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Student Semiotic Representation Skills in Solving Mathematics Problems

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Abstract. *Representational transformation skills significantly influence students' success in problem-solving. Students who struggle with representational transformation skills are often less adept at utilizing mathematical ideas and relationships, and vice versa. Therefore, this study used a qualitative, descriptive-interpretive approach to examine students' semiotic representation skills when solving mathematical problems. The research was conducted in a Year 9 classroom in a public school in Bandung, Indonesia, with 30 participants divided into high, middle, and low-ability groups based on their level of mathematical ability. Data was collected using both test and interview techniques. The results indicated that students in the high and middle ability groups had adequate skills in algebraic treatment and conversion from algebra to geometry and verbal expression skills for constructing algebraic expressions and converting verbal statements into mathematical equations. In contrast, the low-ability group demonstrated a lack of semiotic representation skills in problem-solving. These findings highlight the importance of transformation and conversion skills in mathematical problem-solving activities and can be valuable information for teachers and observers of mathematics education.*

Keywords: *representation skill semiotics, list representation semiotics, mathematical problems*

Introduction

Mathematics is a discipline in which symbols and numbers are fundamental components. Mathematical knowledge can be considered a form of symbolic knowledge, and it cannot be accessed directly like in other sciences. To comprehend mathematical concepts and relationships, individuals must engage in mental processes that rely on specific signs and symbols. Consequently, using representations is an essential aspect of learning mathematics, as emphasized by Duval (2017).

Representation physically records ideas, knowledge, or messages (Danesi, 2004). It can also be described as the use of signs to recreate something that has been perceived, imagined, or felt in physical form. The field of semiotics is concerned with the science of signs and the process of creating meaningful markings, including the systems, rules, and conventions that govern the use of these signs. In Uzun and Arslan's (2009) work, representation comprises a specific set of signs and symbols that adhere to well-defined rules. This approach identifies algebra, colloquialism, graphics, tables, and computer language as examples of register semiotic representations.

Transformation is vital to semiotic representation, which can occur within or between different representations. These transformations are commonly referred to as treatments and conversions. Treatments involve transforming one representation into another within the same register, while conversions involve transforming a representation from one register to another. Paraphrasing or replacing with synonyms in everyday language, computing, or adding the same number to both sides of an equation in an algebraic list; for example, $2x-1 = 4$ is represented by $2x = 5$. Conversion is a transformation that involves changing the form or representation of one register to another, with or without data loss. It can include transferring information from one language to another, transitioning from verbal statements to algebraic operations, presenting problems in various forms, such as drawing graphs based on equations, tabulating data, and developing mathematical models to solve everyday problems (Duval, 2000; McGee & Martinez-Planell, 2014; Pino-Fan, Guzmán, Duval, & Font, 2015).

Students' representation transformation skills greatly impact their success in mathematical problem-solving. Those with poor representation transformation skills are typically less proficient in applying mathematical concepts and relationships (Gagatsis & Shiakalli, 2004; Uzun & Arslan, 2009). Likewise, Various types of representation systems are crucial to enhance problem-solving performance as they can improve the ability to acquire and apply fundamental mathematical concepts, support the interplay between conceptual and procedural understanding, promote conceptual comprehension and problem-solving proficiency, and significantly contribute to the learning of mathematics (Báez, Pérez-González, & Triana-Hernández, 2017; Bassoi, 2006; Lesh, Behr, & Post, 1987; Silva, Santiago, & Dos Santos, 2014). The main goal of teaching mathematics is to enable students to transition seamlessly from one representation to another without conflicts (Hitt, 1998). it is essential to include activities emphasizing the importance of verbal, graphic, and algebraic representations to provide opportunities for students to use different representations of the same mathematical concept and perform transformations between them. According to Gagatsis and Shiakalli (2004), a piece of information in mathematics can be expressed in these three ways. Therefore, incorporating such activities in assessments and lesson plans can facilitate the development of students' representation transformation skills, which are crucial for success in problem-solving. This approach has been supported by Uzun and Arslan (2009), who argued that these activities provide opportunities for students to learn how to utilize different representation systems, support the relationship between conceptual and procedural understanding, and form conceptual understanding that contributes significantly to mathematics learning.

