V(U)= ((U))={d}.

Example 10:

Let X=Y= Z={a ,b ,c ,d}, Ä={X, $,{a},{d},{a d},{c ,d},{ a ,c, d}} Ã= {Y, $,{a},{b},{a, b}, µ= {Z, $,{a},{b},{a ,b},{a ,b .c}}. Define *f* : (X ,Ä) '! (Y ,Ã) by *f*(a)=a ,*f*(b)=d ,*f*(c)=b and *f*(d)=c ,let *g*: (Y , Ã) '!(Z,µ) the identity function . Then, clearly *f* and *g* are a somewhat #rg- irresolute, but *g¿f* is not somewhat #rg- irresolute function .Since for #rg-open set U={c} in (Z ,µ). $(gof)^{-1}$(U)= f^{-1}(g^{-1}(U)) = (({c})) = ({c})={d} . It is observe that there is no non empty #rg-open set V in (X ,Ä) such that V (U)= ((U))={d }.

Remark 3:

If *f* : (X ,Ä) '! (Y ,Ã)) is somewhat #rg-continuous function and *g*: (Y , Ã) '!(Z,µ) is a #rg- irresolute function , then is not necessarily . Then *g¿f* : (X ,Ä) '!(Z ,µ) is somewhat #rg- irresolute . It is easy see that in example(4-10).

The following proposition give the condition unorder to remark(4-3) true**.**

Proposition 8:

Let *f*: (X ,Ä) '! (Y ,Ã) and *g*: (Y , Ã) '!(Z,µ) be any two functions and space (Y ,Ã) is $T_{\#rg}$-space .Then *g¿f* : (X ,Ä) '!(Z ,µ) is somewhat #rg –irresolute function **,** if *f* is a continuous function and *g* is a #rg –irresolute function.

ISSN 0215-9511

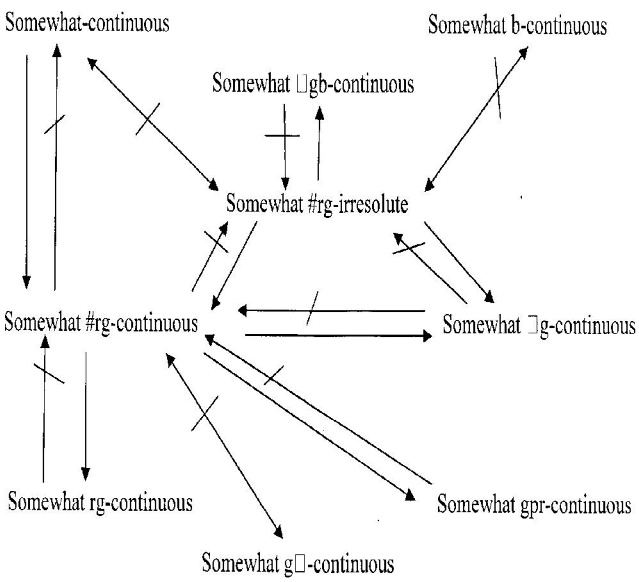
Proof:

Let U be #rg- open set in (Z ,µ) such that $(gof)^{-1}$(U) $\neq \emptyset$ .Since *g* is #rg-irresolute function . Then g^{-1}(U) is #rg- open set (Y ,Ã) . Also , since (Y ,Ã) is $T_{\#rg}$-space and by using proposition(3-3) step-v- we get (U) is an open set in (Y ,Ã) . By hypotheses *f* is a somewhat #rg- continuous function , then there exists a non-empty #rg-open set V in (X ,Ä) such that V $\subseteq$ $f^{-1}(g^{-1}$(U)) . But *(*(U)) = (U) . Hence , V (U) . Therefore , *g¿f i*s somewhat #rg-irresolute function

CONCLUSION

In this these, we defined somewhat #rg-continuous functions, studied its properties and we introduce the relationships of these functions with some other somewhat continuous functions.

Also, from the above discussion and results , we have the following implications.



Diagram(1)

Summarized The Relationships Between Somewhat –continuous functions Types

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