

Last-Survivor Insurance Premium and Benefit Reserve Calculation using Gamma-Gompertz Mortality Law

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Abstract

When the insurance benefit of a last-survivor insurance product is payable at the moment of the last insured death, exploring continuous mortality models is essential to obtain the most appropriate premium and benefit reserve. In this study, the Gamma-Gompertz mortality law was applied to Indonesian population mortality data at adulthood and old age stages to calculate the annual gross premium and gross benefit reserve of a whole life last survivor insurance product for some age scenarios of a husband and a wife. The annual gross premium was computed using the actuarial equivalence principle. Results show that the older the policyholders in our sample purchase the product, the higher the annual gross premium they must pay. The gross benefit reserve needed to be set by the insurance company for the whole life last survivor insurance product was calculated using the prospective method. For the insureds with a specific age in our sample, the value of the gross benefit reserve grows for each valuation year until it approaches the insurance benefit amount.

Keywords: Annual gross premium, Equivalence Principle, Gamma-Gompertz mortality law, Gross benefit reserve, Last-survivor insurance, Prospective Method.

1. INTRODUCTION

Last-survivor insurance became a phenomenal insurance product in the United States in 1993 (Bragg, [2]). At that time, at least 50 companies issued this life insurance product. This product is also known as a second-to-die insurance product. This life insurance policy will pay an insurance benefit after the last death of the insureds. The insurance premiums are usually still payable until the latest death of the insureds has not happened, so all insureds should have the ability to pay the insurance premium even if some have passed away. In practice, this type of insurance product only covers two people who are often married to each other as per the purpose of this product creation. Therefore, this life insurance product is marketed toward adults and the elderly.

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The last survivor insurance is generally bought by a married couple who have worked hard and want to pass a significant inheritance to their children after their death. One of the advantages of the last survivor product is that the price is significantly less expensive than two life insurance products that the husband and wife buy separately for the same total amount of insurance benefits (Kagan, [6]). The married couple can be insured simultaneously in one policy only. In addition, if either the husband or the wife has a bad medical history, they usually cannot buy a single life insurance policy as insurance companies will deny them. Thus, they will buy a last-survivor life insurance product as insurance companies typically allow them to purchase it even though some might have bad medical history records.

Suppose the last survivor insurance benefit is payable at the moment of the last death of the insureds. In that case, the insurance company needs a continuous life insurance model to calculate its premium and benefit reserve. When the insurance company only has mortality data in a mortality table, it could use a model assumption for fractional ages or a continuous mortality distribution. Two of the fractional age assumptions are the uniform distribution of deaths and the constant force of mortality. Two continuous mortality distributions often used to model human mortality are the Gompertz distribution and the Makeham distribution. Riaman, Sukono, and Supian [9] used the Gompertz model to calculate the net premium and the premium reserve for an endowment last survivor insurance product. Hasriati and Nababan [5] used the Pareto distribution to model each participants mortality to calculate premiums for two types of multiple life insurance products. Vaupel, Manton, and Stallard [10] introduced the Gamma-Gompertz mortality law as another mortality distribution to model human mortality.

Missov [7] stated that the Gamma-Gompertz mortality law is the most prevalent parametric model applied for the human mortality data of adults and older people. Since last survivor insurance contracts are usually purchased by adults, the Gamma-Gompertz mortality law is compatible with this life insurance contract. Therefore, this paper calculated the annual gross premium and gross benefit reserve for a last-survivor insurance policy where the Gamma-Gompertz mortality law was used to model adult and old Indonesian mortality, both for males and females. This mortality law was chosen because it can fit the recent adult and old Indonesian mortality data better than the well-known Gompertz and Makeham distributions. This research can provide a continuous model option for insurers to calculate the continuous type of last-survivor insurance premiums and benefit reserves more accurately when they only have a mortality table.

