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Exploration of Students' Epistemological Obstacles in Understanding the Concept of Variables and Expressions

Rosliana Siregar[1], Muhammad Daut Siagian[2*], Tommy Tanu Wijaya[3]
[1,2]Mathematics Education Department, Universitas Islam Sumatera Utara, Indonesia
[3]School of Mathematical Sciences, Beijing Normal University, China
*Email: *mdsiagian@fkip.uisu.ac.id*

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Abstract. *Variables and expressions serve as bridges where students cross from arithmetic to algebra. Although variables and expressions are important concepts in middle school and high school mathematics, they are topics that many students find challenging, and many, in fact, do not develop a thorough understanding of these topics. This study aims to explore the epistemological obstacles students face in understanding and interpreting the concepts of variables and expressions. Based on these objectives, a qualitative research design with a phenomenological approach was chosen to achieve this research objective. The subjects in this study were 8th grade middle school students in the city of Medan, Indonesia. Tests and interviews were used to collect data. The data obtained were analyzed using an inductive approach. The data obtained are presented in narrative and graphical forms. Based on the research results obtained, six types of student errors in solving problems related to the concept of variables and expressions were identified. Overall, students' limitations in understanding and interpreting variables as something unknown is one of the triggers for epistemological obstacles. Teachers should give chances for pupils to debate and explain variables and expressions in the classroom to assist students gain a comprehensive knowledge of these mathematical tools.*

Keywords: *epistemological obstacle, variables, algebraic expression*

Introduction

The NCTM recommends that the notion of "algebra" be expanded to cover a wide range of mathematical activities. because basically all students can learn algebra and children can develop algebraic concepts at an early age (NCTM, 2000). It is essential for students to acquire knowledge of algebra and related procedures and skills if they are to engage in higher mathematics and other disciplines such as science and economics (Kieran, Pang, Schifter, & Ng, 2016). It was further explained that algebra is a gateway for high-level mathematics and opportunities (Kriegler, 2007), and even algebra is a very important material for students to master because it is one of the materials that will be used in daily life activities, both implicitly and explicitly (Suhaedi, 2013). This is in line with the view that algebra emerges from human activity (Kaput, 2008). Algebra also introduces children to variables and various mathematical symbols that can be used to simplify sentences into mathematical models in order to solve problems in everyday life. This is in line with Herstein's statement that algebra is the study of mathematical symbols and the rules for manipulating symbols (Herstein, 1964).

To be able to manipulate mathematical symbols, one needs a comprehensive understanding of the basic knowledge of these algebraic concepts. The basic thing that must be learned is how to interpret and understand the concept of variables and expressions. Understanding variables is a fundamental thing that is very important to pay attention to. With a proper understanding of variables in algebra, students will be able to write expressions correctly. However, in reality, some studies report that students have difficulty understanding algebraic concepts such as variables and algebraic expressions. (Jupri, Drijvers, & van den Heuvel-Panhuizen, 2014; MacGregor & Stacey, 1995; Macgregor & Stacey, 1997; Mengistie, 2020). In addition, another error that students often make is an error in representing a mathematical problem in a mathematical model, in this case related to equivalence of expressions (Tabach & Friedlander, 2017), suppose $3 + 4x$ is equal to $7x$. Such a habit is associated with the student's desire to use algebraic expressions as "closed result" in the same way in obtaining the results of calculations such as $3 + 4 = 7$.

Based on research conducted by Siagian et al. (2021), it appears that students have not been able to understand and solve the question of algebra correctly. One reason is that the student's understanding of variable concepts is not comprehensive; students understand variables as something unknown. The notion of the unknown can become an epistemological obstacle when trying to conceptualize the meaning of variables (Panizza, Sadovskya, & Sessa, 1997). The numerous uses of the term "variable" are often the basis of the difficulty students in understanding it. Research by Schoenfeld and Arcavi (1988) has shown that some students grapple with distinctions between object names (eg, Michael's person), attribute names (eg, Michael's height), or a measurement or quantity (hours). Students can manipulate variables without really understanding the power and flexibility of symbols (Macgregor & Stacey, 1997). Students can interpret letters or algebraic expressions based on intuition, just guess, or compare them with other symbol systems they know (Wagner & Parker, 1993). In addition, there is evidence that shows students' understanding of variables is fixed and unchanged over the years at school, even up to the college level (Macgregor & Stacey, 1997).

