Available online: https://a-mm.org

**Annals of Mathematical Modeling, 1 (2), 2021, 47-63**

# Effectiveness of problem posing and investigation in terms of problem solving abilities, motivation and achievement in mathematics

**Arifah Muzayyanah [a] *, Dhoriva Urwatul Wutsqa [b]**

Universitas Negeri Yogyakarta. Jalan Colombo No. 1, Yogyakarta 55281, Indonesia
* Coressponding Author. E-mail: [a] arifahmuzayyanah86@gmail.com, [b] dhoriva@yahoo.com

**Abstract:** The study aims at describing the comparison between the Problem-Posing Approach and the Investigation Approach in terms of effectiveness from the perspective of Mathematical Problem-Solving Skills, mathematical motivation and Mathematical Learning Achievements. The nature of the study is a quasi-experimental research. During the conduct of the study, the overall number of populations was 206 students from Grade VII of the State 2 Junior High School Gamping. The samples within the study were selected randomly from the population namely 32 students from Grade VII A and 34 students from Grade VII B. Then, the instrument that had been administered within the conduct of the study was the Mathematical Problem-Solving Skills Test, the Mathematical Learning Achievement Test and the Mathematical Learning Motivation Questionnaire. After the data had been gathered, the overall data were analysed by using the one-sample t-test, the $T^2$ Hotteling's Test and the independent sample t-test. The results of the study show that the learning process by means of Investigation Approach has been more effective in comparison to Problem-Posing Approach from the perspective of Mathematical Problem-Solving Skills and Mathematical Learning Achievement but, on the other hand, the results of the study also show that the learning process by means of Problem-Posing Approach has been more effective in comparison to Investigation Approach from the perspective of Mathematical Learning Motivation for the Grade VII students of the State 2 Junior High School Gamping.

**Keyword:** Problem-Posing Approach, Investigation Approach, Mathematical Problem-Solving Skills, Mathematical Learning Achievement, Mathematical Learning Motivation

## INTRODUCTION

Mathematics holds an important role within the daily life. The development of science and technology is not apart from, or cannot be set apart from, the role of Mathematics. Being aware of the significant role that Mathematics holds in multiple aspects of life, Mathematics thus should be provided with the students since the elementary school degree. In order to pursue this end, the learning process should be meaningful for each student. However, the learning process will be meaningful if the students are actively involved in the learning process and are able to develop their mathematical skills and motivation within the Mathematics learning process.

According to the Minister of National Regulation Number 22 of 2006 on the Content Standards, one of the capacities that the students should master is the mathematical problem-solving capacity. By mastering this capacity, the students will be accustomed to be persistent, curious and confident whenever they deal with the less-conducive situation outside Mathematics (NCTM, 2000, p.52). In addition, the Singapore Ministry of Education states that mathematical problem-solving is the core of the Mathematical Learning Process. The reason is that the mathematical problem-solving includes the acceptance and the implementation of mathematical concepts and capacities on numerous situations, including the non-routine problems, the open-ended problems and the real-life problems. With regards to problem-solving skills, the problem that might be encountered is different from one student to another because problem refers to the situation that an individual or a group of individual should solve (Gorman, 1974, pp.293-294; Carson, 2007, p.7; Shumway, 1980, p.287; Karp & Bay-Williams, 2010, p.33). In the context of Mathematical Learning Process, problem is defined as a situation that involves mathematical contents such as number, geometrical shape or algebra relationship (Royer, 2003, pp.70-71). In order to solve this problem, a student should master the Mathematical Problem-Solving Skills.

Then, the components that might be contained in the problem-solving skills are namely: (1) awareness of problem; (2) definition of problem and mental representation for the given problem; (3) exploration toward all possible strategies of solution; (4) implementation of the best strategy; (5) monitoring on the progress toward the objective of the solution; (6) evaluation of solution accuracy; and (7) learning from experience. This process might be conducted more effectively when the students have wide background knowledge with regards to the problems and also wide performance memory (Byrnes, 2008, p.79). In relation to these components, a learning process that focuses on the Mathematical Problem-Solving Skills should contain the following cognitive activities: (1) presenting the problem, including the relevant context of knowledge; (2) looking for solution, including the objective clarity and the action plan development for achieving the given objective; and (3) implementing the solution, including the implementation of the lesson plan and the result evaluation (Kirkley, 2003, p.4).