2. LAST-SURVIVOR INSURANCE

A last-survivor insurance policy theoretically may insure two or more individuals. However, we will only consider a contract covering two individuals. Even though insurance contracts that insure more than two people are also available, these contracts are less common in practice. Suppose the two individuals are aged x and y years when the insurance contract coverage begins. For convenience and simplicity, we use (x) and (y) to denote the two individuals, respectively. Specifically, we observe their last survivor status, which is commonly represented by $\bar{x}y$. If the future lifetimes of (x) and (y) are respectively represented by a random variable T_x and a random variable T_y , then

$$T_{\bar{x}y} = \max(T_x, T_y) \quad (1)$$

is a random variable that expresses the time to failure of the last survivor status of (x) and (y) . In simple terms, $T_{\bar{x}y}$ in Equation (1) is called a random variable of time to the last death (Dickson, Hardy, and Waters, [4]).

In this paper, the life insurance contract discussed is a life insurance contract based on the last survivor status of (x) and (y) . This insurance contract is commonly referred to as last-survivor life insurance. If the agreement is based on two individuals, the insurance benefit will be paid on the last death of (x) and (y) .

3. GAMMA-GOMPERTZ MORTALITY LAW APPLICATION TO LAST-SURVIVOR STATUS

The probability that (x) will survive for at least t years is usually denoted by $S_x(t)$. This S_x function is commonly known as a survival function. Mathematically, $S_x(t) = \Pr(T_x > t)$. The actuarial notation for $S_x(t)$ is ${}_t p_x$. Meanwhile, the probability of (x) will die before age $x + t$ is usually denoted by an actuarial notation of ${}_t q_x$, where ${}_t q_x = \Pr(T_x \leq t) = 1 - {}_t p_x$.

In Gamma-Gompertz mortality law, the probability that a newborn baby will survive at least age t years can be formulated as Equation (2).

$$S_0(t) = \left[1 + \frac{a\sigma^2}{b} (e^{bt} - 1) \right]^{-\frac{1}{\sigma^2}}, \quad \text{for } t > 0, \quad (2)$$

where a , b , and σ^2 are model parameters that characterize mortality rates (Castellares, Patricio, and Lemonte, [3]). Then, using Equation (2), the formula for the survival function can be obtained as Equation (3).

$$S_x(t) = \frac{S_0(x+t)}{S_0(x)} = \left[\frac{1 + \frac{a\sigma^2}{b} (e^{b(x+t)} - 1)}{1 + \frac{a\sigma^2}{b} (e^{bx} - 1)} \right]^{\frac{1}{\sigma^2}}. \quad (3)$$

In addition, using Equation (3), the formula of the force of mortality function at age $x + t$, $t > 0$, denoted by μ_{x+t} or $\mu_x(t)$ becomes Equation (4).

$$\mu_{x+t} = -\frac{d}{dt} \ln S_x(t) = \frac{ae^{b(x+t)}}{1 + \frac{a\sigma^2}{b} (e^{b(x+t)} - 1)}. \quad (4)$$

If random variables T_x and T_y are assumed to be independent, then using Equation (3), the probability of the last survivor status of (x) and (y) will fail in t years, which is traditionally denoted by an actuarial notation of ${}_t q_{\bar{x}y}$ can be calculated using Equation (5).

$${}_t q_{\bar{x}y} = {}_t q_x {}_t q_y = \left(1 - \left[\frac{1 + \frac{a\sigma^2}{b} (e^{b(x+t)} - 1)}{1 + \frac{a\sigma^2}{b} (e^{bx} - 1)} \right] \right) \left(1 - \left[\frac{1 + \frac{a\sigma^2}{b} (e^{b(y+t)} - 1)}{1 + \frac{a\sigma^2}{b} (e^{by} - 1)} \right] \right). \quad (5)$$

This probability may also be stated as the probability that both (x) and (y) will die in t years. Moreover, the probability that the last survivor status of (x) and (y) will survive for at least t years which is commonly denoted with an actuarial notation of ${}_t p_{\bar{x}y}$ can be calculated using the formula of ${}_t p_{\bar{x}y} = 1 - {}_t q_{\bar{x}y}$. This probability can also be defined as the probability that at least one of (x) and (y) is alive in t years. Finally, the force of termination of the last survivor status of (x) and (y) , denoted by $\mu_{\bar{x}y}(t)$, can be computed using Equation (6).