Semiotic representation plays a crucial role in studying and understanding mathematical concepts. However, it has not received the attention it deserves from mathematics teachers and researchers. Duval (2000) pointed out that researchers have neglected the two types of cognitive operations, namely treatment transformation, and conversion, which are essential for mathematical activities, especially in problem-solving situations. Consequently, many students struggle with these skills, as they are rarely asked or accustomed to performing such cognitive activities. Gagatsis and Shiakalli's (2004) research revealed that high school students are seldom asked to produce algebraic expressions based on graphs or write verbal descriptions that match algebraic expressions and vice versa. Additionally, students face difficulties in problem-solving tasks involving verbal, graphic, and algebraic representations, which require them to transform between different registers (McGee & Martinez-Planell, 2014; Uzun & Arslan, 2009). Therefore, improving students' treatment and conversion skills in mathematics education is vital, which can help them overcome these challenges and become proficient problem-solvers.

The difficulties mentioned above are not unique to specific regions but also to students in Indonesia. According to Juandi and Jupri (2013), using representation in learning mathematics is not fully optimized, leading to students' lack of experience in manipulating mathematical objects. This condition also affects the achievement of Indonesian students in the 2015 Trends in International Mathematics and Science Study (TIMSS) in the mathematics content domain, with an average score of 397, much lower than other participating countries such as Singapore, Hong Kong, and the Republic of Korea with an average score above 600 (Mullis & Martin, 2019). Furthermore, Indonesian students' achievement in the Program for International Student Assessment (PISA) in 2018 indicated an average math score of 379 (Dewabrata, 2019).

A person's ability to solve mathematical problems depends on the ability to consider the forms of representation involved in it. This shows that a person's ability to change one representation to another will affect his ability to find solutions to the problem (Johar & Lubis, 2018). Thus, it is necessary to know how students' mathematical representation skills are in solving problems. Therefore, this study aims to analyze students' semiotic representation skills in solving mathematical problems. The research focuses on activities to identify students' skills in transforming conversions and treatments in solving mathematical problems. For this purpose, a test was designed consisting of word problems and covering several types of representations. Different representations, i.e., instructions or activities, perform some conversion transformations, for example, from algebra to geometry and from algebra to graph. Meanwhile, treatment or processing transformation activities in this study are processes or steps of operations in finding the correct answer (Pino-Fan et al., 2015).

This study differs from previous research in terms of the instruments employed. The word problems used in this study assessed students' ability to comprehend and interpret a problem or concept as the initial step in problem-solving. The aim is to evaluate students' capacity to transform from verbal representations (word problems) to other forms of representation, such as algebra, tables, and geometry, as well as from algebra to verbal representations.

Method

This study employed a qualitative research methodology with a descriptive-interpretive approach. The researchers analyzed the written test responses and interview transcripts to understand and interpret the meaning of the phenomena described by the participants (Creswell & Poth, 2018). This method analyzed students' semiotic representation skills in solving mathematical problems. The research was conducted in a Year 9 classroom in a public school in Bandung, Indonesia, involving 30 students (aged 15-16 years) for the 2022/2023 academic year. They were divided into three groups based on their level of mathematical ability. The mathematical ability, used as a reference for the division of groups, is taken from the student's daily test scores in the previous material. The first group, the high-ability group labeled "H", consisted of 11 students. The second group, the middle ability group labeled "M," had 11 students, and the third group, the low ability group labeled "L", comprised nine students.

The research instrument consisted of three questions and interview guidelines. The test consisted of problem-solving items on algebraic adapted from questions on the national exam (UN) and the Program for International Student Assessment (PISA). Before the items were used, they were validated by two supervisors and three senior mathematics teachers. After being validated, the questions were then tested on students in different classes. The results of the instrument try-out were revised and administered to the students participating in this study. The test comprised word problems and adapted to the objectives of this study, namely: (1) determining students' semiotic representation skills in compiling equations or mathematical modeling and skills in making conversions from algebraic to geometric expressions, (2) examining student skills specifically on verbal expressions (studying literacy questions) and conversion transformation skills from tabular expressions to graphics, (3) examining conversion transformation skills from verbal expressions and geometric patterns (square patterns) to inductive-deductive reasoning to support the process of argumentation or logical reasoning (Báez, et al., 2017; Flores, 2002).