Based on the problems described and the urgency of understanding the variable, this study aims to investigate the learning obstacles experienced by students related to their understanding of the concept of variables and algebraic expressions. Obstacles to learning are difficulties that students experience in learning due to external factors. Brousseau (2002) argues that learning obstacles experienced by students come from various sources, including ontogeny obstacles, didactical obstacles, and epistemological obstacles. Ontogenic obstacles are related to student difficulties caused by student unpreparedness. There are three types of the ontogenic obstacles: psychological, instrumental, and conceptual aspects. The psychological aspect is related to the student's readiness to study. Instrumental refers to students' lack of technical or operational knowledge while learning. Conceptual relates to students' unpreparedness in terms of prerequisite materials or designs that the teacher gives that are too easy or difficult. Epistemological obstacles are associated with learning difficulties caused by how students gain understanding of history or their origins. Didactic obstacles are related to learning difficulties caused by teaching materials, such as the suitability of the learning flow that the teacher uses (Sidik, Suryadi, & Turmudi, 2021; Suryadi, 2019).

There are several studies on investigating learning obstacles and students' understanding of algebraic concepts (Blanton et al., 2015; Job & Schneider, 2014; Jupri, Drijvers, & van den Heuvel-Panhuizen, 2015; Maknun et al., 2022; Moss, Boyce, & Lamberg, 2019; Noto, Pramuditya, & Handayani, 2020; Rachma & Rosjanuardi, 2021; Siagian et al., 2022; Sidik et al., 2021), and some of them focus on supporting students' algebraic understanding. As done by Noto et al. (2020), they conducted an analysis of student learning obstacles, especially epistemological obstacles to understanding algebraic concepts. This study involved 17 students in grade 7. And the results found that students generally could not interpret the concept and clarify the elements of algebraic forms. Meanwhile, Jupri et al. (2014) conducted a study by investigating students' difficulties in learning algebra. The findings of this study reveal that mathematization, namely the ability to translate back and forth between the world of problem situations and the world of mathematics and to rearrange the mathematical system itself, is the most frequently observed difficulty in both written tests and interview data. Other difficulties observed relate to understanding algebraic expressions, applying arithmetic operations to numerical and algebraic expressions, understanding the different meanings of the equal sign, and understanding variables.

Considering previous research, this research will explore students' learning obstacles in terms of epistemological obstacles in understanding the concept of variables and expressions. Variables and expressions were chosen as a research topic with the consideration that the concepts of variables and expressions are fundamental things that will lead students to succeed in understanding algebraic comprehensively. However, this is in contrast to the fact that there are still problems experienced by students in interpreting variables. so that this becomes an obstacle for students when expressing algebraic forms. Thus, it is necessary to conduct a search related to learning obstacles, which are the basis of problems for students in understanding variables and expressions. It aims to gain a comprehensive view of the learning obstacles experienced by students in studying algebraic concepts, in particular in understanding and interpreting variable and expressions. Based on this, educational practitioners can later provide an overview of an epistemic learning process in teaching the concept of variables and expressions.

