Position Tracking for Static Target using Burst Signals with Time Difference of Arrival Method

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Abstract—Time difference of arrival (TDOA) is one of transmitter tracking method to find location or position based on the difference of arrival signals from transmitter to receiver in time domain. This work aims to find the position of a transmitter using TDOA method. Operating frequency used is amateur radio 144.100 MHz for sending and receiving burst signal. Time difference is calculated by using cross correlation function. Output parameters are distance and coordinate of Tx toward Rx. The transmitter position is calculated by hyperbolic and trigonometry equations, giving distance error of 30.49%. Further improvement would require two or more tracking position methods in order to obtain more accurate value.

Keywords—*tracking; burst signals; TDOA; cross correlation; amateur radio*

I. INTRODUCTION

Tracking the position of an unknown target position is widely investigated. There are cases such as finding the location of sound source, the location of BTS (Base Transceiver Station), as well as other targets. Some of methods used to determine the location of a target are TOA (Time of Arrival), TDOA (Time Difference of Arrival), AOA (Angle of Arrival), FDOA, (Frequency Difference of Arrival), RSS (Receive Signal Strength), and COO (Cell of Origin) [1].

Example of time difference method application is ITD (Interaural Time Difference) received by two microphones to determine the location of the sound source. Voice signals captured by each microphone will be cross-correlated. Results at the highest peak of respective cross-correlation will show the value of ITD [2]. Other research is determining location coordinate of a radio transmitter (*Tx*) to receiver (*Rx*) station. The study using cross correlation algorithm are conducted to determine the similarity of a signal or calculate the time delay between the signals of the other signal [3].

This research is about tracking simulation for static target position and is focused to detect the location of a target. The target is a static object as a transmitter of unknown position and some objects as receivers that have known position. Distance tracking uses TDOA. This study proposes the use of amateur radio frequency (144100 MHz) to transmit and receive signal or information. Amateur radio equipment can be built or modified according to the desired utility, also can be placed anywhere. In addition, amateur radio is accessible and steady in

any condition such as disaster or other emergency situations [4]. Frequency signals used in this study is 500 Hz due to the radio amateur frequencies can only carry signals with frequencies not exceeding 2 kHz and computational limitations. Beacon signal consists of one or a series of pulse emitted by Tx and the signal received by Rx.

II. METHODS

In order to locate unknown static target (a transmitter) position using TDOA method, specific procedure should be followed. The location of the transmitter (Tx) expressed in coordinates (x, y). There are some necessary parameters: time signal to the receiver (Rx), the time difference until signal reaches between each other Rx, the distance between each Rx, coordinates of each Rx, and angle between the line from Tx to Rx and the Rx line. The main parameter used in this study is time for using TDOA method. Illustration of position scenario can be seen in Fig. 1 below:
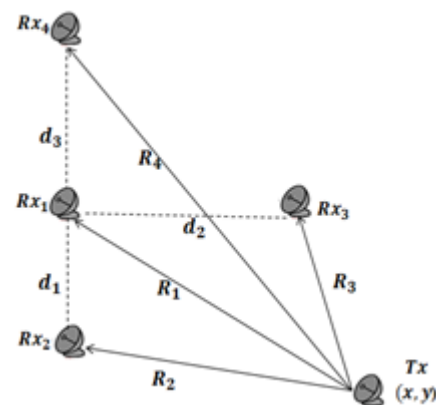

Fig. 1. Location Scenario Rx and Tx

Referring to Fig. 1 above, R_1, R_2, R_3 and R_4 are range between Tx with Rx; while d_1, d_2, and d_3 are the distance between Rx with another Rx; R_{x_1}, R_{x_2}, R_{x_3} and R_{x_4} are *receivers*, T_x is *transmitter*, and x, y is coordinate of Tx to be searched.
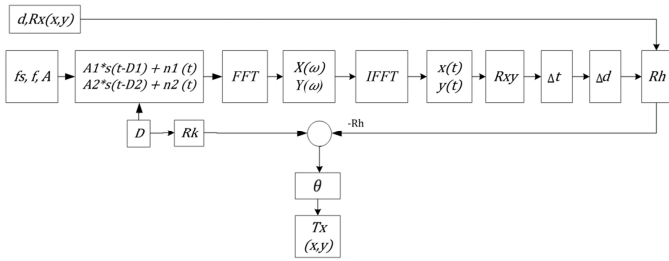
Fig. 2. Block Diagram of the Method

The research for tracking the position of static target is divided into five steps, viz. initial coordinate determination, TDOA method, distance calculations, finding the location coordinates and validation results. All stages are simulated with Matlab 2008b. Block diagram of this method can be seen in Fig. 2.

