Robust Optimization Model for Internet Shopping Online Problems with Endorsement Costs in the Fashion Industry

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Abstract
Online business is a business activity carried out via the internet or digitally. Buying, selling, and advertising are done online through e-commerce, social media, or online shops. The products offered vary, including services, food, household needs, and fashion. Selling online is not limited by time and distance, and consumers can obtain information about products and services that can influence their decisions. At the same time, sellers also have the opportunity to advertise their products in a broader range by making endorsements. An endorsement is a form of advertising using well-known figures who are recognized, trusted, and respected by people. In this paper, a model for optimizing the problem of online internet shopping with endorsement fees is formulated. This optimization model aims to maximize the profits gained by sellers in marketing their products online. In marketing products, there is uncertainty in the number of requests. To overcome this uncertainty, an approach is needed that can handle this uncertainty, namely Robust Optimization. The Robust Optimization model is solved using the polyhedral uncertainty set approach, resulting in a computationally tractable optimal solution. The optimal solution for the Robust Optimization model for online shopping problems with endorsement costs on polyhedral uncertainty sets produces a smaller value than the nominal model.
Keywords: Internet shopping online, endorsement costs, Robust Optimization.

1. Introduction

Digitization transfers media from print, audio, and video to digital forms. Digitalization requires equipment such as computers, scanners, media source operators, and supporting software, according to Sukmana [1]. Business digitalization has resulted in online and offline business growth going online. The online business that is experiencing rapid progress at this time is E-Commerce. Electronic Commerce, abbreviated as E-Commerce, is a commercial

transaction between sellers and buyers or with other parties in the same contractual relationship to send some goods, services, or rights transfers. This commercial transaction is contained in electronic media (digital media), which physically does not require a meeting of the parties, and the existence of this media is in a public network of systems as opposed to a private network (closed system). Moreover, this public network system must consider an open system (Ding, [2]).

One strategy in digital business marketing activities that is widely used is an Instagram endorsement. Social media endorsements currently booming are a form of mutually beneficial cooperation between the two parties. Usually, it happens between online shops and artists, influencers, or celebrities because they have a lot of fans and followers, which increases sales for online shops and specific products and services (Hartini,[3]).

Referring to Błażewicz et al. [4], the problem of internet shopping online discussed aims to obtain a minimum cost of managing shopping lists at several available stores by taking into account production costs and shipping costs. A year later, Błażewicz and Musiał [5] devised the first algorithm and introduced the results in computational form. Furthermore, Chung [6] discusses the problem of internet shopping online by adding a delivery limit, where the problem of internet shopping online considers the costs of purchase and also the delivery time. Then Chaerani et al. [7] consider that Chungs [6] optimization model has a delivery time decision variable that needs to be adjusted, so an adjustable variable is formed, which is added to the Optimization Adjustable Robust Counterpart Optimization model with the longest delivery period uncertainty set.

This research will discuss internet shopping online with a focus on this research to meet the number of requests when buying and selling online, where there is uncertainty in the number of product requests, to maximize the profit earned by the seller. The primary reference for this paper is to use the article "Internet Shopping Optimization Problem with Delivery Constraint" by Chung [6]. There are three differences in this study with Chung [6]. First, the model to be discussed takes the seller's point of view, while the reference article discusses it from the buyer's point of view. Second, the parameters used in this study are product selling prices, production costs, number of product requests, production capacity, and storage capacity. At the same time, reference articles involve product prices, shipping costs, and delivery times. Third, this research will involve endorsement costs, namely costs required by stores when they want to advertise their products through influencer services, while reference articles do not involve endorsement costs. This research can be used for online shop as one of their strategies to overcome the uncertainty data in the number of requests.