Method

The study uses a qualitative research design with a phenomenological approach. Phenomenology is interpreted as a qualitative method that focuses on understanding and interpreting human life experience as a topic according to his own frame of reference, namely with regard to meaning and how that meaning is obtained from experience (Grbich, 2007; Langdridge, 2007; Suryadi, 2019). The phenomenon that will be investigated is middle school students' learning obstacles in understanding the concept of variables and expressions. Furthermore, there is a connection between the phenomenological realities obtained through normal interpretation and relevant theories (pragmatic interpretation). The subjects in this study were 22 students in the 8th grade of middle school in Medan who had learned about the form of algebra. After the students are given the test, the data is sorted or selected from 22 to 5; in the final stage, the data are summarized into 3 students, which are subsequently encoded into S1, S2, and S3. It aims to ensure that the data can be obtained in a thorough and comprehensive manner as needed, as well as to assist researchers in drawing quality conclusions. Data reduction is done so that the student’s experience in understanding and defining variable concepts and expressions can be explored comprehensively.

Data were gathered through tests and interviews designed to elicit information about students' experiences and how those experiences influenced their perceptions of various concepts and expressions. The tests given to students relate to the concept of variables and expressions as presented in Table 1.

Table 1. Test instruments

Problem type	Problem description
1	How many terms are in this expression? $6n + 1$ Mention these terms and their explanations.
2	Rigo borrowed 20 marbles from Egar, then Rigo returned the d marbles to him. Choose an expression that shows how many marbles Rigo still has to pay to Egar. a. $20 + d$ c. $\frac{20}{d}$ b. $20 - d$ d. $20d$
3	Ogi had the following problem to solve: "Find the value for x, in the expression: $x + x + x = 12$" Then Ogi answered in the following way: a. 2, 5, 5 b. 10, 1, 1 c. 4, 4, 4 Which of Ogi's answers is correct? (Circle the correct letters: a, b, and c) and give the right reasons according to you.
4	Observe the following expressions:  Write another form of the expression above, and in your opinion, can the above expression be operated on? If yes or no, give your reasons.

Validation and reliability tests are performed prior to the usage of the test instrument. The validity and reliability tests performed are based on the opinions of Cohen et al (2020). Content validation was chosen and found acceptable to examine the validity of the test instrument used in this study to see the applicability of the context in algebraic material, particularly the notion of variables and expressions both theoretically and practically. While the reliability test was conducted to see how the context of the information supplied affects students' performance in answering questions, the researcher was interested in the level of readability of students' grasp of variables and expressions. The interviews were conducted one-to-one after the researcher analyzed the results of the tests that had been obtained to ascertain the experiences and obstacles encountered by the students. All data obtained was transcribed, and pseudonyms were given to each participant. The data obtained were analyzed using an inductive approach, which combines a systematic method of managing data through reduction, organization, and connection (Dey, 1993; LeCompte, 2000), and the data obtained is presented in narrative and graphical forms. Learning obstacles can be detected by studying student failures in problem solving including the notion of variables and expressions. As a result, in addition to mistakes, this study examines additional phenomena in students' replies. To be able to group the mistakes acquired, the researcher presents themes based on the same patterns and types of errors based on the data produced and evaluates past studies as assumptions about features of methods of thinking and comprehending each form of error. One-on-one task-based interviews were conducted after the classification technique and descriptive analysis of reported student faults were completed. The goal of this interview is to improve the descriptive component of the thinking style as well as the comprehension of the errors discovered.

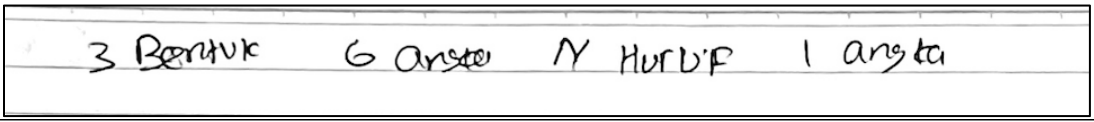
Results and Discussion

The research results are presented in illustrations of students' work in solving problems related to the concept of variables and expressions, which are supported by interview data. The data are presented in themes (see Table 2), which are based on students' mistakes in completing tests related to the concept of variables and expressions.