The test, which lasted approximately 60 minutes, required students to work on the prepared question sheets, followed by semi-structured and recorded interviews for all participant groups. Thirteen students participated in the interviews, including eight from the high-ability group, four from the middle-ability group, and one from the low-ability group. The interviews were conducted over three days following the written test implementation. The interviews aimed to obtain data that was not collected during the test, assess the student's ability to transform and process conversions accurately, and determine their level of comprehension of the given concepts or problems. Students who answered correctly and could explain the answers orally were deemed to have understood the concepts or problems. Conversely, if a student's written response was correct but could not explain orally, it was concluded that the student had written without understanding, indicating the possibility of copying a peer's answer or cheating.

Furthermore, the results of student work and interviews were analyzed and grouped into categories. Students who can do all the questions correctly and can explain correctly orally, both in the treatment transformation and conversion qualitatively, are called the "Skill encountered at" category; if the student's answers are only partially correct, then it is categorized as "Skill partly encountered at" and if all the answers are wrong or blank answer sheets, it is classified as "Skill not encountered at" (Uzun & Arslan, 2009).

Results and Discussion

This section discusses the results of student work related to semiotic representation skills in solving the given problems. The semiotic representation skills that were analyzed were seen based on the group's level of students' mathematical abilities. The results are summed up in the form of categories: "Skill encountered at", "Skill partly encountered at", and "Skill not encountered at".

Analysis of Problem 1

Problem 1 has four questions. Questions 1, 3, and 4 require students' skills in geometric representations based on verbal expressions or descriptions of the context presented. While question 2 requires students' skills to construct equations or mathematical models based on verbal expressions (questions) or conversions from verbal to algebraic expressions. High and medium-ability students have demonstrated their ability to construct mathematical equations or models and draw rectangles relevant to the first and third questions. The form of the equation created $S = Q – P = 104 – 80 = 24$. Then, 24 was converted (conversion transformation) into size $p \times l$ (p: length and p: width), making it possible to meet the size of the remaining newly tilled land as an expansion or addition of the area previously used. The remaining available land size was (10 x 8) m, adjusted to the resulting expansion area of 24 m^2. Next, 24 was represented in size ($p \times l$): 8x3, 6x4, and 12x2 (transform treatment). However, for the 12x2 size, it was not possible because the remaining land was only 10x8. Sizes 8x3 and 6x4 were asked to be drawn geometrically (conversion from algebraic expressions to geometric), but none of the students could do it correctly (see Figure 1). It can be inferred that the students have competence in Algebra problem-solving but struggle with accurately converting geometric expressions. The following example highlights the problem-solving ability of the high and medium groups when tackling Problem 1.

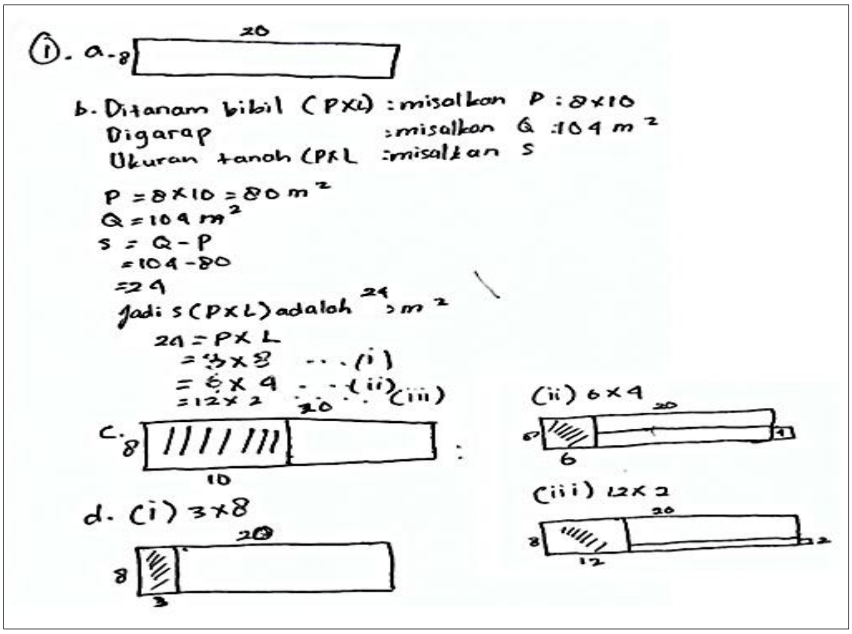


Figure 1. Example of student work on problem 1 at high and medium-ability levels

The low-ability group of students exhibited a lack of skill in composing mathematical equations and an inability to translate algebraic expressions into geometric expressions in all aspects of problem 1. The findings from the analysis of Problem 1 are presented in Table 1.