A. Initial Rx Coordinate Determination

Rx coordinate is expressed in Cartesian coordinate. The detailed coordinates of each predetermined Rx can be seen in Table I. Based on the block diagram (Fig. 2), signal receiver is adapted to distance toward transmitter. It is assumed that location of receivers toward transmitter is known. The distance of receiver to transmitter can be calculated using Eq. (1) below [5]:

$$d = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \qquad (1)$$

where d is the distance between Rx; x_i y_i is coordinate of Tx; x_j y_j is coordinate of Rx. Range Rx to Tx can be seen in Table I. The time of signal is found using Eq. (2) below.

$$t = \frac{R}{c} \qquad (2)$$

The use of Eq. (2) is to obtain signal timing parameters, which can be seen in Table I.

TABLE I. COORDINATES OF Rx

Rx	Coordinate (x, y)	Range (m)	Time of signal (μS)	A (V)
1	1;4.5	265.377	0.8	1.8
2	1;1	201.246	0.67	2.48
3	5.5;4.5	200.125	0.66	2.49
4	1;7	324.5	1	1.54

The amplitude of the signal is a random data adjusted to the distance of Rx against Tx on the scenario. The more remote the location Rx against Tx the smaller the amplitude or signal strength reaching Rx. There are many methods to find the amplitude of the waves, one of which is based on the Eq. (3) below:

$$A = \frac{F}{D} \qquad (3)$$

A is amplitude (dB), D is range (meter), F is frequency (Hz). Based on the Eq. (3) signal amplitude is determined as in Table I.

The distance between Rx is sought based on coordinates of each Rx. Distance between Rx based on their coordinates can be seen in Table II as follows:

TABLE II. DISTANCE BETWEEN RECEIVERS

Rx	Distance between Rx (meter)
1 with 2	105
1 with 3	135
1 with 4	75

values in Table II above are determined using Eq. (1) as well as the distance parameter Tx to Rx.

B. Time Difference of Arrival Method

The difference in arrival time or arrival time of a signal from the moment Tx emits a signal to each Rx called TDOA (Time Difference of Arrival). This time difference indicates the difference in distance between Rx with each other in respective to Tx.

TDOA is used to calculate the cross-correlation between the signals that arrive at each Rx. Cross-correlation function is employed because the location and time Tx initial transmit signal is unknown. As an alternative to the transmit time of the unknown, cross-correlation is used to determine the time difference between the signal reaches each other Rx [6].

Rx timing differences between one another depict slices between two Rx hyperbole. In this hyperbole line, there is Tx location possibility, assuming Rx and Tx are co-planar [7]. Location time difference can be illustrated as in Fig. 3 below:
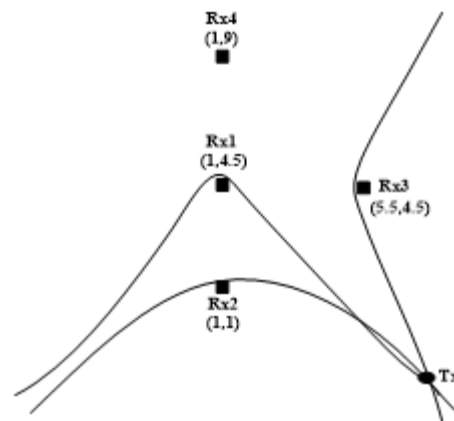

Fig. 3. Location Line Scenario

The general model for estimation of the time difference between signals received at two Rx, $x(t)$ and $y(t)$, is given by [8]:

$$x(t) = A_1 s(t - d_1) + n_1(t) \qquad (4)$$

$$y(t) = A_2 s(t - d_2) + n_2(t) \qquad (5)$$

Cross-correlation method to obtain TDOA (τ in Eq. (6)) is translated into Expectations $x(t)$ and $y(t)$ in the following equation [8]:

$$R_{xy}(\tau) = E\{x(t)y(t - \tau)\} \qquad (6)$$

Based on the Eq. (6) there is the maximum value of the amplitude of the cross-correlation argument or a D as timing difference function [8]:

$$D = -argmax|R_{xy}(\tau)| \qquad (7)$$

C. Calculation of Distance

1) Range Difference between Receiver

Difference of distance between the sought Rx or TDOA is adjusted to the time difference between the two Rx. The relationship between distance and difference TDOA between Rx can be calculated using Eq. (8) as follows [5]:

$$d = \Delta t \times c \qquad (8)$$

where d is range between Rx (meter), Δt is time difference (second), and c is speed of light ($3 \times 10^8 \, m/s$).