Table 2. Themes of student errors in solving problems

	Error Theme	Number of students
1.	Limitations in interpreting algebraic forms	16
2.	Using algebraic expressions as “closed results”	17
3.	Has limited understanding of algebraic expressions	16
4.	View variables as quantities	16
5.	Holding the misconception that the same letter does not necessarily represent the same number	18
6.	The tendency to think arithmetic	20

Based on Table 2, an explanation will be carried out related to each theme of student errors in solving the three problems presented in Table 1. Students make mistakes when solving problems because they have a limited understanding of interpreting algebraic forms. This can be seen from the results of student work in Figure 1.



It consists of 3 shapes, 6 numbers, N letters, and 1 number

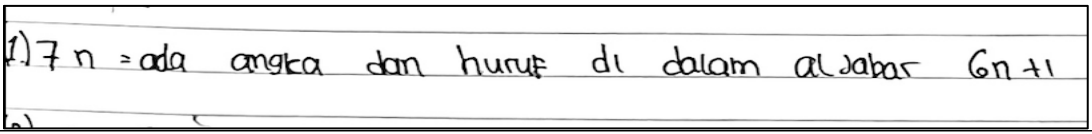
Figure 1. Illustration of S1 students' answers to problem one

Based on Figure 1, it shows that students do not have a comprehensive picture of the algebraic terms contained in the $6n + 1$ expression (see interview excerpts from Table 3). In contrast, students interpret the terms in the expression $6n + 1$, which is composed of 3 forms, namely 6 as a number, n as a letter, and 1 as a number.

Table 3. Interview snippet

Researcher	Respondents
What do you think of when you see algebraic forms?	Yes, like my answer, there are letters and numbers
Have you ever heard or read the terms coefficients, variables, or constants?	It looks like it has variables, which, if I'm not mistaken, are the letters. Like x, y, and that!
What variables do you understand?	Variables are marked with letters
What do the letters mean here?	Ehmmm…

On the other hand, it was also found that students solved problem one by operating the $6n + 1$ algebraic expression like general addition; this can be seen in Figure 2.



7n = There are numbers and letters in algebra: 6n + 1

Figure 2. Illustration of S1 students' answers to problem one

Figure 2 shows that in solving problem 1, there are misconceptions in the students' understanding of the problem given. However, when viewed from the student's answers above, it can be stated that students use algebraic expressions as "closed results," namely, $6n + 1$ equals $7n$. This is clarified by interview data with students presented in Table 4.

Table 4. Interview snippet

Researcher	Respondents
In problem one, what is asked is how many terms are in the expression $6n + 1$. So, how come your answer is $7n$?	Yes, I think there are numbers and letters in $6n + 1$ algebra
Then what does 7n mean?	Yes, the result of $6n + 1$
Can $6n + 1$ add up?	Yes
Explain how it can be added up!	Yes, $6 + 1$ equals 7, and because 6 has n, the result is $7n$

Based on the interview excerpts from Table 4 above, students' understanding of algebraic concepts is not yet comprehensive, so they have a tendency to view algebraic expressions as "closed results." The interview snippet above also shows that students do not understand the elements contained in algebraic expressions, such as coefficients, variables, and constants. Based on this, it can also be concluded that students have a limited understanding of algebraic expressions. Students' limitations regarding algebraic expressions can also be seen based on their answers to problem 2. In problem 2, students are unable to determine the exact algebraic expression that describes the situation. The following is an illustration of the form of student answers to problem 2.