2. Methods

2.1. **Internet Shopping Online Optimization Model from The Buyer's Side.** The mathematical formulation for the Internet Shopping Online problem from the buyer's side with a delivery time limit has two sets, namely $M = 1, \ldots, m$, which is the set of shops to choose from. The set $N = 1, \ldots, n$ is the set of products that can be selected from m stores. Then give p_{ij} as the price of product i at store j, and f_j as the shipping costs at store j. Then notice that f_j is a fixed costs regardless of the number of products to be purchased. Then there is d_{ij}, which is the estimated delivery time for the product i from store j to the buyer after ordering. The goal of this problem is to buy n products from m stores with minimum cost and delivery time. Then there is the variable x_{ij} as a binary decision variable with a value of 1 if product i is selected from store j and 0 if the product i is not selected from store j. Then w_j is the binary decision variable whether there is a shipping fee at shop j or not. Because the optimization problem has two objective functions, namely costs and delivery time, the internet shopping online optimization model can be written mathematically as a multi-objective optimization

problem formulated as follows (Chung, [6]).

$$\min \sum_i \sum_j p_{ij} x_{ij} + \sum_j f_j w_j,$$

$$\min \max_{i,j} \; (d_{ij} x_{ij})$$

$$\text{s.t.} \; \sum_j x_{ij} = 1, \; \forall i = 1, \dots, n, \tag{1}$$

$$\sum_i x_{ij} \le n w_j, \; j = 1, \dots, m,$$

$$x_{ij}, w_j = \{0, 1\}.$$

2.2. **Integer Programming.** Integer programming is an optimization problem used to find a variable value that is integer. Optimization problems requiring all variables to have integer values are called all-integer programming problems. In contrast, optimization problems that require that some variables have integer values are called mixed-integer programming problems. Integer programming problems can be changed to 0-1 or Binary Integer Programming. The model formulation for Integer Programming is as follows (Rao, [8]).

$$\min Z = \sum_{j=1}^m c_j x_j$$

$$\text{s.t.} \; \sum_{j=1}^m a_{ij} x_j \ge b_i, \; i = 1, \dots, n, \tag{2}$$

$$x_j \ge 0, \; j = 1, \dots, m,$$

$$x_j \; \text{integer}.$$

2.3. **Strong Duality Theory.** If there is an optimal solution for a symmetrical primal or dual program, then the other program has an optimal solution, and both objective functions have the same optimal value (Bronson and Naadimuthu, [9]). The rules for contructing the dual problem can be seen in Table 1.

TABLE 1. Rules for construction the dual problem.

	Primal	Dual
Objective Function	$\min c^T x$	$\max b^T w$
Variable	$x_i \ge 0$	Constraint-i:$A_i^T w \le c_i$
Variable	x_i free	Constraint-i:$A_i^T w = c_i$
Constraint	$A_i w = b_j$	Variable-j free
Constraint	$A_i w \ge b_j$	Variable-j:$w_j \ge 0$
Coefficient Matrix	$A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{bmatrix}$	$A^T = \begin{bmatrix} A_1 & A_2 & \cdots & A_m \end{bmatrix}$
Right Side	Vektor b	Vektor c
Coefficient of the objective	Vektor c	Vektor b

2.4. **Robust Optimization.** Robust Optimization is a method to solve Robust Optimization problems for parameter data uncertainty. The uncertainty is assumed to be in an uncertainty set, referring to Ben-Tal and Nemirovski [10]. The following is a general model of an uncertain linear optimization problem (Gorissen, Yanikoğlu and den Hertog, [11]).

$$\min_{\mathbf{x}} \left\{ \mathbf{c}^T \mathbf{x} : \mathbf{A}\mathbf{x} \le \mathbf{b} | (\mathbf{c}, \mathbf{A}, \mathbf{b}) \in \mathcal{U} \right\}, \tag{3}$$

where $\mathbf{c} \in \mathbb{R}^n$, $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{A} \in \mathbb{R}^{m \times n}$, $\mathbf{b} \in \mathbb{R}^m$ and $\mathbf{U}$ is primitive uncertainty set. Furthermore, the constraint matrix $\mathbf{A}$ is expressed in terms of a primitive uncertainty parameter $\zeta \in \mathcal{Z}$, where $\mathcal{Z} \subset \mathbb{R}^L$ is a primitive uncertainty set, thus obtaining:

$$\min_{\mathbf{x}} \left\{ \mathbf{c}^T \mathbf{x} : \mathbf{A}(\zeta)\mathbf{x} \leq \mathbf{b} | \zeta \in \mathcal{Z} \right\} \leftrightarrow \min_{\mathbf{x}} \left\{ \mathbf{c}^T \mathbf{x} : \mathbf{A}(\zeta)\mathbf{x} \leq \mathbf{b} | \mathbf{a}_i^T(\zeta)\mathbf{x} \leq \mathbf{text}_i, \ i = 1, ..., m, \ \forall \zeta \in \mathcal{Z} \right\}. \tag{4}$$

