DETERMINING INITIAL ASSIGNMENT FOR SOLVING QUADRATIC ASSIGNMENT PROBLEMS

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Abstract

The Quadratic Assignment Problem is a combinatorial problem of deciding the placement of facilities in specified locatians in such a way that will to minimize an objective function expressed in terms of distance each location and flows each facilities. First they a simple heuristic approach is used to obtain an initial assignment point.. Heuristic Tabu search method is the implemented in other to get another assignment point, which in turn will be used as a starting point for Nonlinier Programing approach. The continuous result of nonlinier programing problem are the prosesed using another heuristic method to obtain an integer feasible solution to the quadratic assignment problem. The non convex behavior of the problem suggests that a good starting is necessary in order to obtain global solution. The procedure is then implemented on Backboard Wiring problem with dimension 20 x 20.

Keywords; Quadratic Assignment problems; Tabu Search.

1. Introduction

Have old recognized in economics that there is raw material which indivisible that can generate the serious problema when wishing reached the efficient raw material allocation. indivisibility represent a special case from problema of allocation of raw material which is found the non-convection problem. One of problema related to indivisible situation or integer from decision variable and existence of non-konveksitas from objective function is Quadratic Assignment Problem (QAP) which is developed by Lawler (1963). this Problema model play an important role in a lot of usage, for example location theory, scheduling, backboard wiring, control panel, lay out and urban planning.

Look the problem of location of n facility in n location, which every assignable location for one facility, and every assignable facility for only one location. If f_{ik} represent the path between facility of i and facility of k and C_{jq} is fare of transportation perunit between location of j and q. hence prblem model can be writed as :

QAP is more difficult to be finished compared with linear assignation problem. unlinear Assignation problem can be solved by relaxing the integer program to the linear program

$$Min \quad \varphi = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{q=1}^{n} f_{ik} c_{jq} x_{ij} x_{kq} \qquad (1).$$

s.t

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1, \ldots, n \qquad (2)$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad j = 1, \ldots, n \qquad (3)$$

$$0 \le x_{ij} \le 1, \qquad x_{ij} : \text{integer}$$

and later will be finished. The result is always in integer. QAP is not only non-linier, but also not unicapital.

As case study of method applying will be finished by network of installation of communications cable (backboard wiring) with the variable amount 20 x 20.
By using model QAP hence long amount of cable will be minimized. Distance of each every location to dihungkan expressed with the matrix of this C matrix and represent the symmetry matrix of where number its especial diagonal is zero. Distance between location of i and j expressed by Cij. Flow from every facility to be connected to be expressed with the matrix of this F matrix and represent the symmetry matrix. where number its especial diagonal is zero. Flow of between facility of i and j expressed by f_{ij}.

QAP known as by]NP- hard problem. up to now not yet there is no efficient algorithm which can finish the QAP with the dimension N >15 in an optimal fashion. Generally in usage of dimension larger ones will emerge. Because conduct of target function is nonkonveks result needing of a[n procedure to obtain;get the starting points correct solution. So that obtainable global optimum solution.

Procedure of starting points weared at solution QAP is method of heuristik of Taboo Search.
Method of Taboo search is a n better method heuristik compared to by method heuristik other; dissimilar because Taboo Search can avoid the sort-time cycling and infinite loop so that expected can yield the optimum solution coming near global optimum.
Growth recently in program of is non linear have big scale to have made way to treat the QAP as a quadratic program have big scale from at hence method combinatorial.

As a method application case which is application method weared to finish the problema of linear count number use that is Branch and Bound (BB) and method of Cutting Plane have ever been raised by all researcher (see the Bazarra and Elshafei [1979]; Gendron And Crainine [1994]; Anstreicher And Baixius [1999]; Baixius And Anstreicher [2000]; Anstreicher Et al [2002]), but because QAP is difficult problem non polinomial hence it is not a wonder that method raised the beneficial only for the problema of with the small dimension. A N important factor in performance of algorithm B & B is method of boundary election under.