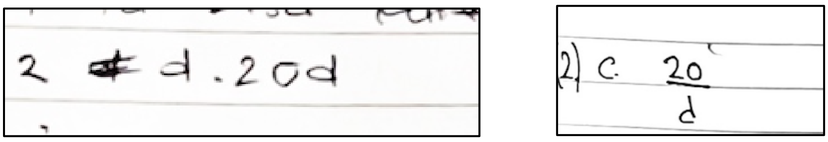
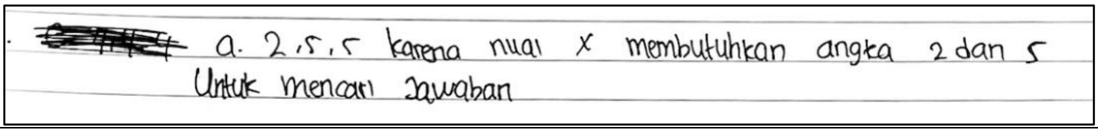


Figure 3. Illustration of S2 students' answers to problem two

On the other hand, students also view variables as quantities. This can be seen from the pattern of answers given by students in solving Problem 4 in Figure 4.



a. 2, 5, 5 because the value of x requires the numbers 2 and 5 to find the answer

Figure 4. Illustration of S3 students' answers to problem three

Based on the pattern of student answers in Figure 4 above, it shows that students cannot understand the meaning of variables precisely. Instead of viewing x as a variable that should have the same value, Students actually view x as only a quantity whose final result is 12 (see the interview excerpt in Table 5).

Table 5. Interview snippet

Researcher	Respondents
Look again at problem four. Why do you think option (a) represents the expression?	Because the sum of $2 + 5 + 5$ equals 12
Ok, options (b) and (c) also add up to 12!	Yes, I think all three choices are correct
Oh, I see! What do you think about x in the expression $x + x + x = 12$?	x is a letter whose value is unknown
Ok, what do you think the variable is?	Variables are lettering whose values are not known, such as x and y
If n is in the expression in problem one, is it also a variable?	As far as I know, only the x and y variables
In problem 4, the variable is x, right? If the variables are the same, are the values not the same?	No, what matters is that the total is 12

The interview snippet in Table 5 above shows that students understand the variable as a letter whose value is unknown. In the context of problem three, students have not been able to understand the meaning of variables precisely. so that students only concentrate on the number of values that must be combined to form the result 12. Based on the interview excerpt in Table 5 above, it can also be concluded that students hold the misconception that the same letter does not necessarily represent the same number. Overall, only 27.3% of the total respondents were able to provide appropriate arguments for solving problem three. Figure 5 below is one form of student answer that can solve Problem 3 correctly.

c. 4,4,4 , alasan x+x+x=12 karena huruf x berarti angka huruf x sama

c. 4, 4, 4, The reason $x + x + x = 12$ is because the letter x means the number of the letter x is the same.

Figure 5. Illustration of S3 students' answers to problem three

According to students with answers in Figure 5, because the expression in the three-letter problem (variable) is all x, then the numbers in the letter x must all be the same, namely 4. If we look closely, the tendency of students to view variables as quantities is because students also have a tendency to think arithmetically. This can also be seen from the results of the students' work in solving Problem 4. Where 90.9% of the respondents could not solve Problem 4 correctly. Students cannot translate the expressions presented in problem four into other forms of expression, as shown in Figure 6.

ekspresi di atas tidak dapat di operasikan karena buahnya tidak sama banyak

The expression above cannot be operated because the fruit is not equal.

(a)

4 karena mereka di jumlahkan senilai 3 Apel dan 1 jeruk

4, because they add up to equal 3 apples and 1 orange.

(b)

ya
3 apel + 1 lemon = 4 buah

Yes, 3 apples + 1 orange are equal to 4 pieces.

(c)

Figure 6. (a), (b) and (c) illustration of S1, S2, and S3 students' answers to problem four

According to the answer in Figure 6(a), the expression in the problem cannot be operated because the fruit is not the same. In other words, students tend to think arithmetic and only focus on quantity (see the interview excerpt in Table 6), but they cannot perceive that in an algebraic context, if apples and oranges are represented in algebraic expressions, they will have different variables.