Table 1. Skills expected in problem 1

Skill	Activity/ Semiotic Representation Indicator	Groups of Students Based on Their Level of Mathematical Ability		
		High	Medium	Low
Presenting story problems (verbal) in the form of geometric images	Question 1 conversion from verbal expressions to geometric expressions	Skill encountered at	Skill encountered at	Skill not encountered at
Create a mathematical model and determine the size of the expanded land	Question 2 conversion from verbal expression to algebraic expression, and treatment in algebra	Skill encountered at	Skill encountered at	Skill not encountered at
Make shading on land that has been planted with vegetables	Question 3 conversion from verbal expressions to geometric expressions	Skill encountered at	Skill encountered at	Skill not encountered at
Draw a section of expanded ground of any possible size	Question 4 conversion from algebraic expressions to geometric ones	Skill partly encountered at	Skill partly encountered at	Skill not encountered at

Analysis of Problem 2

Problem 2 also has four questions. Questions 1, 2, and 3 require skills in transforming treatment in algebra based on verbal expressions or descriptions of the stories presented. While

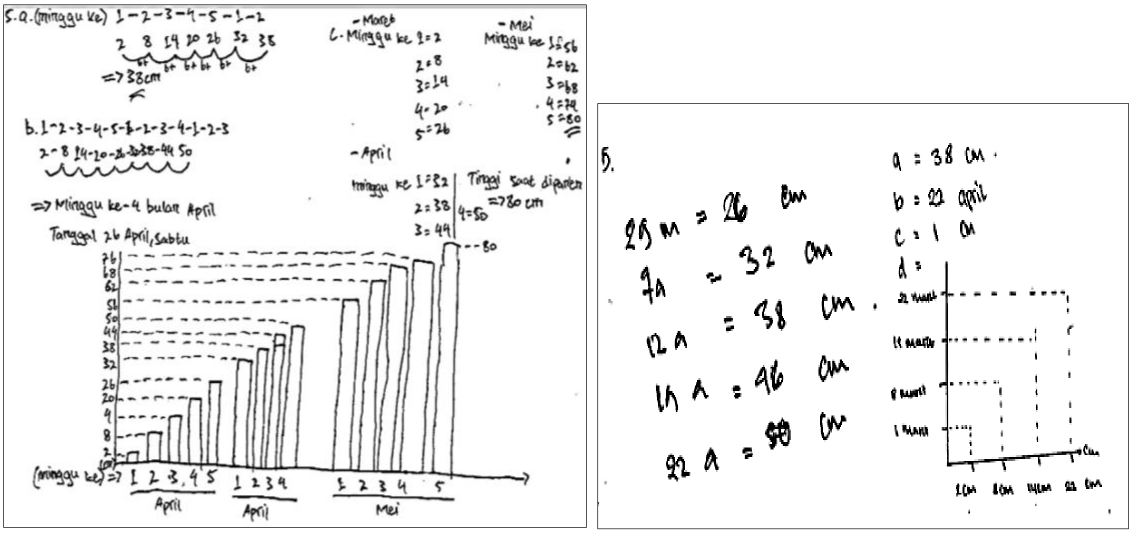
Question 4 requires students' skills to carry out conversion transformations from tables to graphics. The high-ability group solved Problem 2 by writing weeks 1, 2, 3, … respectively with their equivalents: 2, 8, 14, 20, … as the height of the beanstalk. Students wrote 6+ as a symbol for the height of the peanut tree by 6 cm per week. Subsequently, the students determined the height of the peanut tree on April 12, which was 38 cm. To arrive at this conclusion, they considered that there were five weeks in March, and by the end of the fifth week, the tree had reached a height of 26 cm. Therefore, in the first week of April, the tree had grown to 32 cm, and by the second week of April (specifically on April 12, Saturday), it had reached 38 cm. The second question required the students to determine when the peanut tree would reach a height of 50 cm. The students did the necessary calculations and determined that the tree would reach 50 cm in the fourth week of April (April 26). The third question asked the students to calculate the height of the peanut tree at three months from the initial measurement. The high-ability group responded by providing algebraic expressions to answer questions 1 to 3 and determined that the tree would reach a height of 80 cm on May 31 by using a provided calendar as a reference.