solution $\mathcal{Z} \in \mathbb{R}^L$ is called Robust feasible if it satisfies all the uncertain constraints $[\mathbf{A}(\zeta)\mathbf{x} \leq \mathbf{b}]$ for all realizations of $\zeta \in \mathbf{Z}$. Given the constraint-wise assumption of uncertainty in Robust Optimization, the problem model in (4) can be focused on a single constraint as follows.

$$(\bar{\mathbf{a}} + \mathbf{P}\zeta)^T \mathbf{x} \leq \mathbf{b}, \ \forall \zeta \in \mathcal{Z}, \tag{5}$$

where $(\bar{\mathbf{a}} + \mathbf{P}\zeta)$ is an affine function over primitive uncertain parameters $\zeta \in \mathcal{Z}$, $\bar{\mathbf{a}} \in \mathbb{R}^n$, and $\mathbf{P} \in \mathbf{M}_{n,L}(\mathbb{R})$.

Theorem 2.1 (Ben-Tal and Nemirovski, [10]). *Assume that the uncertainty set $\mathcal{U}$ is an affine image of a bounded set $\mathcal{Z} = \{\zeta\} \subset \mathbb{R}^n$, and $\mathcal{Z}$ is given either by:*

1) A system of linear inequality constraints

$$\mathbf{Q}\zeta \leq \mathbf{q}. \tag{6}$$

2) A system of Coniq Quadratic inequalities

$$\|\mathbf{Q}\zeta - \mathbf{q}_i\|_2 \leq \mathbf{q} + i^T \zeta - \mathbf{r}_i, \ i = 1, ..., N. \tag{7}$$

3) A system of Linear Matrix Inequalities

$$\mathbf{q}_0 + \sum_{i=1}^{\dim\zeta} \zeta_i \mathbf{Q}_i \geq \mathbf{0}. \tag{8}$$

In the case 2) and 3) assume also that the system of constraints defining $\mathcal{U}$ is strictly feasible. Then the Robust Counterpart of the uncertain Linear Programming (3) is equivalent to:

1) A Linear Programming problem in case 1)
2) A Conic Quadratic problem in case 2)
3) A Semidefinite program in case 3)

The set of polyhedral uncertainties can be expressed as follows:

$$\mathcal{Z} = \{\zeta : \mathbf{d} - \mathbf{D}\zeta \geq \mathbf{0}\}, \tag{9}$$

where $\mathbf{D} \in \mathbb{R}^{m \times L}$, $\zeta \in \mathbb{R}^L$, and $\mathbf{d} \in \mathbb{R}^m$. The set $\mathcal{U}$ can be defined as:

$$\mathcal{U} = \{\mathbf{a} \| \bar{\mathbf{a}} + \mathbf{P}\zeta, \ \mathbf{d} - \mathbf{D} - \zeta \geq \mathbf{0}\}. \tag{10}$$

In order to obtain the Robust Counterpart formulation with polyhedral uncertainty set, the polyhedral uncertainty set is applied to the inequality (9) as follows.

$$(\bar{\mathbf{a}} + \mathbf{P}\zeta)^T \mathbf{x} \leq \mathbf{b}, \ \forall \zeta : \mathbf{d} - \mathbf{D}\zeta \geq \mathbf{0}. \tag{11}$$

Referring to Gorissen, Yanikoğlu, and den Hertog [11], the formulation of the Robust Counterpart with the set of polyhedral uncertainties is obtained in three steps, namely as follows:

1) The reformulation of the inequality constraint in (11) is equivalent to the worst-case formulation as follows:

$$\max_{\zeta : \mathbf{d} - \mathbf{D}\zeta \geq 0} \left(\mathbf{P}^T \mathbf{x}\right)^T \zeta \leq \mathbf{b} = \bar{\mathbf{a}}^T \mathbf{x} + \max_{\zeta : \mathbf{d} - \mathbf{D}\zeta \geq 0} \left(\mathbf{P}^T \mathbf{x}\right)^T \zeta \leq \mathbf{b}. \tag{12}$$

