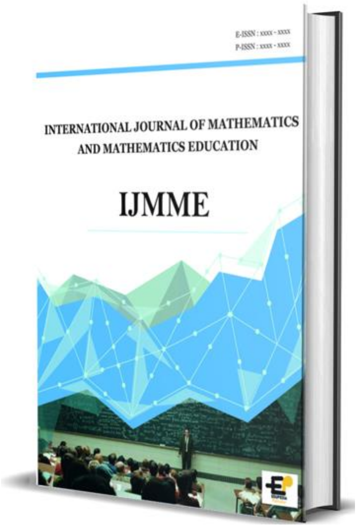

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The SS Method to Obtain an Optimal Solution of Transportation Problem

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Abstract

The transportation problem is a problem of distributing goods from several sources to several destinations so as to minimize transportation costs. Solving the transportation problem is needed to determine how many items must be sent from each source to each destination that must be met with minimum shipping costs. The purpose of this research is to examine, investigate, and apply the SS method to the minimum case transportation problem, both balanced and unbalanced. This research method is in the form of literature study and literature review from various sources that are relevant to the research. The results showed that the SS (Sheethalakshmy Srinivasan) method proposed by A. Seethalakshmy and Dr. N. Srinivasan can provide the optimal solution without having to find an initial feasible solution to the transportation problem so as to reduce the calculation time. This method provides an optimal solution for the minimum case transport problem for both balanced and unbalanced cases.

Introduction

Transportation is used to regulate distribution from sources that provide the same product, to places that need it optimally. Transportation is used to solve business problems, capital expenditure, allocation of funds for investment, location analysis, assembly line balancing and production planning and scheduling (Candra, 2016). The case of transportation arises when someone tries to determine the method of delivery (distribution) of a type of goods (item) from several sources (locations of supply) to several destinations (locations of demand) that minimize costs. The goal in this transportation problem is to allocate goods at the source in such a way that all needs at the destination (demand location) are met. According to Adi Nugroho et.al (2015), transportation is the delivery an item and people from the place of origin to the destination. Whereas according to Harvanda et. Al (2023), transportation is basically a means of moving people or items from one place to another. So that it can be concluded that the transportation problem is distributing an item from several sources to several destinations so that it can be minimize transportation costs.

To obtain a feasible solution in solving transportation problems, there are several methods, namely the North West Corner Method, Least Cost Method, and Vogel's Approximation Method. After obtaining the feasible solution, the next step is to perform an optimality test to obtain an optimal solution with using the Stepping Stone Method or the Modified Distribution Method (Utami & Dewi, 2019). Along with the development of the times, several new methods have emerged that immediately get optimal solutions including the Zero Suffix Method (Ngastiti & Surasoro, 2018), Zero Point Method (Pratiwi, 2016), Zero Neighbouring Method (Aneja & Bhatia, 2018), Exponential Approach Method (Hidayat, 2016) and others.

Zero Suffix Method is one of the methods of optimizing transportation problems that directly tests the optimumancy of the transportation problem table without having to determine the initial solution (Karnila et. al, 2019). Improved Zero Point Method is very useful method to solve all kinds of transportation problems, this method provides an optimal solution without help of any other modification method (Utami & Dewi, 2019). Some of these methods focus on the cost of a reduction result that is worth zero. Most of those direct methods manage to provide the optimal solution on the issue of balanced transportation, while on the issue of unbalanced transportation does not necessarily result in an optimal solution.

The SS method (Sheethalakshmy & Srinivasan, 2016) is a new method in solving transportation problems so it is necessary to study whether the method it provides optimal solutions to transportation problems both balanced and unbalanced. In this research discusses about steps to find the optimum solution on the transportation problem with using the SS Method (Sheethalakshmy & Srinivasan, 2016) so as to determine the optimal quantity of items that must be distributed from several sources to several destinations for a minimum cost transportation.

Transportation problem first raised by F. L. Hitchcook (1941) and is known for the Hitchcook distribution problem which is a problem arrangement of the distribution of similar items from a number of sources to a number of places which requires optimally. The model of transportation problem is as follows (Karnila, et. Al, 2019):

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$

const.