$$\mu_{\bar{x}y}(t) = -\frac{d}{dt} \ln {}_t p_{\bar{x}y}. \quad (6)$$

4. LIFE ANNUITY AND INSURANCE BENEFIT FORMULATION ON SINGLE STATUS AND LAST-SURVIVOR STATUS

Let the effective compound interest rate per annum be constant and equal to i , then the discount factor, which is generally denoted by v , can be calculated using the formula of $v = (1 + i)^{-1}$. Suppose that a series of payments of one unit is made at the beginning of each year while (x) is still alive. Then, the actuarial present value of this series of payments is denoted by an actuarial notation of \ddot{a}_x and can be calculated using Equation (7).

$$\ddot{a}_x = \sum_{t=0}^{\infty} v^t {}_t p_x. \quad (7)$$

Suppose that a series of payments of one unit is made at the beginning of each year while at least one of (x) and (y) is still alive. Then, the actuarial present value of this series of payments

is denoted by an actuarial notation of $\ddot{a}_{\bar{x}y}$ and can be computed using Equation (8).

$$\ddot{a}_{\bar{x}y} = \sum_{t=0}^{\infty} v^t {}_t p_{\bar{x}y}. \tag{8}$$

Suppose that an insurance benefit of one unit is paid immediately upon the death of (x) whenever it occurs in the future (whole life). Then, the actuarial present value of this payment which is denoted by an actuarial notation of \bar{A}_x becomes Equation (9).

$$\bar{A}_x = \int_0^{\infty} v^t {}_t p_x \mu_x(t) dt. \tag{9}$$

Suppose that an insurance benefit of one unit is paid immediately upon the death of the second to die of (x) and (y) whenever it occurs in the future (whole life). Then, the actuarial present value of this payment which is denoted by an actuarial notation of $\bar{A}_{\bar{x}y}$ becomes Equation (10).

$$\bar{A}_{\bar{x}y} = \int_0^{\infty} v^t {}_t p_{\bar{x}y} \mu_{\bar{x}y}(t) dt. \tag{10}$$

5. ANNUAL GROSS PREMIUM CALCULATION

In this paper, the last survivor insurance is applied to a married couple where the husband's age is x years old, and the wife's age is y years old when the insurance contract begins. The annual gross premium will be computed for five age scenarios:

- Scenario 1: $x = y$ (they have the same age).
- Scenario 2: $x = y + 5$ (the husband is five years older than the wife).
- Scenario 3: $x = y + 3$ (the husband is three years older than the wife).
- Scenario 4: $x = y - 3$ (the husband is three years younger than the wife).
- Scenario 5: $x = y - 1$ (the husband is a year younger than the wife).

The assumptions used in the gross premium calculation are as follows:

- (1) The effective compound interest rate used is constant at 6% per annum.
- (2) The future lifetime of (x) follows the Gamma-Gompertz mortality law, where its parameter values are estimated using the death probability data for males aged 20 years old and above on the Indonesian Mortality Table IV (AAJI, [1]).
- (3) The future lifetime of (y) follows the Gamma-Gompertz mortality law, where its parameter values are estimated using the death probability data for females aged 20 years old and above on the Indonesian Mortality Table IV (AAJI, [1]).
- (4) The future lifetime random variables of (x) and (y) are independent.
- (5) An insurance benefit of 100 million Indonesian rupiahs will be paid immediately upon the second death of (x) and (y) whenever it happens in the future.
- (6) The annual gross premium of G will be paid at the beginning of each year, while at least one of (x) or (y) is still alive.
- (7) The operating expenses of the insurance company are described in Tables 1, 2 and 3.
- (8) The gross premium is calculated using the actuarial equivalence principle.