2) The dual formulation of the maximization problem on inequality (12), the dual form formulation is used to convert the min-max problem into a min-min problem. The primal form of equation (12) is as follows:

$$\max_{\zeta} \left\{ \left(\mathbf{P}^T \mathbf{x}\right)^T \zeta : \mathbf{d} - \mathbf{D}\zeta \geq 0 \right\}. \tag{13}$$

3) Next, the primal form is changed to the dual form. The objective maximization function in the primal form is changed to an objective minimization function in the dual form. The coefficient of the objective function in the primal form becomes a right-hand side constant in the dual form, and the right-hand side constant in the primal form becomes the coefficient of the objective function in the dual form. The primal variable ζ is unsigned, so the constraint function in the dual form is equal to (=). The constraint function in the primal form is inequality, so the y variable becomes a dual variable with a non-negative value. The dual form of equation (13) is as follows.

$$\min_{y} \left\{ \mathbf{d}^T \mathbf{y} : \mathbf{D}^T \mathbf{y} = \mathbf{P}^T \mathbf{x}, \mathbf{y} \geq 0 \right\}. \tag{14}$$

Based on the Strong Duality Theorem, the objective function value of inequality (13) and its dual form (14) have the same objective function value, so equation (12) is equivalent to:

$$\bar{\mathbf{a}}^T \mathbf{x} + \min_{y} \left\{ \mathbf{d}^T \mathbf{y} : \mathbf{D}^T \mathbf{y} = \mathbf{P}^T \mathbf{x}, \mathbf{y} \geq \mathbf{0} \right\} \leq \mathbf{b}. \tag{15}$$

Note that the constraint function in (13) is satisfied for a feasible solution $\mathbf{y}$ contained in the feasible set $\mathcal{F} = \mathbf{y} : \mathbf{D}^T \mathbf{y} = \mathbf{P}^T \mathbf{x}, \mathbf{y} \geq \mathbf{0}$. The constraint function can be guaranteed to satisfy the minimum value of $\mathbf{y}$. The final formulation of the Robust Counterpart is as follows:

$$\exists \mathbf{y} : \bar{\mathbf{a}}^T \mathbf{x} + \mathbf{d}^T \mathbf{y} \leq \mathbf{b}, \ \mathbf{D}^T \mathbf{y} = \mathbf{P}^T \mathbf{x}, \mathbf{y} \geq \mathbf{0}. \tag{16}$$

The constraints in equation (16) are in the form of Linear Programming (LP), thus referring to Ben-Tal and Nemirovski [10] and Chaerani *et al.* [7] RC is guaranteed to be computationally tractable. The formulation for polyhedral uncertainty can be seen in Table 2.

TABLE 2. Rules for construction the dual problem.

Uncertainty Set	$\mathcal{Z}$	Robust Counterpart	Tractability
Polyhedral	$\mathrm{d} - \mathbf{D}\zeta \geq \mathbf{0}$	$\begin{cases} \bar{\mathbf{a}}^T \mathbf{x} + \mathbf{d}^T \mathbf{y} \leq \mathbf{b} \\ \mathbf{D}^T \mathbf{y} = \mathbf{P}^T \mathbf{x} \\ \mathbf{y} \geq \mathbf{0} \end{cases}$	LP

3. Results and Discussion

3.1. **Optimization Model Formulation for Internet Shopping Online with Endorsement Costs.** Internet Shopping Online Optimization Problems from the seller's point of view have an objective function to maximize profits, namely sales results minus production costs. The decision variable of the model for online shopping internet optimization problems by taking the seller's point of view is (x_{ij}) quantity of item i from online shop j to be produced. The parameters used in the model for the internet shopping online optimization problem by taking the seller's point of view are product selling prices (p_{ij}), product production costs (e_{ij}), number of product requests (d_{ij}), production capacity (f_{ij}), storage capacity (k_j). The costs incurred for the endorsement are added to the constraint function, provided that the selling price of the product multiplied by the number of goods produced minus the production costs of the number of goods must be greater or equal to the cost of the endorsement. Then the model for the problem of optimizing internet shopping online by considering the cost of endorsement (e_j) is as follows:

$$\max \sum_i \sum_j \left(p_{ij} x_{ij} - c_{ij} x_{ij} \right)$$

$$\text{s.t} \sum_i x_{ij} \le k_j, \ \forall \, j = 1, \ldots m$$

$$x_{ij} \ge d_{ij}, \ \forall \, i = 1, \ldots, n, \ j = 1, \ldots, m$$

$$x_{ij} \le f_{ij}, \ \forall \, i = 1, \ldots, n, \ j = 1, \ldots, m \qquad (17)$$

$$\sum_i \sum_j \left(p_{ij} x_{ij} - c_{ij} x_{ij} \right) \ge e_j, \ \forall \, i = 1, \ldots, n, \ j = 1, \ldots, m$$

$$x_{ij} \ge 0, \ \forall \, i = 1, \ldots, n, \ j = 1, \ldots, m \, x_{ij} \text{ integer}.$$

3.2. Robust Optimization Model Formulation for Internet Shopping Online with Endorsement Costs. In the problem of online internet shopping with endorsement costs, the uncertainty parameter lies in the number of requests (d_{ij}). The number of requests is assumed to be uncertain according to the buying interest of the customer. Therefore, the uncertainty is in the constraint function and the right-hand side, so the variable y_{ij} is defined where $y_{ij} = 1$, $i = 1, \ldots n, \ j = 1, \ldots m$ so that the constraints are obtained.

$$x_{ij} \ge d_{ij} y_{ij}, \ \forall \, i = 1, \ldots, n, \ j = 1, \ldots, m. \qquad (18)$$

Constraints can be reformulated as follows.

$$x_{ij} - d_{ij} y_{ij} \ge 0, \ \forall \, i = 1, \ldots, n, \ j = 1, \ldots, m. \qquad (19)$$

The variable number of requests is assumed to be $d_{ij} \in \mathcal{U}$, so the parameters d_{ij} can be written as:

$$x_{ij} - \left(\bar{d}_{ij} + \mathrm{P}_{ij} \zeta \right) y_{ij} \ge 0, \ \forall i = 1, \ldots, n, \ j = 1, \ldots, m, \zeta \in \mathcal{Z}. \qquad (20)$$

Then the uncertainty optimization model obtained from the internet shopping online problem with endorsement costs is as follows:

$$\max \sum_i \sum_j \left(p_{ij} x_{ij} - c_{ij} x_{ij} \right)$$

$$\text{s.t} \sum_i x_{ij} - k_j \le 0, \ \forall \, j = 1, \ldots m$$

$$x_{ij} - \left(\bar{d}_{ij} + \mathrm{P}_{ij} \zeta \right) y_{ij} \ge 0, \ \forall i = 1, \ldots, n, \ j = 1, \ldots, m$$

$$x_{ij} - f_{ij} \le 0, \ \forall i = 1, \ldots, n, \ j = 1, \ldots, m$$

$$\sum_i \sum_j \left(p_{ij} x_{ij} - c_{ij} x_{ij} \right) - e_j \ge 0, \ \forall j = 1, \ldots, m \qquad (21)$$

$$y_{ij} = 1, \ \forall i = 1, \ldots n, \ j = 1, \ldots, m$$

$$x_{ij} \ge 0, \ \forall \, i = 1, \ldots, n, \ j = 1, \ldots, m$$

$$x_{ij} \text{ Integer}$$

$$\zeta \in \mathcal{Z}.$$

3.3. Robust Optimization Model Formulation with Polyhedral Uncertainty for Internet Shopping Online with Endorsement Costs.

Step 1: Reformulation of the inequality constraint containing the uncertainty ζ so that it is equivalent to the worst-case formulation.

In the indeterminate optimization model, constraint (20), which is a constraint that contains an uncertain parameter ζ, can be expressed in vector form, namely:

$$\left(\mathbf{x}^T \right) - \left(\bar{d} + \mathrm{P}\zeta \right)^T y \ge 0. \qquad (22)$$

Constraints in the form of vectors in (22) are reformulated in the worst case to obtain:

$$\left(\mathbf{x}^T \right) - \bar{d}^T y + \max_{\zeta} \left(\mathrm{P}\zeta \right)^T y \ge 0, \qquad (23)$$

which is equivalent to

$$\left(\mathbf{x}^T\right) - \bar{d}^T y + \max_{\zeta} \left(\mathrm{P}^T y\right)^T \zeta \geq 0. \tag{24}$$

Step 2: Formulation of the dual form based on the Duality Theory of the maximization problem contained in the final result of step 1.