$\sum_{j=1}^{n} x_{ij} = a_i, i = 1,2, \dots, m$ (1)

$\sum_{i=1}^{m} x_{ij} = b_j, j = 1,2, \dots, m$ (2)

$x_{ij} \geq 0, i = 1,2, \dots, m; j = 1,2, \dots, n$ (3)

In the transportation model, the ability of sources to provide an items $(\sum_{i=1}^{m} a_i)$ is not necessarily equal to the level of demand from a number of objectives $(\sum_{j=1}^{n} b_i)$ so there are three possibilities that will occur namely (Karnila, et. Al, 2019):

1. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$
2. $\sum_{i=1}^{m} a_i \leq \sum_{j=1}^{n} b_j$

3. $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j$

The first possibility is balanced transportation, while the second and third possibilities are unbalanced transportation. A transportation model is said to be balanced if the amount of supply must be equal to the number of demand. While transportation problems said to be unbalanced if the amount of supply is not equal the amount of demand. But every transportation problems can be made balanced by inserting artificial variable.

If the amount of demand exceeds the amount of supply, then a dummy source that will supply the shortage, i.e. as much as $\sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i$. Conversely, if the amount of supply exceeds the amount of demand, then a dummy goal that will absorb the excess is a much as $\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$. Transportation cost per unit (c_{ij}) from dummy source and dummy destination is zero. It causes in reality from the source of dummy does not occur delivery. In order to understand transportation problems properly and precisely, transportation problems can be described in the form of Table 1. as follows (Dimyati, A. & Dimyati, A., 2010):

Table 1. Transportation Table

Sources		Destination (j)				Supply (a_i)
		1	2	..	n	
Source (i)	1	x_{11} c_{11}	c_{12} x_{12}	...	c_{1n} x_{1n}	a_1
	2	c_{21} $x21$	c_{22} x_{22}	...	c_{2n} x_{2n}	a_2
	...	...	...	...	...	...
	m	c_{m1} x_{m1}	c_{m2} $xm2$	...	cmn xmn	a_m
Demand (b_j)		b_1	b_2	...	b_n	

Notation:

Z = Total cost of transportation

x_{ij} = The number of items shipped from source i to destination j

c_{ij} = Shipping cost per unit of items shipped from source i to destination j

a_i = The large amount of inventory in the i source

b_j = The large demand for items at the destination j

m = Number of source

n = Number of objectives

Method

The method used by the author in this study is:

1. Conduct a literature study, namely by describing basic materials related to transportation problems, such as understanding transportation problems, transportation problem models, methods for finding feasible solutions, methods for determining optimal solutions, then discussing the SS Method algorithm to solve the problem transportation.
2. Reviewing journals regarding the new method, namely the SS Method in determine the optimal solution to the transportation problem and other bibliography that underlies the theory of transportation problems
3. Investigate the optimization of the SS method
4. Applying the SS Method to transportation problems

Results

Transportation problems are problems with the distribution of an item or product from multiple sources to multiple destination with a view to minimizing transportation costs or maximizing profits. There are steps to get the optimal solution to the transportation problem by searching the feasible solution then performs an optimization test. Along with the development of the times, several new simply methods appeared, one of which was the SS Method proposed by A. Seethalakshmy and Dr. N. Srinivasan (Sheethalakshmy & Srinivasan, 2016). The SS Method can be provided quick steps to get the optimal solution of the transportation problem.

The following is a definition of a feasible solution and optimal solution to the transportation problem.

Definition 1. (Mohanaseivi & Ganesan, 2012) *Non negative set* $X = \{x_{ij} \geq 0 \mid i = 1,2,\dots,m; j = 1,2,\dots,n\} \subseteq R\}$ *that satisfies equation 1 and equation 2 on the transportation problem is called a feasible solution. So the feasible solution space of the transportation problem is*

$X = \{x_{ij} \geq 0 \mid \sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,\dots,m;\ \sum_{i=1}^{m} x_{ij} = b_j, j = 1,2,\dots,n\}$

Definition 2. (Mohanaseivi & Ganesan, 2012) *Known* $X = \{x_{ij} \geq 0 \mid \sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,\dots,m;\ \sum_{i=1}^{m} x_{ij} = b_j, j = 1,2,\dots,n\}$ *is a feasible solution of transportation problems. Feasible solution* $x_{ij}^{*} \in X$ *is said to be an optimal solution of transportation problem (minimum cases) if* $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}^{*} \leq \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}, \forall x_{ij} \in X$.