2) Range of Receiver to Transmitter

Rx to Tx range is derived using hyperbolic equations. Based on the elaboration of hyperbolic equations, to facilitate the search for Tx position, certain scheme is illustrated in Fig. 4 below:
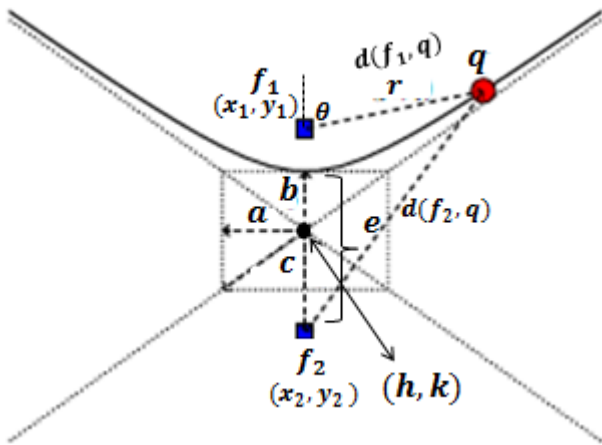

Fig. 4. Hyperbolic Solving

Based on hyperbolic illustration, some equations as parameters to determine the distance from Rx to Tx are described as follows [9]:

$$c = \frac{1}{2}d(f_2, f_1) \qquad (9)$$

$$b = \frac{1}{2}[d(f_2, q) - d(f_1, q)] \qquad (10)$$

$$a = \sqrt{c^2 - b^2} \qquad (11)$$

$$e = \frac{c}{b} \qquad (12)$$

$$h = \frac{x_1 + x_2}{2} \qquad (13)$$

$$k = \frac{y_1 + y_2}{2} \qquad (14)$$

$$\theta = \begin{cases} 0, for \ y_1 = y_2, x_1 > x_2 \\ 180, for \ y_1 = y_2, x_1 < x_2 \\ arccos\frac{y_1 - y_2}{d(f_1, f_2)}, for \ y_1 \neq y_2, x_1 \leq x_2 \\ -arccos\frac{y_1 - y_2}{d(f_1, f_2)}, for \ y_1 \neq y_2, x_1 > x_2 \end{cases} \qquad (15)$$

Tx to Rx distance value is the radius of the cone. Therefore, these distances can be searched by polar equation of the cone as follows [10]:

$$r = \frac{b(e^2 - 1)}{1 - cos\theta} \qquad (16)$$

Equation (16) applies for vertical directrix where the main axis or transverse axis of the conical (depending on type) is vertical, $\theta = 0$. Parametric equations of hyperbolic curve branches are as follows:

$$x = bcosht \qquad (17)$$

$$y = asinht \qquad (18)$$

where x is a hyperbolic cosine function, y is a hyperbolic sine function and $t \in (-\pi, \pi)$ [11][12].

D. Coordinate Transmitter

Tx coordinate can be found using trigonometric equations and conversion of polar coordinates into Cartesian coordinates. Illustration of angle can be seen in Fig. 5 below:
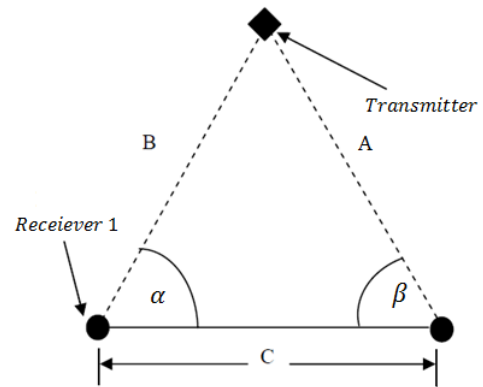

Fig. 5. Illustration of Angle

Trigonometric equations used are the law of cosines as follows:

$$\alpha = cos^{-1}\frac{B^2 + C^2 - A^2}{2BC} \qquad (19)$$

where B and C is the line that form angle α; while A is the line facing the angle α. The same way is used for another angle. Tx coordinate is defined by the following equation [13]:

$$x = rcos\theta \qquad (20)$$

$$y = rsin\theta \qquad (21)$$

E. Validation

The percentage of error is found by comparing the distance from Rx to Tx obtained from the calculation using hyperbolic equations with distance from receiver to Tx based on timing until the signal is determined. The equation used is:

$$Error = \left|\frac{Theory - calculation}{Theory}\right| \times 100 \qquad (22)$$

Theoretical data are obtained by the specified parameters and the calculation data are obtained from equation results.