Some method Heuristik, for example algorithm Genetika, Taboo Search, Ant Colonies have ever been raised by all researcher to finish this problema see the, Bruijs [1984]; Mawengkang And Murtagh [1985]; Burkard Et al.
[1997]; Gambardella Et al. [1999]; Taillard [1991]; Thoremann And Bolte [1999]; and Hahn [2000]). But so far not yet earned to give the optimal solution for the problema of QAP have big scale n > 15

2. Heuristik Starting Points

Problem Combinatorial which is often became of the application like facility location, plan the layout and backboard wiring. By paying attention to problema from location of n facility of n location. Taking example f_{ik} represent the flow
between facility of i and facility of k and cjq is expense evacuation of per unit of between location of j and q so that its problem become

$$Min \quad \varphi = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{q=1}^{n} f_{ik} c_{jq} x_{ij} x_{kq} \qquad (4)$$

$$s.t$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1, \ldots, n \qquad (5)$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad j = 1, \ldots, n \qquad (6)$$

$$0 \leq x_{ij} \leq 1, \qquad x_{ij} : \text{integer}$$

With the definition that each;every location can only be taken possession of by one just facility and each;every facility can only take possession at one just just location

3. Determination of Starting Points is Solving of

Square Form from objective function (4) more tend to do not konveks so that optimal solution of kontinu obtained by through method program the non linear do not represent global optimal. Become one of effort to obtain;get the good solution a lot of influenced by starting points.

Determination of starting points of QAP done by heuristik, in general to obtain;get a assignation of early facility at location by placing facility stired at prima facie location, after tables of matrix of facility stream (f) and matrik apart (c) formed Hence starting points solution of pursuant to algorithm of following

Algorithm 1

1. *Calculate* $F_i = \sum_{k=1}^{n} f_{ik}$ *for every location* i, $i = 1, ..., n$

2. Forming facility list which sort by descending than Fi

3. *Calculate* $C_j = \sum_{l=1}^{n} c_{jl}$ *for every location* j, $j = 1, ..., n$

4. Forming location list which sort by ascending the than Cj
5. Assigning first facility from facility sequence at first location from list location, later;then next facility until all facility assigned. This algorithm is simple enough and easy to for the implementation of in getting assignation of early.

4. Starting Points Development.

Starting points of Assignation yielded by pursuant to algorithm 1 it is not representing result of solution coming near optimal or possible also non representing result of good solution therefore the solving of starting points still require to be developed till get a[n competent solution result. strategy of Development of starting points presented by following using method of heuristik of taboo of searc of Taboo search need the starting points, starting points obtained from algorithm 1 above..
taboo Search have to define the move set that is {single comlement move} and taboo of same single complement move.
the following Taboo Search procedure :

Algorithm 2

1. Select;choose the x E X to start the process.
2. Look for the x ' E N(x) in such a way till f(x') < f(x).
3. Otherwise there is x ' , x become the optimum local dot and stop.
4. On the contrary if found by x ' hence will become the new x and continue to step 2

After starting points obtained from taboo search is later;then finished by relaksasi from QAP with the mentioned starting points become the starting points for the solution of Nonlinier Programing (NLP). other Method Heuristik will be utilized to solving integer from QAP.

5. Heuristik of To Result of Nonlinier Programming.

Optimum Solution from model of nonlinier programming still not yet formed the circular solution to peubah. For that require to be obtained by a n strategy which can determine the variable which can be valuable one and which valuable zero
Strategy to obtain; get the solution that way as described stages;steps heuristik hereunder.

Algorithm 3

1. Paying attention to all peubah which not yet obtained the count value, for example Xij *
2. Calculate the $\delta_{ij} = 1 - X_{ij}^*$, i =1, ..., n ; j = 1, ..., n
3. Determining set i with value δ_{ij} the smallest, example i = i'
4. Make the variabel Xij by δ_{ij} smallest become 1, hence $\delta_{i'j} = 0$, for example for the j = j'
5. Make all other peubah x ij, j ≠ j', in last gathering become 0 and δ_{ij} become 1 (all location for the facility of this)