Here is a theorem to guarantee that transportation problems have a feasible solution:

Theorem 1. *Transportation problem both balanced and unbalanced have a feasible solution. The transportation problem is said to be unbalanced if the amount of inventory is not the same with the number of requests is* $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$. *Transportation problems not balanced can occur in two different forms,* $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$ and $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$.

Theorem 2. (Pandian & Natarajan, 2010) *For any optimal solution to transportation problems (P1) is the optimal solution of the transportation problem (P).*

Theorem 3. (Pandian & Natarajan, 2010) *If* $\{x_{ij}^0 \geq 0 \mid i = 1,2,\dots,m; j = 1,2,\dots,n\}$ *is a feasible solution of transportation problem (P) and* $(c_{ij} - u_i - v_j) \geq 0$ *for all i and j where* u_i *and* v_j *are real numbers, so that the minimum of the transportation problem (P1) is worth 0, then* $\{x_{ij}^0 \geq 0 \mid i = 1,2,\dots,m; j = 1,2,\dots,n\}$ *is an optimal solution of transportation problem (P).*

The SS Method is a direct method to obtain an optimal solution of transportation problem without finding initial feasible solution. SS Method steps to determine the optimum solution of transportation problems (minimum cases) are follows:

1. Form a transportation table
   For transportation issues case minimum cost (c_{ij}). For unbalanced transportation problem needs to be balanced by adding rows or a dummy column.
2. Row Reduction
   Reducing the row by specifying the smallest element of the u_i on each row, then subtract each row element by the smallest element on that row, which is $(c_{ij} - u_i)$.
3. Column Reduction
   Reducing the column by determining the smallest element v_j on each column, then subtract each column element by the smallest element in that column, namely $(c_{ij} - u_i - v_j)$.
4. Allocate
   In a transportation table there will be at least one zero cell on each rows and columns, which is $(c_{ij} - u_i - v_j) = 0$, then calculate the cost S_{ij} reduction, i.e. S_{ij} is the sum of each element on the *i*-th rows and the *j*-th column of the *ij* cell that is zero. Next allocate the value demand or supply on selected cells that have a cost value the largest S_{ij} reduction. If there is the largest summation result equal to or more than one, then zero cells are selected that have a minimum supply or demand.
5. Improvements to the transportation table
   Create a new transportation table by ignoring or marking rows or a column in which supply or demand has been fulfilled. Checks whether the new transportation table has at least one zero in every row and column. If not, go back to steps 2 and 3, but if already have at least one zero on each row and column, go to step 6.
6. Repeat step 4 and 5 until all rows and columns the demand is fulfilled.
7. Calculating the optimal cost, i.e. $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$.

As for the theorem that guarantees that any solution of the transportation problem the minimum case with the use of the SS Method will be obtained optimal solution, they are as follows:

Theorem 4. *Solution that obtained by the SS Method for any problem minimum cases transportation both balanced and unbalanced constitutes optimal solution*.

Proof.

Given any balanced transportation problem with the function of the destination minimize. Analogous to the unbalanced transportation problem of minimum cases. For the unbalanced transportation problem, it needs to be balanced with adding a dummy row or column. Drawing up a preliminary transportation table of minimum cases transportation issue given.

Provided that (u_i) is the smallest value of the i-th row of the table (c_{ij}). Then subtract any i-th row element with (u_i) so that a table is obtained $(c_{ij} - u_i)$.

Provided that (v_j) is the smallest value of the j-th column of the table $(c_{ij} - u_i)$. Then subtract each element of the j-th column with (v_j) so that a table is obtained $(c_{ij} - u_i - v_j)$.

Then calculate the cost of reducing S_{ij}, which S_{ij} is the sum of each element on the i-th row and j-th column on the zero valued ij cell and allocates a demand or supply value on the selected cell that has the largest S_{ij} reduction cost value. If there is the largest summation result which is equal to or more than one, then a zero cell is selected that has an inventory or minimum demand.