III. RESULT AND DISCUSSION

The signals received by each Rx are simulated arrival time. These timing are adjusted to the distance between each Rx to Tx (Table III). The farther the distance between Rx to Tx, the longer the time arrives at Rx. Burst signals in the Matlab simulation are depicted.
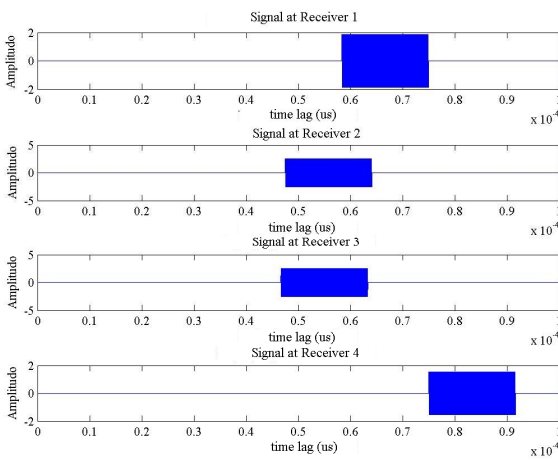

Fig. 6. The Signals Received by Each Rx

Attenuation is deliberately taken because of the distance between Tx and Rx. For a greater distance, the signal received by Rx is weaker.

A. Calculation of Distance

1) Distance between Receiver

Time difference is obtained from the cross-correlation between the received signals at Rx relative to another receiver. Furthermore, distance between the two receivers is calculated as in Eq. (1), so Table III can be formulated.

TABLE III.   DISTANCES BETWEEN RECEIVERS

Rx	T (us)	Distance between Rx(meter)
1 with 2	0.21	63
1 with 3	0.22	66
1 with 4	0.19	57

2) Distance of Receiver to Transmitter

The parameters used to find the distance from the receiver to transmitter are: distance difference between the two receivers, distance between two receivers, and receiver coordinates. Tracking transmitter position is based on the time difference between signals to each receiver. The distance of receiver to transmitter is determined using hyperbolic equations.

The following example is tracking distance of the receiver to the transmitter based on the timing difference between the receiver 1 and receiver 2. Locating distance from the transmitter to the receivers is illustrated.
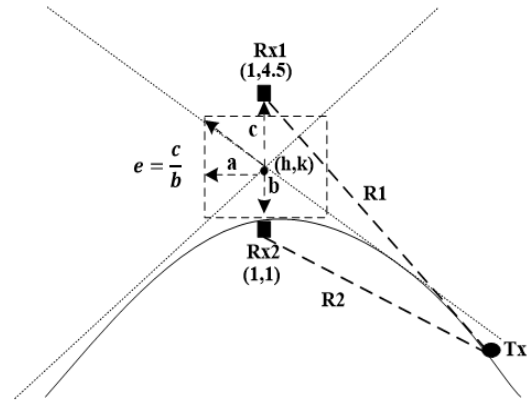

Fig. 7. Locating Position of Tx, with Configuration of Rx1 and Rx2

where Rx_1 and Rx_2 is receiver 1 and receiver 2, T_x is transmitter at unknown position, R_1 is distance between Rx_1 with T_x, R_2 is distance between Rx_2 with T_x, giving value of r is R_2. If $Rx_2 = (x_1, y_1) = (1,1)$ and $Rx_1 = (x_2, y_2) = (1,4.5)$; distance between Rx_1 and Rx_2 is 105 m; difference distance to transmitter between Rx_1 and Rx_2 is 63 m; difference of time arrival is 0.21 μs. Referring to Eq. (9) to Eq. (16), the value of c is 52.5 m; b is 31.5 m; a is 42 m; e is 1.67 m; midpoint coordinate (h, k) is (1, 2.75); $\theta(y_1 \neq y_2, x_1 \leq x_2)$ is 91.91°; distance between transmitter and receiver 2 is r = 279.638m.

If the time arrival of the signal is considered to be known and true, then the distance between transmitter to receiver 2 is 201.246 m (see Table I). Based on the Eq. (22) the percentage of error distance between transmitter to receiver 2 (simulation versus calculation) is:

$$\%Error = \left|\frac{201.246 - 279.638}{201.246}\right| \times 100 = 38.95$$

Percentage error above represents the difference by the initial parameters i.e. the distance from the transmitter to the receiver which have been determined based on the time arrival of signal.