TABLE 1. Company operating expenses in the first year (initial expenses)

Initial expense type	% of the premium	A fixed amount (IDR)
Policy underwriting	5%	20,000
Contract preparation	-	100,000
Commission to sales agents	10%	-
Total	15%	120,000

TABLE 2. Company operating expenses in the second year and after (renewal expenses)

Renewal expense type	% of the premium	A fixed amount (IDR)
Cost to receive premiums	3%	-
Administrative costs	2%	10,000
Renewal commission to agents	5%	-
Total	10%	10,000

TABLE 3. Company operating expenses at the time of benefit payment (claim expenses)

Claim expense type	A fixed amount (IDR)
Payment of claim	100,000
Claim underwriting	20,000
Administrative costs	10,000
Total	130,000

Unlike Nadjafi [8], who used the maximum likelihood method to estimate the parameters of Gamma-Gompertz distribution, we estimated the parameter values in the Gamma-Gompertz model by minimizing the quadratic loss function (QLF) as in Equation (11).

$$QLF(\hat{a}, \hat{b}, \hat{\sigma}^2) = \sum_{x=20}^{110} \left(q_x - \left(1 - \left[\frac{1 + \frac{\hat{a}\hat{\sigma}^2}{\hat{b}} (e^{\hat{b}(x+1)} - 1)}{1 + \frac{\hat{a}\hat{\sigma}^2}{\hat{b}} (e^{\hat{b}x} - 1)} \right]^{-\frac{1}{\hat{\sigma}^2}} \right) \right)^2, \quad (11)$$

where q_x is the death probability on the Indonesian Mortality Table IV. The estimated parameter values are presented in Table 4.

TABLE 4. Estimated Gamma-Gompertz model parameter values

Parameter	Estimated value for	
	males 20 years old and above	females 20 years old and above
a	0.00009540922	0.00009212443
b	0.1066328	0.09977545
σ^2	0.1191104	0.04750706

The male and female death probabilities then are estimated using the Gamma-Gompertz model with the estimated parameter values in Table 4. A comparison between the death probabilities obtained from the Gamma-Gompertz mortality law and the Indonesian Mortality Table IV for males and females of 20 and above are shown in Figures 1 and 2, respectively. From these two figures, we may assume that the Gamma-Gompertz mortality law is suitable to model the mortality of the Indonesian adult and elderly population for both men and women. When we compare the Gamma-Gompertz model to the Gompertz and Makeham models, it fits the data better since it has the smallest residual standard error, as we can see in Table 5.

TABLE 5. Residual standard error of Gompertz, Makeham, and Gamma-Gompertz models

Models	Residual standard error	
	Male	Female
Gompertz	0.149	0.050
Makeham	0.152	0.054
Gamma-Gompertz	0.072	0.030

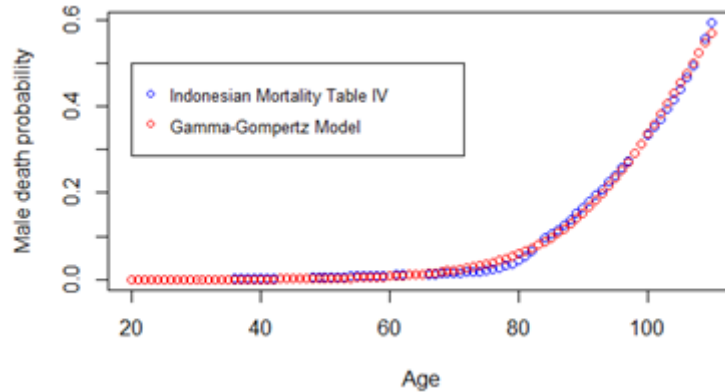


FIGURE 1. Male death probabilities from the Indonesian Mortality Table IV and the Gamma-Gompertz model