Create a new transportation table by deleting or marking rows or columns for which supply or demand has been fulfilled. Then check whether the new transportation table has at least one zero on each row and column. Otherwise, go back to the row and column reduction step.

If there is already at least one zero cell, back to the step of calculating the cost of reducing S_{ij}. Repeat the above steps until all request rows and supply column fulfilled. Obtained solution $\{x_{ij} \geq 0 \,|\, i = 1,2,\ldots,m; j = 1,2,\ldots,n\}$ for all transportation problem with the cost matrix $(c_{ij} - u_i - v_j)$ so $x_{ij} = 0$ for $\left(c_{ij} - u_i - v_j\right) \geq 0$ and $x_{ij} \geq 0$ for $\left(c_{ij} - u_i - v_j\right) = 0$. Therefore, $\min \sum_{i=1}^{m} \sum_{j=1}^{n} \left(c_{ij} - u_i - v_j\right) x_{ij}$ with consraints 1 and 2 fulfilled is worth 0.

Based on Theorem 3, the solution $\{x_{ij} \geq 0 \,|\, i = 1,2,\ldots,m; j = 1,2,\ldots,n\}$ is the optimal solution of the transportation problem.

Based on Theorem 4, using the SS Method will be obtained optimal solution to any well balanced and unbalanced transportation problem with minimum cases.

Discussion

Given examples of balanced transportation problem solved by SS Method

Table 2. Step 1 for Example of Balanced Transportation Table

Source	Destination (in ten of thousands of rupiah)				Supply
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	
Factory A	8 x_{11}	6 x_{12}	10 x_{13}	12 x_{14}	25
Factory B	5 x_{21}	4 x_{22}	10 x_{23}	6 x_{24}	40
Factory C	4 x_{31}	8 x_{32}	9 x_{33}	5 x_{34}	25
Factory D	5 x_{41}	6 x_{42}	11 x_{43}	7 x_{44}	10
Demand	20	30	10	40	100

Step 2: After forming the initial transportation table, the next step is to perform row reduction by specifying the smallest element on each row, then subtracts each row element by the smallest element on that row. At this step, taken the smallest element on the first row which is 6, then subtract element on the first row by 6 and do the same thing on the other row so obtained the table as follows:

Table 3. Step 2 for Example of Balanced Transportation Table

Source	Destination (in ten of thousands of rupiah)				Supply
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	
Factory A	2	0	4	6	25
Factory B	1	0	6	2	40
Factory C	0	4	5	1	25
Factory D	0	1	6	2	10
Demand	20	30	10	40	100

Step 3: Reducing columns by specifying the smallest element in each column, then subtract each column element by the smallest element on the column. The smallest element in the third column is 4, then subtract each element in the third column by 4. Similarly, column fourth, the smallest element in the fourth column is 1, then subtracts any element in the fourth column with 1 so that a table is obtained:

Table 4. Step 3 for Example of Balanced Transportation Table

Source	Destination (in ten of thousands of rupiah)				Supply
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	
Factory A	2	0	0	5	25
Factory B	1	0	2	1	40
Factory C	0	4	1	0	25
Factory D	0	1	2	1	10
Demand	20	30	10	40	100

Step 4: Cause there is already at least one zero cell in each row and column, then calculates the cost of S_{ij} reduction, S_{ij} is the sum of each element on i-th row and jth column of the zero valued ij cells. Then allocate demand or supply value on selected cells that have a cost value the largest S_{ij} reduction. If there is the largest summation result equal to or more from one, a zero cell is selected that has a supply or demand that minimum cells with zero value include cell (1,2), (1,3), (2,2), (3,1), (3,4), (4,1) with S_{ij} reduction cost, are:

1. Cell (1,2) = 2 + 5 + 4 + 1 = 12
2. Cell (1,3) = 2 + 5 + 2 + 1 + 2 = 12
3. Cell (2,2) = 1 + 2 + 1 + 4 + 1 = 9
4. Cell (3,1) = 2 + 1 + 4 + 1 = 8
5. Cell (3,4) = 5 + 1 + 1 + 4 + 1 = 12
6. Cell (4,1) = 2 + 1 + 1 + 2 + 1 = 7