Still in relation to the problem-solving skills, Mayer (Kirkley, 2003, p.4) states that there are three characteristics of problem-solving skills namely: (1) problem-solving is a cognitive process but problem-solving is concluded from the behaviours; (2) problem-solving generates results within the boundaries of the behaviours that lead to the solution; and (3) problem-solving is a process that involves manipulation or operation from the knowledge that has been internalized previously. These characteristics imply that through the Mathematical Problem-Solving Skills the students have the opportunity to establish their own knowledge.

The development of the Mathematical Problem-Solving Skills will be better if the students are assigned into small groups so that the students will be more actively involved in the learning process. According to Adams & Hamm (2010, p.59), the mathematical problem-solving activities that involve group interaction and mutual dependence among the students have been proven to be an effective manner in engaging the students into the real-life assignments and experiences. In addition, through such procedure the students will not be reluctant to raise questions if they have difficulties since they are involved in the discussions within the small groups.

In addition to the Mathematical Problem-Solving Skills, another important aspect that should be given attention within the Mathematical Learning Process is the students' Mathematical Learning Motivation. The statement is in line with the argument proposed by Elliot et al. (2002, p.332), which states that both learning and motivation have been equally important for the complentation of achievements. Learning enables us to attain novel knowledge and capacities, whereas motivation encourages us to show what we have learned. Ambrose (2010, pp.68-69) strengthens this statement by arguing that within the learning context motivation has influence on the direction, the intensity, the persistence and the quality of the learning process that has been conducted. Similarly, Woody, Lavoie & Epps (1992, p.112), Dembo (2004, p.10) and Omrod (2003, p.368) altogether state that motivation refers to the internal process that provides energy and that also directs and maintains certain behaviours in order to meet certain objectives and achievements. Therefore, the learning process should also pay attention to the motivation that the students have.

Brophy (2010, p.208) & Woolfolk (2005, p.370) state that students' motivation to learn refers to the tendency of finding meaningful and useful academic activities and the tendency of striving to find academic benefits from such academic activities. The statement implies that the students who have the motivation to learn will be actively involved in the learning process, exert the higher cognitive process, pay attention to the instruction that the teachers provide, be willing to exercise all materials that have been learned, raise questions when they do not understand the learning materials, absorb and master more learning materials. This kind of students will also exert more enthusiasm and efforts for achieving the learning objectives, both the learning objectives that they have set and the learning objectives that the teachers have set, for the learning materials of each subject (Slavin, 2006, p.317; Santrock, 2011, p.437). Specific to the case of Mathematical Learning Motivation, there are five elements that might affect the students' Mathematical Learning Motivation, namely: (1) students; (2) teachers; (3) material contents; (4) method/process; and (5) students' learning environment (Williams & Williams, 2010, p.2). Thus, it is apparent that motivation might be influenced by both the students' external and internal aspects.

Woolfolk (2005, p.341) states that internal motivation is related to needs, interest and curiosity. In addition, Cohen & Swedlik (2009, p.576) state that intrinsic motivation occurs when the students are involved into certain activities for their own sake without any force such as satisfaction, attraction learning and challenge. On the other hand, Alderman (2004, p.247) & Woolfolk (2005, p.341) state that external motivation refers to the factors of environment, appreciation, social pressure, punishment, appraisal, privilege, certificate or other financial benefit. By paying attention to the learning motivation that the students have, specifically in the case of Mathematical Learning Process, the Mathematical Learning Process will be more meaningful and thus the students' mathematical achievements will improve. This statement is in line with the argument by Elliot et al. (2002, p.332), which states that motivation has influence on learning process and achievement at least in the four aspects namely: (1) the motivation to improve the students' energy and activity level; (2) the motivation to direct someone to certain objectives; and (3) the motivation to promote initiation from certain activities and from the persistence of performing the activities.