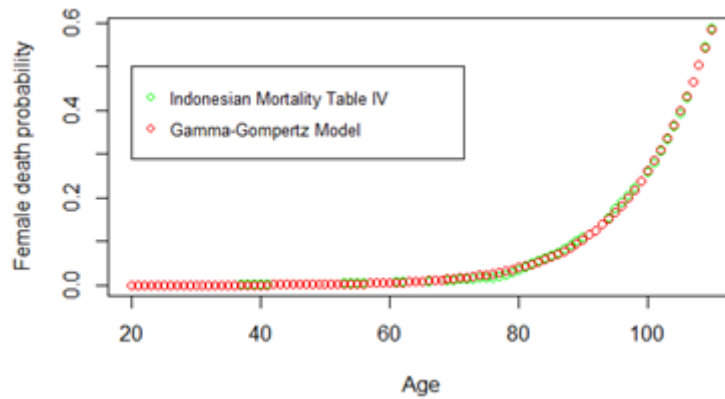


FIGURE 2. Female death probabilities from the Indonesian Mortality Table IV and the Gamma-Gompertz model

Based on the determined assumptions, the actuarial present value (APV) of premiums, benefits, and expenses, respectively, can be formulated as in Equations (12), (13) and (14).

$$APV_{@0} \text{ of premiums on } [0, \infty) = G\ddot{a}_{\overline{x}y} \tag{12}$$

$$APV_{@0} \text{ of benefits on } (0, \infty) = 100,000,000\bar{A}_{\overline{x}y} \tag{13}$$

$$APV_{@0} \text{ of expenses on } [0, \infty) = 110,000 + 0.05G + (10,000 + 0.1G)\ddot{a}_{\overline{x}y} + 130,000\bar{A}_{\overline{x}y} \tag{14}$$

Using the actuarial equivalence principle, which states that the actuarial present value of future premiums must be equal to the total actuarial present value of future benefits and future expenses, the annual gross premium formula becomes Equation (15)

$$G = \frac{100,130,000\bar{A}_{\overline{x}y} + 110,000 + 10,000\ddot{a}_{\overline{x}y}}{0.9\ddot{a}_{\overline{x}y} - 0.05} \tag{15}$$

The amount of annual gross premiums when both husband and wife have the same age for some possible ages at the start of the insurance contract are summarized in Table 6. It is apparent that the older the insurance policyholders purchase the policy, the higher the annual gross premium that they must pay. This result makes sense because the more senior the insurance participants, the smaller the survival probability of their last survivor status. As a result, the actuarial present value of the insurance benefit increases, and the premium payment frequencies, on average, decrease.

TABLE 6. The annual gross premium for scenario 1

Husband's age = wife's age	Annual gross premium amount (IDR)
25	587,801.97
26	624,015.34
27	662,626.80
28	703,804.44
29	747,728.89
30	794,594.37
31	844,609.76
32	897,999.76
33	955,006.14
34	1,015,889.14

The amount of annual gross premiums when the husband is five years and three years older than the wife for some possible ages at the start of the insurance contract are summarized in Tables 7 and 8, respectively. If we compare the premium amounts in Tables 6, 7, and 8, it is visible that the premium will be higher when the husband is older for the same wife's age. This result is reasonable for a reason mentioned earlier. Moreover, the significant husband's age differences in Table 7 and Table 6 cause a notable increase in insurance premiums.

TABLE 7. The annual gross premium for scenario 2

Husband's age	Wife's age	Annual gross premium amount (IDR)
30	25	662,563.66
31	26	703,643.87
32	27	747,450.25
33	28	794,173.54
34	29	844,018.60
35	30	897,205.49
36	31	953,970.67
37	32	1,014,568.25
38	33	1,079,271.41
39	34	1,148,373.79

TABLE 8. The annual gross premium for scenario 3

Husband's age	Wife's age	Annual gross premium amount (IDR)
28	25	633,379.26
29	26	672,568.09
30	27	714,357.09
31	28	758,928.53
32	29	806,478.23
33	30	857,216.69
34	31	911,370.16
35	32	969,181.94
36	33	1,030,913.70
37	34	1,096,846.94

The amount of annual gross premiums when the husband is three years and one year younger than the wife for some possible ages at the start of the insurance contract are summarized in Tables 9 and 10, respectively. Comparing the premium amounts in Tables 6, 9, and

10 shows that the premium will be higher when the wife is older for the same husband's age. This result is coherent with previous observations. Furthermore, by comparing the premium amounts in Tables 8 and 9, where the age difference is the same, we can note that the premium will be higher when the wife is older. This result can be caused by the fact that in our data, females at the age of 20 and above have a higher survival probability than males at the age of 20 and above.