On the other hands, tracking of Tx is as follows:

TABLE IV. SEARCHING TX POSITION

Rx	Coordinate		Distance between Rx	Difference Distance between Rx	$\theta(°)$	r (m)	%Error
	x	y					
Rx1 with Rx3							
Rx3	5.5	4.5	135	66	0	105.06	47.49
Rx1	1	4.5					
Rx1 with Rx4							
Rx	Coordinate		Distance between Rx	Difference Distance between Rx	$\theta(°)$	r (m)	%Error
	x	y					
Rx1	1	4.5	75	57	91.91	239.34	9.81
Rx4	1	7					

TABLE V. CONDITION 2 (SEARCH POSITION TX BETWEEN Rx_1 AND Rx_3)

Distance Rx1 to Tx (R1) (m)	208.34	Coordinate Tx to Rx2		
		α	x	y
		23.45	191.13	82.908
Distance Rx2 to Tx (R2) (m)	100.126	Coordinate Tx to Rx2		
		β	x	y
		124.09	-56.12	82.92
Distance between Rx (m)	162			

B. Determining Coordinate Value

Tx coordinate is searched by using the law of cosines as in Eq. (19) to get the angle formed between line Tx to Rx with line of the two Rx. The angle direction of signal coming from the Tx to Rx is used to decide Tx coordinate employing Eq. (20) and Eq. (21) in changing the polar coordinates into Cartesian coordinates. There are two conditions for search Tx coordinate.

Based on the calculations have been done, Tx coordinates can be searched as follows: Condition 1 (location Tx between Rx_1 and Rx_2), distance Rx_1 to Tx ($R1$) is 208,34 m; distance Rx_2 to Tx ($R2$) is 279,638 m; distance between two Rx (d) is 105 m. Condition 1 can be seen as Fig. 8 below:

TDOA geometry results can be seen in figure below:

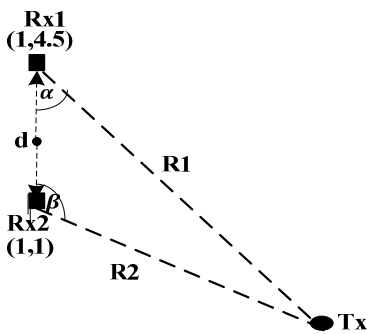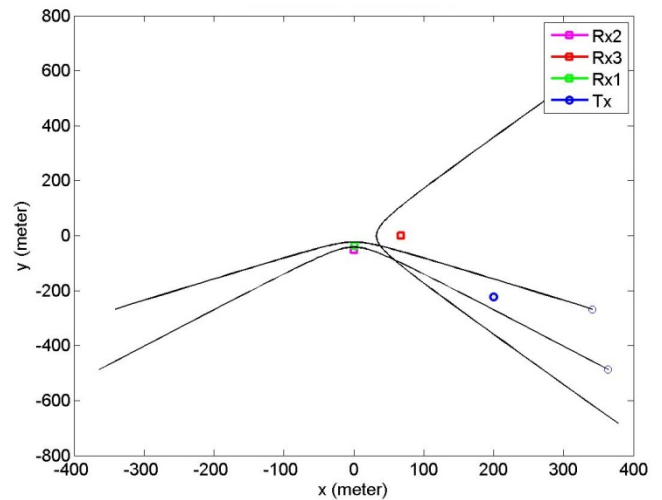Fig. 9. The Tracking Results from the Simulation

Fig. 8. Condition 1 search coordinates Tx

Based on Fig. 8 above coordinates Tx on condition 1 is: Coordinates Tx to Rx_1 searched by Equation (22) to Equation (24): $= 122.9°$; $x = -113.16$; $y = 174.92$; $(x, y) = (-113.16; 174.92)$. Coordinates Tx to Rx_2 searched by Equation (22) to Equation (24): $= 38.72°$; $x = 218.17$; $y = 174.91$; $(x, y) = (218.17; 174.91)$

Furthermore to find Tx coordinates respective to Rx3 (condition 2) in the same way, gives Table V:

IV. CONCLUSION

Position tracking for static target based TDOA has been simulated. This research gives performance indicated by the average percentage error Tx Rx distance of ± 30.49% in comparison with analytical method. It is shown that burst signals can be applied to the radar system at frequency of 144.100 MHz amateur radio. Accuracy of the distance measurement depends on the frequency or bandwidth of signal being delivered. Additional methods are needed (at least two methods to triangulate the position on static targets), such as AOA (Angle of Arrival), RSSI (Received Signal Strength Indicator), TOA (Time of Arrival), etc. in order to improve the detection approach.

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