Defined the set of polyhedral indeterminacy as $\mathcal{Z} = \zeta : \mathbf{d} - \mathbf{D}\zeta - \geq 0$, where $\mathbf{D} \in M_{m,l}(\mathbb{R})$, $\zeta \in \mathbb{R}^L$, and $\mathbf{d} \in \mathbb{R}^m$, so (24) becomes:

$$\left(\mathbf{x}^T\right) - \bar{d}^T y + \max_{\zeta : \mathbf{d} - \mathbf{D}\zeta \geq 0} \left(\mathbf{P}^T y\right)^T \zeta \geq 0. \tag{25}$$

Then focus on the third term of the left-hand side in (25), namely:

$$\max_{\zeta : \mathbf{d} - \mathbf{D}\zeta \geq 0} \left(\mathbf{P}^T y\right)^T \zeta, \tag{26}$$

which can be expressed as:

$$\begin{aligned} \max_{\zeta : \mathbf{d} - \mathbf{D}\zeta \geq 0} & \left(\mathbf{P}^T y\right)^T \zeta, \\ \text{s.t } & \mathbf{D}\zeta \leq \mathbf{d}, \ \forall \zeta \in \mathcal{Z}. \end{aligned} \tag{27}$$

The primal problem (27) is a maximization problem with an inequality constraint function $\leq$ and has an indefinite variable ζ which has no limits. Therefore, according to Strong Duality Theory, the dual problem of (27) is:

$$\begin{aligned} \min & \ \mathbf{d}^\gamma, \\ \text{s.t } & \mathbf{D}^T \gamma = \mathbf{P}^T \gamma \\ & \gamma \geq 0. \end{aligned} \tag{28}$$

The primal-dual relationship used is strong duality. Therefore, the optimum value for the primal problem (27) is the same as the optimum value for the dual problem (28) so that:

$$\max_{\zeta} \left\{ \left(\mathbf{P}^T y\right)^T \zeta : \mathbf{D}\zeta \leq \mathbf{d} \right\} = \min_{\gamma} \left\{ \mathbf{d}^T \gamma : \mathbf{D}^T \gamma = \mathbf{P}^T y, \gamma \geq 0 \right\}. \tag{29}$$

Substitute (29) in (25) to obtain:

$$\left(\mathbf{x}^T\right) - \bar{d}^T y + \min_{\gamma} \left\{ \mathbf{d}^T \gamma : \mathbf{D}^T \gamma = \mathbf{P}^T y, \gamma \geq 0 \right\} \geq 0. \tag{30}$$

Step 3: The dual formulation in (30) is satisfied for a feasible solution γ contained in the feasible set $\mathcal{F} = \left\{ \gamma | \mathbf{D}^T \gamma = \mathbf{P}^T y, \gamma \geq 0 \right\}$, so that $\exists \gamma \geq 0 \ni \mathbf{D}^T \gamma = \mathbf{P}^T y$. Since the existence of a solution γ is guaranteed, the constraint on (30) can be written as

$$\left(\mathbf{x}^T\right) - \bar{d}^T y + \mathbf{d}^T \gamma \geq 0, \tag{31}$$

with additional constraints:

$$\begin{aligned} \mathbf{D}^T \gamma &= \mathbf{P}^T y, \\ \gamma &\geq 0. \end{aligned} \tag{32}$$

Constraints (31) and (32) can be restated in the form of an index, namely:

$$x_{ij} - \bar{d}_{ij} y_{ij} + \sum_{h \in H} d_h \gamma_h \geq 0, \ \forall \ i = 1, \dots, n, \ j = 1, \dots, m \tag{33}$$

$$\sum_{h \in H} D_{zh} \gamma_h = \sum_{k \in K} \sum_{j \in J} P_{zj} y_{ij}, \ \forall j = 1, \dots, m \tag{34}$$

$$\gamma_h \geq 0, \ \forall h \in H. \tag{35}$$

It can be seen that (33) and (34) are linear constraint functions. The limit (35) is a non-negative limit for all variables γ.