TABLE 9. The annual gross premium for scenario 4

Husband's age	Wife's age	Annual gross premium amount (IDR)
25	28	646,914.96
26	29	687,074.31
27	30	729,912.78
28	31	775,620.09
29	32	824,400.31
30	33	876,473.10
31	34	932,074.84
32	35	991,460.12
33	36	1,054,903.09
34	37	1,122,699.13

TABLE 10. The annual gross premium for scenario 5

Husband's age	Wife's age	Annual gross premium amount (IDR)
25	26	607,445.05
26	27	644,967.75
27	28	684,981.65
28	29	727,662.02
29	30	773,197.26
30	31	821,790.00
31	32	873,658.26
32	33	929,036.62
33	34	988,177.60
34	35	1,051,353.09

6. GROSS BENEFIT RESERVE CALCULATION

This paper calculated the gross benefit reserve using the prospective method. We subtract the actuarial present value of all future outgo (benefits and expenses) by the actuarial present value of all future income (premiums). Then, the gross benefit reserve can be formulated as in Equation (16).

$$\begin{aligned} &\text{Gross benefit reserve at the end of year } t \\ &= \text{APV}_{@t} \text{ of benefits on } (t, \infty) + \text{APV}_{@t} \text{ of benefits on } [t, \infty) \\ &\quad - \text{APV}_{@t} \text{ of premiums on } [t, \infty) \end{aligned} \quad (16)$$

Suppose the last survivor status remains active at the end of year t . Then, we must compose that event into three disjoint events: both (x) and (y) are alive; (x) is alive, but (y) is not alive; and (y) is alive, but (x) is not alive. Using the same assumptions as to the annual gross premium calculation and using the annual gross premium obtained by Equation (15), we obtain the formulas in Equations (18)-(25).

- If (x) and (y) survive to the end of year t , then

$$\text{APV}_{@t} \text{ of premiums on } [t, \infty) = G\ddot{a}_{x+\bar{t};y+t} \quad (17)$$

$$\text{APV}_{@t} \text{ of benefits on } (t, \infty) = 100,000,000\bar{A}_{x+\bar{t};y+t} \quad (18)$$

$$\text{APV}_{@t} \text{ of expenses on } [t, \infty) = (10,000 + 0.1G)\ddot{a}_{x+\bar{t};y+t} + 130,000\bar{A}_{x+\bar{t};y+t} \quad (19)$$

This event will occur with a probability of ${}_t p_{xt} p_y$.

- If (x) survives to the end of year t , but (y) dies before the end of year t

$$\text{APV}_{@t} \text{ of premiums on } [t, \infty) = G\ddot{a}_{x\bar{+}t} \quad (20)$$

$$\text{APV}_{@t} \text{ of benefits on } (t, \infty) = 100,000,000\bar{A}_{x\bar{+}t} \quad (21)$$

$$\text{APV}_{@t} \text{ of expenses on } [t, \infty) = (10,000 + 0.1G)\ddot{a}_{x\bar{+}t} + 130,000\bar{A}_{x\bar{+}t} \quad (22)$$

This event will occur with a probability of ${}_t p_{xt} q_y$.