In addition, question 4 requires converting data from tables to a graph. The high-ability group demonstrated this skill by creating a bar graph with some spacing (although with some misconceptions). However, the moderate-ability group only provided answers without clearly describing the steps to derive the algebraic expressions. Similarly, they lacked the skill to draw a growth chart for the peanut tree in question 4. These students appear to be answering without a deep understanding of the concepts and questions, as evidenced by their explanations during the interviews. For example, during an interview with a student from the moderate-ability group, M2, the following response was recorded.

Q : *What is the difference in the height of the peanut trees each week?*
M2 : *The difference is seven because every week, the difference is seven days*
Q : *How tall was the peanut tree from the first measurement in March?*
[on the questions written in the table]
M2 : *2, 3, 6*
Q : *Are the differences different or the same?*
M2 : *Changed, because every week it will increase to 6 cm [misconception]*
Q : *The question is, does the difference change or not?*
M2 : *Changes - oh, I'm wrong, but the change is still 6cm every week*
Q : *First question, how tall was Cindy's peanut tree on Saturday, April 12?*
M2 : *The answer is 38 cm*
Q : *How do you get the result of 38 cm?*
M2 : *So that adds up, adding up 6 cm each week*
Q : *Then the second question, when will the height reach 50 cm?*
M2 : *April 22*
Q : *What day is April 22?*
M2 : *Tuesday*
Q : *If you look at the questions, what does Cindy measure every day?*

M2 : *Measure it on Saturday.*
Q : *This means the day should change or not*
M2 : *No, it shouldn't change. It means my answer is wrong [confused]*

Figure 2 below is an example of solving Problem 2, representing high-ability (a) and medium-ability (b) groups.



(a) (b)

Figure 2. Example of student work on problem 2 at high and medium-ability group

The findings from the analysis of student work revealed that the low-ability group was unsuccessful in their treatment and conversion transformation skills for Problem 2. The low-ability group showed a lack of proficiency in algebraic expressions (tasks in algebra) for questions 1 to 3 and could not convert table data to graphics for question 4, leading to their inability to complete question number 2. The findings, including the skills of the high, medium, and low-ability groups related to treatment and conversion transformation, are presented in Table 2.

Table 2. Skills expected in problem 2

Skills	Activity/ Semiotic Representation Indicator	Groups of Students Based on Their Level of Mathematical Ability		
		High	Medium	Low
Determining the height of the beanstalk on April 12	Question 1 Treatment in algebra	Skill encountered at	Skill partly encountered at	Skill not encountered at
Determines the date when the peanut tree will reach 50 cm in height	Question 2 Treatment in algebra	Skill encountered at	Skill partly encountered at	Skill not encountered at
Determining the height of the peanut tree at harvest (3 months of age)	Question 3 Treatment in algebra	Skill encountered at	Skill partly encountered at	Skill not encountered at
Draw a pea tree growth chart	Question 4 Conversion from tables to graphics	Skill encountered at	Skill not encountered at	Skill not encountered at

Analysis of Problem 3

Problem 3 comprises three distinct questions that require various mathematical skills. The first question necessitates the ability to convert pictorial expressions into algebraic expressions, while the second question involves applying algebraic manipulation techniques. Finally, the third question demands the capacity to translate algebraic expressions into verbal expressions and draw conclusions based on them. In solving Problem 3, both the high and moderate-ability groups employed a combination of inductive and deductive reasoning to validate their conjecture regarding the general formula or the formula for determining the number of conifer trees and apples in each row.

Inductive reasoning is demonstrated by students writing: the first row of conifer trees is 8, the second is 16, the third is 24 and the fourth is 32. It can be seen that the formula is n x 8. For an Apple tree, the first row is 1x1 = 1, the second row is 2 x2 = 4, the third row is 3 x3=9, and the fourth row is 4x4=16 so the general formula found is n^2. To determine the number of apple trees and conifers in row 8 (first question), students did a task in Algebra by replacing n with 8. Students interpreted the symbol n for an unknown number, an integer. Both groups concluded that the most numerous trees were conifers (question 3). They argued that having more conifer trees would protect the apple trees, making them more beneficial and, therefore, should be present in greater numbers. However, this argument lacked a foundation in algebraic manipulation and transformation, rendering it inappropriate. Consequently, this instance, the ability to convert algebraic expressions into verbal expressions was not exhibited. During the interviews, the students ultimately revised their initial conclusions and determined that more apple trees were present. This revision was prompted by their recognition of the error in their reasoning, leading to a correction in their thought process.