Table 6. Interview snippet

Researcher	Respondents
Try to explain again the solution to these four problems	In my opinion, it cannot be operated because the number of pieces is not the same
Oh, I see! What do you know about algebraic addition operations?	Ehmmm… I don't remember
What do you think about other expressions of the form of expression in Problem 4?	$3 + 1$

In Figures 6(b) and 6(c), students are convinced that the expression in Question 4 can be operated on or summed up to three apples and one orange. This suggests that students only focus on the amount of fruit presented in question 4 and tend to think about arithmetic as they did in elementary school. Based on their exposure to the research results, it can be concluded that students have not yet developed a comprehensive understanding of the concepts and meanings of variables and expressions.

This is triggered by epistemological obstacles experienced by students. Students have a limited understanding of the meaning of variables. The tendency of students to understand and interpret variables as something that is unknown and represented by letters such as x and y, such an understanding can be said to be an incomplete understanding of the concept of variables. The notion of the unknown can become an epistemological obstacle when students try to conceptualize the meaning of variables (Panizza et al., 1997; Siagian et al., 2021). There are several definitions of the term "variable," which frequently contribute to students' confusion. According to Schoenfeld and Arcavi's research, some pupils struggle to distinguish between the name of an item (e.g., Michael's person), the attribute name (e.g., Michael's height), and the measurement or amount (h units) (Schoenfeld & Arcavi, 1988). According to MacGregor and Stacey (1997), children might modify variables without fully comprehending the power and flexibility of symbols research, some pupils struggle to distinguish between the name of an item (e.g., Michael's person), the attribute name (e.g., Michael's height), and the measurement or amount (h units) (Schoenfeld & Arcavi, 1988). According to MacGregor and Stacey (1997), children might modify variables without fully comprehending the power and flexibility of symbols. Students can read letters or algebraic formulas using intuition, guessing, or comparing them to other symbol systems they are familiar with (Wagner & Parker, 1993). Furthermore, there is evidence that pupils' comprehension of variables remains constant or unchanging throughout their school careers, even up to the college level (Macgregor & Stacey, 1997).

In this study, it was found that students' errors in identifying components in algebraic form were caused by their limited understanding of the meanings of coefficients, variables, and constants. Students only remember coefficients as numbers and variables as letters whose values are unknown. Research conducted by Yansa, Retnawati, and Janna (2021) found something similar. That is, variables are always represented by letter symbols, while coefficients and constants are represented by numbers. The student's misconception is that numbers followed by letters are also called variables, and if a number stands alone or does not contain letters, it is also called a coefficient. Variables and constants are basic concepts used in mathematical modeling and formulas. Understanding the role of variables and constants enables students to become skilled in algebraic manipulation, which is important in mathematical reasoning and for succeeding in math exams (Watson, Jones, & Pratt, 2013).

It is the job of teachers to guarantee that students' earliest encounters with utilizing letters in algebra create the groundwork for a cohesive structure of algebraic knowledge (Macgregor & Stacey, 1997). It is especially important for students to have a strong foundation in algebra, emphasizing that the variable (letter) represents numbers and the equation reflects balance, i.e., the value on the left is equal to the value on the right. Students are exposed to variables, or pronumerals, as a method of encoding numbers with letters. Students may have seen symbols like squares used to represent unknown numbers in the past, but they will now be exposed to popular pronumerals like x, y, and t. In algebra, students might quickly become confused about the meaning of letters. They may believe that the letters indicate a unit of measurement or a secret code to be deciphered. Students can comprehend that the variable represents an unknown quantity if symbols are used to teach the notion of variables, such as $6+? = 9$, and then the statement is written as $6 + x = 9$. Students will understand that these letters mean "for a number" if the origin of the term "pronumeral" is emphasized.