- If (y) survives to the end of year t , but (x) dies before the end of year t

$$\text{APV}_{@t} \text{ of premiums on } [t, \infty) = G\ddot{a}_{y\bar{+}t} \quad (23)$$

$$\text{APV}_{@t} \text{ of benefits on } (t, \infty) = 100,000,000\bar{A}_{y\bar{+}t} \quad (24)$$

$$\text{APV}_{@t} \text{ of expenses on } [t, \infty) = (10,000 + 0.1G)\ddot{a}_{y\bar{+}t} + 130,000\bar{A}_{y\bar{+}t} \quad (25)$$

This event will occur with a probability of ${}_t q_{xt} p_y$.

Thus, the gross benefit reserve at the end of year t can be computed using the formulas in Equation (26)-(6).

$$\text{Gross benefit reserve at the end of year } t = \frac{c}{d} \quad (26)$$

where

$$\begin{aligned} c = & 100,130,000 (\bar{A}_{x+\bar{t};y+tt} p_{xt} q_y + \bar{A}_{x+tt} q_{xt} q_y + \bar{A}_{y+tt} q_{xt} p_y) \\ & + (10,000 - 0.9G) (\ddot{a}_{x+\bar{t};y+tt} p_{xt} q_y + \ddot{a}_{x+tt} q_{xt} q_y + \ddot{a}_{y+tt} q_{xt} p_y) \end{aligned} \quad (27)$$

and

$$d = {}_t p_{xt} p_y + {}_t p_{xt} q_y + {}_t q_{xt} p_y. \quad (28)$$

As an illustration, the gross benefit reserves are computed only for $x = 30$ dan $y = 25$, which is one of the possible ages in scenario 2. Gross benefit reserves from the end of year 1 to the end of year 80 are illustrated in Figure 3. The value of the gross benefit reserve began to increase rapidly for the first 40 years of the insurance coverage. However, it became slower while getting closer to the insurance benefit value of 100 million Indonesian rupiahs. This result is commonsensical as the longer the insurance coverage runs, the higher the failure probability of the last survivor status. Therefore, the fund prepared by the insurance company to pay for insurance benefits must be increased every year, as shown in Figure 3.

7. CONCLUSION

The Gamma-Gompertz mortality law is suitable to model the Indonesian population mortality of adults and elderly, both males and females. Hence, the Gamma-Gompertz mortality law can be used in annual gross premium and gross benefit reserve calculations for a whole life last survivor insurance product when its benefit is paid at the moment of the last death of the insureds. The annual gross premium calculation can utilize the actuarial equivalence principle. According to the entry age of the insureds on some specific ages, the annual gross premium they must pay increases as they get older. The benefit reserve that an insurance company must prepare for last-survivor insurance at the end of each year increases annually, and its value approaches the insurance benefit amount. The longer the insurance coverage runs, the greater the fund the insurance company must prepare to pay for the insurance benefit.

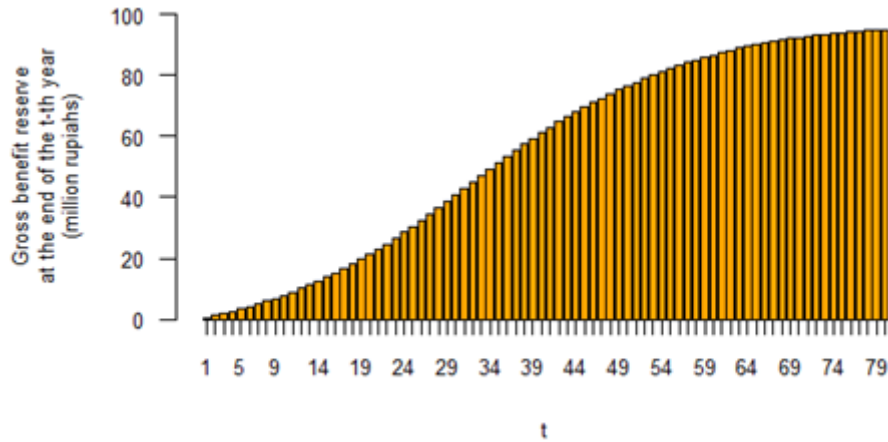


FIGURE 3. Gross benefit reserve for $x = 30$ and $y = 25$

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