The interview was conducted with student representatives from the high-ability group (H6) and the medium-ability group (M1). H6 revised its conclusion because after replacing the value of n with 9 (9th row), he found more apple trees (induction reasoning). While M1 used deductive reasoning that n^2 (the formula for many apple trees) is greater than n x 8 (the formula for the number of conifer trees). The following is an excerpt from an interview with H6 and M1).

Interview with H6 students

Q : *So, your conclusion is that the most numerous trees are conifer trees? [looking at the answer sheet]*
H6 : *Apple trees are the most numerous [H6 revised its conclusion]*
Q : *Why are there so many apple trees?*
H6 : *You see, the conifer trees in the ninth row are 72, while the apple trees in the ninth row are 81.*

Interview with M1 students

Q : Between 8 x n and n x n. Which one is more?
M1 : n x n
Q : Then how?
M1 : It turns out there are more apples.

As for the low-ability group, no skills were found for conversion from pictorial expressions to algebra (question 1), treatment in algebra (question 2), or conversion from algebraic expressions to verbal expressions (question 3). Table 3 presents the results of the analysis of student answers.

Table 3. Skills expected in problem 3

Skills	Activity/ Semiotic Representation Indicator	Groups of Students Based on Their Level of Mathematical Ability		
		High	Mediu m	Low
Defines a lot of conifers and apple trees in the 8th row	Question 1 conversion from image expression to algebra	Skill encountered at	Skill encountered at	Skill not encountered at
Specifies the number of conifers and apple trees in the nth row	Question 2 Tritmen in algebra	Skill encountered at	Skill encountered at	Skill not encountered at
Make inferences about what Trees are the most	Question 3 conversion from algebraic expressions to verbal expressions	Skill partly encountered at	Skill partly encountered at	Skill not encountered at

This study investigated the effectiveness of four mathematical representations types: verbal, geometric, graphic, and algebraic, in solving algebraic problems. The participants were categorized into three groups based on their mathematical abilities: high, medium, and low. The analysis of the results of each problem focused on the expected skills exhibited by each group.

Students' Skills in Compiling Equations or Mathematical Modeling and Making Conversions from Algebraic Expressions to Geometric Ones

The skills of compiling mathematical equations were found in students in the high and medium-ability groups, as well as conversion skills from algebraic expressions to geometric ones, namely drawing rectangles, even though they did not match the size or elements that had been determined. Students have difficulty determining the size (pxl) that must be adjusted to the available land area. Students cannot interpret transformations in different representational systems (Gagatsis & Shiakalli, 2004; Lesh, et al., 1987). In this case, students can carry out tasks in algebra, namely making measurements suitable as the area of the available land. However, these measurements cannot be described or translated into the rectangular images they draw. The low-ability group exhibited difficulties in initiating mathematical modeling as the first step in problem-solving. The analysis revealed that the students had difficulty interpreting the problem (verbal transformation). This led to a cascade of difficulties, such as converting verbal expressions into mathematical modeling, performing algebraic triangulation, and transforming algebraic models into geometric shapes (i.e., drawing a rectangle of a specific dimension).

Students' Skills in Verbal Expression (Examining Literacy Questions) and Conversion Skills from Tabular Expressions to Graphics

The high-ability student group could complete all questions, including converting algebraic expressions presented in tabular form into graphical representations. Conversely, the medium-ability group showed inadequacies in converting verbal into algebraic expressions, as their answers lacked a clear description of the steps involved in the process. Additionally, the graphic drawing skills of this group were also lacking, as evidenced by their inability to complete the fourth question. The root cause of these inadequacies appears to stem from an inability to comprehend verbal expressions, a critical component of literacy questions. These types of questions demand students' proficiency in a range of skills, including identifying, understanding, analyzing, communicating ideas, calculating, and interpreting mathematical problems in different contexts (Fery, et al., 2017).

During the interview with M2, it was observed that students struggled with solving problem 2. Specifically, M2 struggled with understanding the variation in the height of the peanut tree, mistakenly assuming that the height difference was always changing. Additionally, M2 incorrectly interpreted the frequency of measurements, as the written questions clearly stated that measurements were taken every Saturday, but M2's calculation of the peanut tree's height was based on measurements taken on a Tuesday (April 22). This suggests that the student's weaknesses in identifying and interpreting verbal expressions significantly impact their ability to communicate, perform calculations, and apply algebraic treatment skills or conversion of algebraic expressions into graphical representations. In this instance, middle-ability students were unable to create a growth chart for the peanut tree, despite having the tabulated data, highlighting their difficulties with graphing data.