Then select the cell with the largest summation result. Of the sixth cell, there is more than one cell with the same largest number, namely cells (1,2), (1,3), and (3,4) so that a cell that has a supply or demand is selected that minimum, namely:

1. Cell (1,2) have a supply of 25 and demand of 30
2. Cell (1,3) have a supply of 25 and a demand of 10
3. Cell (3,4) have a supply of 25 and a demand of 40

Of the three cells above, which have a supply or demand value that the minimum is a cell (1,3) because the cells has the most minimum value which a demand value of 10, then allocating the supply value or demand on such cells so as to obtain:

Table 5. Step 4 for Example of Balanced Transportation Table

Source	Destination (in ten of thousands of rupiah)				Supply
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	
Factory A	[2]	[0]	[0] 10	[5]	25
Factory B	[1]	[0]	[2]	[1]	40
Factory C	[0]	[4]	[1]	[0]	25
Factory D	[0]	[1]	[2]	[1]	10
Demand	20	30	~~10~~ 0	40	100

Step 5: Create a new transportation table by deleting or marking inventory rows and or the request column that has been fulfilled. Then check if the new transportation table has at least one zero cell on each row and column. If no, go back to the row and column reduction step.
Third column (Warehouse 3) has been fulfilled so the third column needs to be marked or ignored, then obtained the transportation table as follow:

Table 6. Step 5 for Example of Balanced Transportation Table

Source	Destination (in ten of thousands of rupiah)				Supply
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	
Factory A	[2]	[0]	[0] 10	[5]	~~25~~ 15
Factory B	[1]	[0]	[2]	[1]	40
Factory C	[0]	[4]	[1]	[0]	25
Factory D	[0]	[1]	[2]	[1]	10
Demand	20	30	~~10~~ 0	40	100

Step 6: Cause it still has at least one zero cell in each row and column, it can skip to the next step which is to repeat the fourth and fifth steps until the demand row and supply column are fulfilled.
The cell with zero value include cells (1,2), (2,2), (3,1), (3,4), (4,1) then calculated the cost of S_{ij} reduction

by summing each of the elements that has not been marked on the cell.

1. Cell (1,2) = 2 + 5 + 0 + 4 + 1 = 12
2. Cell (2,2) = 1 + 1 + 0 + 4 + 1 =7
3. Cell (3,1) = 2 + 1 + 0 + 4 + 0 =7
4. Cell (3,4) = 5 + 1 + 1 + 0 + 4 = 11
5. Cell (4,1) = 2 + 1 + 0 + 1 + 1 =5

Then select the cell with the largest summation result. Of the five cells, the cell that has the largest summation result is the cell (1,2) so that it can directly allocate supply or demand value to the cell then mark the supply row or demand column that has been fulfilled and obtained table as follow:

Table 7. Step 6 for Example of Balanced Transportation Table

Source	Destination (in ten of thousands of rupiah)				Supply
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	
Factory A	2	0 15	0 10	5	~~25~~ ~~15~~ 0
Factory B	1	0	2	1	40
Factory C	0	4	1	0	25
Factory D	0	1	2	1	10
Demand	20	~~30~~ 15	~~10~~ 0	40	100

Step 7: Cause it still has at least one zero cell in each row and column, it can skip to the next step which is to repeat the fourth and fifth steps until the demand row and supply column are fulfilled.

The cell with zero value include cells (2,2), (3,1), (3,4), and (4,1) then calculated the cost of S_{ij} reduction by summing each of the elements that has not been marked on the cell.