6. Calculation of Quadratic Assignment Problem

Problem of Qudratic Assignment Problem is borrowed ideas from Christoper E.Nugent (1968) and N. Christofides (1989). To finish this problem is used by determination procedure of starting points use the method heuristik (algorithm1) as starting points to finish the method of Taboo Search (algorithm 2), by using language of C ++ what its result represent the starting points from solution of Nonlinier Programming by using method of Tapping and Demarcation use the language of Fortran is later; then continued with the method of the other heuristik (algorithm 3).
Problema finished is problems Christofides of have Dimension N= 20

7. Solution QAP for the problema of N. Christofides of have Dimension N=20

Problema QAP of have dimension N=20 represent the big enough count number problem where binary variable amount is 400

In the following is presented by the tables of matrix groove the F (f_{ik}) expressing stream from couple of facility of i and k and also matrik of fare of transport C(c_{jq}) expressing distance of between location of j and q

8. Determination of starting points use the algorithm 1

1. Calculating Fi to each;every facility i , i = 1 n. from matrix flow (F) the tables of 1
2. Forming facility list which sort by descending the than Fi is as :

Tables 1. Descendingl Location list for the N of = 20

No	$F_I = \Sigma f_{ik}$	Sequence Descending
1	101	10
2	226	1
3	82	15
4	204	2
5	148	6
6	145	7
7	19	19
8	24	18
9	113	9
10	172	3
11	91	13
12	128	8
13	6	20
14	168	4
15	157	5
16	92	12
17	51	16
18	93	11
19	84	14
20	40	17

3.Calculating Cj to each every location j, j = 1 ..., n from matrix apart the (C)

4 Forming of location list which sort by ascending the than Cj shall be as follows

Tables 2. Ascendingl Facility List for the N=20 of

No	$C_j = \Sigma c_j$	Sequence Ascending
1	101	14
2	91	7
3	106	18
4	82	3
5	110	20
6	97	12
7	105	17
8	104	16
9	104	15

10	80	2
11	93	9
12	92	8
13	87	5
14	98	13
15	94	10
16	87	4
17	97	11
18	74	1
19	107	19
20	89	6

5. Assigning first facility from tables 1 with the first location from tables 2 obtained starting points

i : 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
j : 15 18 9 10 20 2 19 3 11 4 14 12 5 16 13 6 8 17 1 6

Pursuant to facility assignation to location like above hence objective function Z obtained is Z = 4819

9. Determination of Starting Points use the Taboo Search Algorithm 2

1. Starting points of solution which is obtained from algorithm 1
   x (0) = 15 18 9 10 20 2 19 3 11 4 14 12 5 16 13 6 8 17 1 6
2. Move set N(x) the{transfer two position)
3. Taboo Move

Accretion Edge that is active taboo during one iterasi and reduction edge that is active taboo during two iterasi
By doing algorithm step 2 using language C++ by using starting points
x (0) by iterasi 81840 obtained by a facility assignation shall be as follows:

i: 1 2 3 4 5 6 7 8 9 10 11 12: 13 14 15 16 17 18 19 20
j: 4 10 6 18 8 12 15 19 20 11 16 7 14 9 2 5 3 1 13 17

representing starting points from Nonlinier Programing. Pursuant to facility assignation to location obtained by a objective function of Z = 2232

Facility assignation to location is above converted in the form of fairish matrik assignation 20 x 20 where valuable xij is 0 if there no assignation from fasillitas i to j and valuable 1 if there are assignation of facility i to location j later;then the matrik assignation finished by using Nonlinier Programming with the language Fortran and continued with the algorithm 3 to obtain;get the better objective function value of Z.Optimum = 1096 where its facility assignation shall be as follows:

i:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
j:	3	20	7	18	9	12	19	4	10	11	1	6	15	8	2	5	14	16	13	17

10. Conclusion

Problema Quadratic Assignment Problem (QAP) represent the difficult problems of nonpolinomial. Solution of early from problema conducted by heuristik that is method of Taboo Search is hereinafter used by Nonlinier Programming By determining starting points use the heuristik of Taboo Search, procedure of solving of obtainable problem of optimal result .

Nature of do not konveks from QAP result each;every solution of depend on starting points assignation. To process the development hereinafter. computing experience also show to each;every different starting points hence final solution also differ. So that final conclusion is starting points of assignation represent the important problem in optimal solution determination

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