The Robust Counterpart model with a set of polyhedral uncertainty for internet shopping online problems with endorsement costs is as follows:

$$
\begin{aligned}
\max \ & \sum_i \sum_j \left(p_{ij}x_{ij} - c_{ij}x_{ij}\right) \\
\text{s.t} \ & \sum_i x_{ij} - k_j \le 0, \ \forall j = 1,...,m \\
& x_{ij} - \bar{d_{ij}}y_{ij} + \sum_{h \in H} \mathbf{d}_h \gamma_j \ge 0, \ \forall i = 1,...,n, \ j = 1,...,m \\
& \sum_{h \in H} \mathbf{D}_{zh} \gamma_h = \sum_{k \in K} \sum_{j \in J} \mathbf{P}_{zj} y_{ij}, \ \forall j = 1,...,m, \ z = 1,2,...,L \\
& x_{ij} - f_{ij} \le 0, \ \forall i = 1,...,n, \ j = 1,...,m \\
& \sum_i \sum_j \left(p_{ij}x_{ij} - c_{ij}x_{ij}\right) - e_j \ge 0, \ \forall j = 1,\ldots,m, \\
& y_j = 1, \ \forall j = 1,\ldots,m \\
& x_{ij} \ge 0, \ \forall i = 1,\ldots,n, \ j = 1,\ldots,m \\
& x_{ij} \ \text{Integer} \\
& \gamma_h \ge 0, \ \forall h \in H.
\end{aligned}
\tag{36}
$$

3.4. **Numerical Experiment.** Numerical experiments in this study were carried out with data obtained from online stores on Instagram. Data were obtained on 25 November 2022 from an online store with the username cievno.id. The data used in this research is secondary data. The secondary data obtained are product prices and product costs. These data can be seen in Table 3.

TABLE 3. Product prices, production costs, production capacity, demand, storage capacity, endorsement costs.

Product-n	Product Prices	Production Costs	Production Capacity	Demand	Storage	Endorsement Costs
Product-1	155.000	130.000	700	592		
Product-2	92.000	80.000	300	116		
Product-3	140.000	120.000	200	56		
Product-4	87.000	77.000	100	44	1200	5.000.000
Product-5	125.000	115.000	100	55		
Product-6	55.000	40.000	100	30		
Product-7	82.000	72.000	50	8		

The numerical experiment of optimization internet shopping online problem model with endorsement costs in equation (21) and the numerical experiment for the Robust Optimization model of an internet shopping online problem with endorsement costs in equation (36) was done using *Python*. Substitute the data in Table 3 into equation (21) and equation (36). Calculations can be completed with integer programming method and Robust Optimization method using *Python*, which produces optimal solutions.

This study uses *Google Colab* as a means of using *Python*. The following is a Robust Optimization model solving algorithm for internet shopping online problems with endorsement costs:

(1) Create a new notebook on the *Google Colab* online platform.
(2) Install the library used, namely the *pulp* library.
(3) The required import libraries are the *pulp*, *pandas*, and *numpy* libraries.

(4) Upload existing research data on *Google Drive*.

(5) Import the previously uploaded spreadsheet data into *Google Colab*.

(6) The Robust Optimization model experiment for the problem of internet shopping online with endorsement costs in the form of pseudocode is as follows:

> *For all $i \in M$ do*
>> *Initiate $p_i, c_i, f_i, r_i, e, k \in \mathbb{R}$*
>> *Initiate $x_i, d_i \in \mathbb{Z}$*
> *End for*
> *Initiate $\gamma \geq 0$*
> *Initiate $\mathbf{d1}, \mathbf{D}, \mathbf{P} \in \mathbb{R}$*
> *For all $i \in M$ do*
>> *Read $p_i, c_i, f_i, r_i, e, x_i, d_i$*
> *End for*
> *Use the pulp module to initiate robust_model*
> *Read robust_model*
> *Use the pulp module to solve robust_model*
> *For all $i \in M$ do*
>> *Print x_i*
> *End for*
> *Print x_i*
> *Print Z*

Using the *Python* programming language, the results of a numerical experimental optimization model for the problem of online internet shopping with endorsement costs can be seen in Table 4.