1. Cell (2,2) = 1 + 1 + 4 + 1 = 7
2. Cell (3,1) = 1 + 4 = 5
3. Cell (3,4) = 1 + 1 + 4 = 6
4. Cell (4,1) = 1 + 1 + 1 = 3

Then select the cell with the largest summation result. Then mark the supply row or demand column that has been fulfilled and obtained table as follow:

Table 8. Step 7 for Example of Balanced Transportation Table

Source	Destination (in ten of thousands of rupiah)				Supply
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	
Factory A	2	15 / 0	10 / 0	5	~~25~~ ~~15~~ 0
Factory B	1	15 / 0	2	1	~~40~~ 25
Factory C	0	4	1	0	25
Factory D	0	1	2	1	10
Demand	20	~~30~~ ~~15~~ 0	~~10~~ 0	40	100

Step 8: Repeat step 2 to 4 until all demand rows and supply column are fulfilled, so that using the SS Method the following final transportation table is obtained:

Table 9. Final Transportation Table using SS Method

Source	Destination (in ten of thousands of rupiah)				Supply
	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	
Factory A	0 / 8	15 / 6	10 / 10	0 / 12	25
Factory B	0 / 5	15 / 4	0 / 10	25 / 6	40
Factory C	10 / 4	0 / 8	0 / 9	15 / 5	25
Factory D	10 / 5	0 / 6	0 / 11	0 / 7	10
Demand	20	30	10	40	100

From Tabel 3, the optimal allocation is obtained with total minimum cost:

Z = Σi4=1 Σj4=1 cij xij

= 6 (x_{12}) + 10 (x_{13}) + 4 (x_{22}) + 6 (x_{24}) + 4 (x_{31}) + 5 (x_{34}) + 5 (x_{41})

= 6 (15) + 10 (10) + 4 (15) + 6 (25) + 4 (10) + 5 (15) + 5 (10)

= 90 + 100 + 60 + 150 + 40 + 75 + 50

= 565.

So that the minimum distribution cost is Rp5.650.000,00.

Given examples of balanced transportation problem solved by SS Method.

A company engaged in the field of beverage production located in the Rungkut Industry area, Surabaya. The company has several depots, namely Sier, Tandes, and Gempol with successive production capacities of 3600 bottles, 2100 bottles, and 1500 bottles. This items are distributed from the depot to several supermarkets A by 2500 bottles, B 850 bottles, C 1800 bottles, and D for 2000 bottles.

As for the distribution fee (in ten rupiah) from a depot to several supermarkets varies according to the distance traveled. By data obtained, problems that occur in this beverage company are unbalanced transportation problem. The amount of demand for beverage is smaller than the number of beverafe that this company provides. From the problem the minimum cost of distributing goods to this company will be determined. Data on the problem contained in this beverage company as follow:

Table 10. Unbalanced Transportation Table using SS Method

Source	Destination (in ten rupiah)								Supply
	A		B		C		D		
Sier	0	8	15	6	10	10	0	12	3600
Tandes	0	5	15	4	0	10	25	6	2100
Gempol	10	4	0	8	0	9	15	5	1500
Demand	2500		850		1800		2000		7200 / 7150

Table 4 is unbalanced transportation problem $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$ where the amount of inventory to be distributed exceeds the amount of demand so it needs to be balanced by adding a *dummy* column of $\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$ in order to solve transportation problems so that the table becomes:

Table 11. Step 1 for Transportation Table using SS Method

Source	Destination (in ten rupiah)					Supply
	1	2	3	4	*Dummy*	
Sier	3	2	4	5	0	3600
Tandes	4	7	5	3	0	2100

Gempol	18	14	12	20	0	1500
Demand	2500	850	1800	2000	50	7200

Step 2: After adding a *dummy* column, the next step is to do reduction rows by specifying the smallest element on each row, then substracts each line element by the smallest element on that row. The smallest element in each row is 0 so that when a row reduction is performed, the table will not change and go directly to the next step.

Step 3: Reducing column by specifying the smallest element in each column, then subtract each column element by the smallest element. The smallest element in the first column is 3, then subtracts each element in the first column with 3 and do the same thing on the other column so that the following table is obtained:

Table 12. Step 2 for Destination of Transportation Table using SS Method

Source	Destination (in ten rupiah)					Supply
	1	2	3	4	*Dummy*	
Sier	0	0	0	2	0	3600
Tandes	1	5	1	0	0	2100
Gempol	15	12	8	17	0	1500
Demand	2500	850	1800	2000	50	7200

Step 4: Calculating the cost of reducing S_{ij} is the sum of each element on the i-th row and the j-th column of the zero value ij cell. Then allocate demand or supply value in zero cells that have S_{ij} reduction cost value biggest. If there is the largest summation result equal to or more than one, then a zero cell is selected that has a minimum supply or demand.