Another student tendency is to frequently use algebraic expressions as "closed results." This tendency is usually associated with students' desire to bring algebraic expressions to "closed results," in a manner similar to the final results obtained in arithmetic exercises. In arithmetic, the final solution is a number, not a mathematical sentence, such as $6 + 1$. In fact, students need to "forget" their arithmetic way of doing math in order to see $6n + 1$ as an acceptable symbolic final answer (Tabach & Friedlander, 2017). Tabach and Friedlander (2008) proposed using the potential of meaningful situations to overcome this obstacle. They stated that their experience building a context-based algebra curriculum demonstrates that this method provides an important bridge between arithmetic and algebra, between physical and abstract things, and allows students to acquire algebraic ideas in a more relevant way.

In addition, the student's paradigm of the meaning of variables as something unknown has an impact on other meanings that are formed in students' minds when faced with contextual problems such as problem four. Students tend to think arithmetic and are not even able to express four-problem sets in the form of algebraic expressions; in other words, students tend to view variables as quantities. Therefore, the teacher needs to emphasize the boundaries of the transition from arithmetic to algebra. It is quite interesting if we observe how students have no difficulty with addition (or subtraction) operations that contain "..." or "• ," but they get confused when the "..." or "• " symbols are replaced with letters like "x" or "y." This shows that variables in the form of letters are not easy for students. To overcome these difficulties, we can use arithmetic operations as a starting point for learning algebra. The transition from arithmetic operations to algebraic operations is done more smoothly. An example is directing students to understand that the letters "x" or "y" have the same position as "..." or "• ." Difficulties in algebraic operations are not only experienced by students who are just learning algebra. Several studies (Linchevski, 1995; Linchevski & Herscovics, 1996) showed that many 13-year-old students could not perform addition and subtraction of algebraic forms.

Variables and expressions serve as bridges by which students cross from arithmetic to algebra (Schoenfeld & Arcavi, 1988). Although variables and expressions are important concepts in middle school and high school mathematics, they are topics that many students find challenging, and many, in fact, do not develop a thorough understanding of these topics. Marpa (2019) stated that some of the students' errors in algebra stem from misunderstanding variables. The main reason for this misconception is a lack of understanding of the basic concept of variables in different contexts. The abstract structure of algebraic expressions poses many problems for students, such as understanding or manipulating them according to accepted rules, procedures, or algorithms. Other reasons for the difficulties students experience in learning algebra are that they do not know the difference between the uses of variables, do not know the role of variables in making generalizations, are unable to interpret variables, and fail to perform operations with variables. To support students in building a deeper understanding of these important concepts, teachers must allow students to work on multiple assignments set in multiple contexts and engage students in working with and discussing multiple meanings of variables.

Conclusion

The results of the study showed six types of student mistakes in solving questions related to variable concepts and expressions. Overall, the limitation of students in understanding and interpreting variables as something unknown is one of the triggers of epistemological obstacles. The limitations of students in understanding and interpreting such variable concepts affect the problem-solving related to the variable concept and expression. On the other hand, the popular use of pronumerals such as x and y also triggered the emergence of epistemological obstacles to understanding the meaning of variables. The limitations of their understanding of the meanings of coefficients, variables, and constants are also one of the epistemological obstacles to identifying the components of algebraic expression. Understanding variables and expressions is important for students to make the leap from arithmetic to algebra, but students generally have misunderstandings about this topic. Students, for example, often use previous experience with alphabet letters used in mathematics as abbreviations of units, and this relationship often stops them from reasoning about variables and expressions as numbers. When understanding and evaluating expressions, students sometimes misinterpret the meaning of coefficients. In addition, while looking at expressions, students can observe a set of discrete sizes without any connection to a single number referenced by the phrase.

Teachers should consider the typical misunderstandings highlighted in this topic and try to correct them through the selection and application of carefully selected mathematical activities. The findings of this study were obtained through the exploration of student experiences in learning algebraic concepts, in particular variable concepts and expressions, through testing. Thus, this becomes one of the limitations of this research, which opens up opportunities for future researchers to look comprehensively by evaluating the learning process, conducting textbook analysis as a survey of aspects of didactic obstacles, and investigating the influence of student development as an overview of the ontogenic obstacles aspects.

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