TABLE 4. Numerical experimental results of internet shopping online models with endorsement costs.

Product-n	Internet Shopping Online Problem Optimization Model with Endorsement Costs	Robust Optimization Model Problems of Internet Shopping Online with Endorsement Costs
Product-1	700	700
Product-2	300	200
Product-3	200	200
Product-4	100	0
Product-5	100	0
Product-6	100	100
Product-7	50	0
Profit	29.100.000	25.400.000

The numerical experimental results in Table 4 show a comparison of the calculation of the nominal model with the Robust Optimization model. The optimal solution of the Robust Optimization model for the online internet shopping problem with endorsement costs is smaller than the optimal solution for the nominal model. It is because the Robust Optimization considers the uncertainty factor assumed in the polyhedral uncertainty set. The Robust Optimization model guarantees the fulfillment of the number of product requests if the data uncertainty lies in the given set. The Robust Optimization model produces the best worst-case solution, the best solution from the worst possibility in the number of requests data.

4. CONCLUSION

The Robust Optimization model for internet shopping online with endorsement costs considers the uncertainty parameter: the number of product requests. This Robust Optimization

model for internet shopping online with endorsement costs can be solved using the polyhedral uncertainty set approach. Then a computationally tractable model is obtained, and the problem can be solved computationally in polynomial time. It is obtained that the internet shopping online problem model can be formulated into a Robust internet shopping online optimization model with endorsement costs in the set of polyhedral uncertainties contained in equation 36. The results of the numerical experiment of the Robust Optimization model for the problem of internet shopping online with endorsement costs obtained the optimal solution. The optimal solution for the Robust Optimization model for online shopping problems with endorsement costs on polyhedral uncertainty sets produces a smaller value than the nominal model. Subsequent research can be developed using another set of uncertainties, namely the box uncertainties, which contain various realizations for each uncertainty ζ, which is the most Robust and guarantees the constraints are met. Then the ellipsoidal uncertainty set, which contains the set of smaller uncertainties, guarantees the fulfillment of constraints as discussed by (Gorissen, Yanikoğlu, and den Hertog, [8]). For future research, you can also consider the endorsement period following the duration of posting on social media to obtain a model close to the problem being discussed.

References

[1] Sukmana, E., 2005, *Digitalisasi Pustaka, in Makalah at Seminar Peran Pustakawan pada era digital.*

[2] Ding, J., 1999, *E-commerce: Law & practice*, Sweet & Maxwell Asia.

[3] Hartini, S., 2016, Efektifitas Endorsment Pada Media Sosial Instagram pada Produk Skin Care, *Bina Insani Ict Journal, Volume 3, Issue 1, Pages 44-50.*

[4] Błażewicz, J., Kovalyov, M.Y., Musiał, J., Urbański, A.P., and Wojciechowski, A., 2010, Internet Shopping Optimization Problem, *International Journal of Applied Mathematics and Computer Science, Volume 20, Issue 2, Pages 385-390.*

[5] Błażewicz, J. and Musiał, J., 2011, *E-Commerce Evaluation – Multi-Item Internet Shopping Optimization and Heuristic Algorithms, Pages 149154.*

[6] Chung, J.B., 2017, Internet Shopping Optimization Problem with Delivery Constraints, *Journal of Distribution Science, Volume 15, Issue 2, Pages 1520.*

[7] Chaerani, D., Rusyaman, E., Mahrudinda, Marcia, A., and Fridayana, A., 2021, Adjustable Robust Counterpart Optimization Model for Internet Shopping Online Problem, in *Journal of Physics: Conference Series*, IOP Publishing Ltd.

[8] Rao, S.S., 2009, *Engineering Optimization: Theory and Practice*, Fourth Edition.

[9] Bronson, R. and Naadimuthu, G., 1997, *Schaums Outline of Operations Research*, McGraw-Hill Education.

[10] Ben-Tal, A. and Nemirovski, A., 2002, Robust Optimization - Methodology and Applications, *Mathematical Programming, Series B, Volume 92, Issue 3, Pages 453-480.*

[11] Gorissen, B.L., Yanıkoğlu, I., and den Hertog, D., 2015, A Practical Guide to Robust Optimization, {em Omega, Volume 53, Pages 124137}.