The cell with zero value are cells (1,1), (1,2), (1,3), (1,5), (2,4), (2,5), (3,5) with S_{ij} reduction costs are:

1. Cell (1,1) = 2 + 1 + 15 = 18
2. Cell (1,2) = 2 + 5 + 12 = 19
3. Cell (1,3) = 2 + 1 + 8 = 11
4. Cell (1,5) = 2
5. Cell (2,4) = 1 + 5 + 1 + 2 + 17 = 26
6. Cell (2,5) = 1 + 5 + 1 = 7
7. Cell (3,5) = 15 + 12 + 8 + 17 = 52

The cell that has the largest summation result is (3,5) cell and obtained table as follow:

Table 12. Step 3 for Destination of Transportation Table using SS Method

Source	Destination (in ten rupiah)					Supply
	1	2	3	4	*Dummy*	
Sier	0	0	0	2	0	3600
Tandes	1	5	1	0	0	2100
Gempol	15	12	8	17	0 50	~~1500~~ 1450
Demand	2500	850	1800	2000	~~50~~ 50	7200

Step 5: Check whether the table already has at least one zero cell in the row and column. If so, you can go directly to the next step. But if no, it is necessary to carry out the reduction of rows and columns.

Cause there are still no zero cell in the row that have not been marked, namely the third row, it is necessary to reduce the row by subtracting each row element with the smallest element in the row. The smallest element on the third row that has not been marked is 8, then subtracts each element of the third row that has not been marked with 8, so that a table is obtained:

Table 13. Step 3 for Destination of Transportation Table using SS Method

Source	Destination (in ten rupiah)					Supply
	1	2	3	4	*Dummy*	
Sier	0	0	0	2	0	3600
Tandes	1	5	1	0	0	2100
Gempol	7	4	0	9	0 50	~~1500~~ 1450
Demand	2500	850	1800	2000	~~50~~ 50	7200

Step 6: Repeat step 4 until all demand rows and supply column are fulfilled, so that using the SS Method the following final transportation table is obtained:

Table 14. Final Transportation Table using SS Method

Source	Destination (in ten rupiah)					Supply
	1	2	3	4	*Dummy*	
Sier	2500 (3)	850 (2)	250 (4)	0 (5)	0 (0)	3600
Tandes	0 (4)	0 (7)	100 (5)	2000 (3)	0 (0)	2100
Gempol	0 (18)	0 (14)	1450 (12)	0 (20)	50 (0)	1500
Demand	2500	850	1800	2000	50	7200

From Table 6, the optimal allocation is obtained with total minimum cost:

$Z = \sum_{i=1}^{3} \sum_{j=1}^{5} c_{ij} x_{ij}$

$= 3\ (x_{11}) + 2\ (x_{12}) + 4\ (x_{13}) + 5\ (x_{23}) + 3\ (x_{24}) + 12\ (x_{33}) + 0\ (x_{35})$

= 3 (2500) + 2 (850) + 4 (250) + 5 (100) + 3 (2000) + 12 (1450) + 0 (50)

= 7500 + 1700 + 1000 + 500 + 6000 + 17400 + 0

= 34100.

So that, the minimum distribution cost is Rp341.000,00.

Conclusion

Based on the discussion, conclusions can be drawn that SS Method is an alternative method that can be used to solving transportation problems. This method provides steps that simple in determining the optimal solution without having to look for a feasible solution on transportation problem so that it is easier to understand and faster calculations determine its optimal solution. Completion of this method begins with perform row and column reduction then calculate the cost of reducing S_{ij} by summing each element in the *i*-th row and the *j*-th column on *ij* cells that are worth zero. Next allocate the value of the inventory or query on the selected zero cell that has the largest summation value. If there is the same largest summation result, then a zero cell is selected which have a minimum supply value or demand value. The results obtained using the SS Method on cases transportation issues minimum both balanced and unbalance is the optimal solution.

Recommendations

This research is limited to the problem of minimum case transportation, whether balanced or not balanced. further research can continue to be developed in other cases that occur in a company more specifically

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