Students are not aware that graphs and charts are alternative ways of representing the same concept (Gagatsis & Shiakalli, 2004). Additionally, for low-ability students, the required skills in problem 2 were not exhibited. Some students left their answer sheets blank, while others provided answers without explaining the correct solution. These students struggle to interpret and comprehend verbal expressions within the questions, hindering their problem-solving ability. According to Lesh, et al. (1987) and Gagatsis and Shiakalli (2004), the inability to interpret or translate a concept or problem is a significant factor affecting the learning and performance of mathematical problem-solving. Although problem 2 could also be approached with the concept of number patterns, students tended to rely on procedural knowledge to solve it

Student's Skills in the Conversion of Verbal Expressions and Square Geometric Patterns to Inductive-Deductive Reasoning

Students in the high and middle-ability groups solved Problem 3 using inductive-deductive reasoning alternately to prove their conjectures about the number pattern represented by a square-shaped picture (square pattern). The square pattern has sides containing conifers on the outside and apples on the inside and will increase in size with each row or syllable. In this particular task, students were given square patterns and asked to determine the number of trees in each row. Students used inductive reasoning to transform pictorial patterns into numerical patterns. While some students relied on deductive reasoning to determine the number of trees in the next row (line 8). High- and medium-ability students could generate general formulas or equations (deductive reasoning). However, neither group was able to conclude which type of tree would dominate if the pattern were extended to a larger area.

Some students initially concluded that conifer trees were the most based on the argument that they were protective, without providing any algebraic analysis or expression to support their claim. However, during the interview, they revised their conclusion. This suggests that students may struggle with converting algebraic expressions into verbal expressions. This finding is consistent with research by Gagatsis and Shiakalli (2004) that high school students are rarely asked to write verbal expressions corresponding to algebraic expressions, and vice versa. Similarly, Duval (2000) noted that students encounter difficulties learning mathematics when faced with proof tasks or verbal problems in arithmetic or algebra.

The research instrument used in this study was adapted from National Examination and PISA questions. It was intended to assess students' semiotic representation skills and mastery of previously learned mathematical content. However, it became apparent that many students did not have a strong grasp of the concepts they were expected to master. It was evidenced by their difficulties in solving the questions presented to them. For example, when asked about the meaning of the symbol "m" (representing meters) in question 1, a student from the low-ability group (L4) responded, "It just follows the example of a practice question." Similarly, several students could not explain properly when asked to interpret or explain the meaning of the questions in their own words. Many of these students admitted to never having encountered questions like these before.

In addition, the researchers gathered information from one of the teachers (D), who reported that most students were not keen on being challenged during the learning process. For instance, students were less responsive in finding formulas and tended to seek final results or formulas already available without showing interest in understanding the formula-discovering process. Similarly, when presented with non-routine questions, students avoided finding the solutions and instead relied on the teacher to explain the solutions. The teacher acknowledges the challenges of dealing with students who lack motivation and prefer to have instant answers, such as copying their peers' responses. This situation poses a problem for almost all mathematics teachers, particularly given the student's technological dependence on cell phones and gadgets. These are issues that need to be addressed by mathematics teachers and education observers. The insights were gained through discussions multiple teachers at the research site.

Conclusion

Students' semiotic representation skills vary considerably, depending on the type and complexity of the representation involved. The analysis of the test and interview results, revealed that only the high-ability group displayed conversion and transformation skills from algebra to graphics. Conversely, the low-ability group struggled with semiotic representation skills across all three problems. They experienced challenges in transforming verbal descriptions into equations or mathematical models.

This study found that high- and middle-ability students tend to have verbal expression skills, enabling them to interpret word problems and convert them into algebraic expressions or mathematical models. However, it is essential to note that possessing specific transformation skills does not necessarily imply proficiency in other skills, as various factors can affect their development. Therefore, it was observed that although high and medium-ability students exhibited conversion skills from verbal to algebra and treatment skills in algebra, they struggled with the proper conversion from algebra to geometry or graphics and from algebra to verbal expressions, such as making general conclusions or generalizations. These results highlight the importance of semiotic representation for students, as verbal and non-verbal representations are necessary for problem-solving activities. Teachers and observers of mathematics education should take note of these findings to ensure that students develop proficiency in all types of semiotic representation skills.

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