Within the learning process, knowledge expansion occurs in the affective, cognitive and psychomotor aspect and the expansion will be different from one student to another. The benchmark for measuring the size of the expansion might be the students' learning achievements. This statement is in accordance to the argument by Hawkin, Florian & Rouse (2007, p.22) and also Arends & Kilcher (2010, p.59), which altogether states that achievement refers to the satisfaction when the students strive to learn certain subjects or strive to attain difficult skills and they succeed mastering the subjects or the skills.

Furthermore, the factor that influences the meaningfulness of the learning process is teacher. In this regard, teacher as the learning process facilitator should prepare a learning process that accomodates the students' diversity. One of the efforts might be selecting variative approach that might be adjusted to the individual differences within the students and the learning materials. However, in the practice the learning process less facilitates the students to grow and develop their Mathematical Problem-Solving Skills, motivation and learning achievement. The reason is that the teachers have inclined to use the conventional approach in which they have been lecturing the students all the time. In such conventional approach, the teacher serves as the learning subject – explaining the materials and becoming the learning sources – while the students serve as the learning object – paying attention and taking notes on what the teachers have explained.

According to Cotton (2010, p.223), the learning focus for the students is the solution of exercise items that have been provided instead of the learning process for facilitating their understanding toward Mathematics. The teachers tend to pay attentionl to the achievement of the curriculum targets and thus distributes the National Examination-oriented test items to the students. As a result, the students tend to memorize the mathematical formula without understanding how to apply the mathematical principles into the daily life. Consequently, the learning objectives for developing the students' mathematical capacity and motivation.

The role of the teachers as the sole source of the learning process within the conventional approach has caused the students to be reluctant in looking for alternative sources. Thus, the solution for the test items will always refer to what the teachers have taught. Not to mention, the test items that the teachers have administered are the routine ones. Consequently, the students have less practice for solving the test items that demand the problem-solving skills. The impact is that the students' capacities in solving problems have been less developed and the students' learning motivation and even achievement have been low as well.

One of the ways that might be afforded for dealing with such situation is implementing the variative and student-oriented learning approach and providing opportunities for the students to develop their skills in accordance to the characteristics of the topics that have been presented. Several learning approaches that might be adopted in order to develop the students' problem-solving skills and also to improve the students' learning motivation and achievement are the Problem-Posing Approach and the Investigation Approach. According to Silver et al. (1996, p.293), Problem-Posing is the significant centre within the discipline of Mathematics and within the mathematical thinking process. Furthermore, Silver et al. (1996, p.294) also states that Problem-Posing includes several definitions namely: (1) formulation or re-formulation of test items that have been administered by means of several changes in order to be more facilitative; and (2) formulation of test items that have been related to the requirements of the test items that have been completed within the problem-solving process.

Lavy & Shriki (1996, p.293) states that within the learning process by means of Problem-Posing Approach the students are advised to go through three learning levels. In the first level, the students are asked to define the list of the problem attributes. Then, in the second level the students should discuss the questions of "what," "if" and "how" and suggest the alternatives for the attributes that have been put on the list. Last but not the least, in the third level the students should raise new questions that have been inspired from the alternatives that have been found. In relation to the Problem-Posing Approach within the Mathematical Learning Process, Bonotto (2010, p.21) states that the mathematical Problem-Posing Approach refers to the process that has been based on the mathematical experience. The students establish their own interpretation from the real situations and turn the real situations into the meaningful mathematical problems. Then, according to Brown & Walter (2005, p.18) within the proposal of the mathematical problem there are two important aspects namely accepting and challenging. The aspects of accepting are related to the students' capacity in understanding the situations that the teachers have given. On the contrary, the aspects of challenging are related to how far the students are challenged by the given situations so that the capacity of proposing mathematical problems will be born.

Departing from the above elaboration, it might be concluded that Problem-Posing is a learning approach in which the students are asked to formulate the problems over a situation or to re-formulate a problem into several new questions that are simpler, more understandable and might be solved based on the knowledge that the students have internalized. According to Silver & Cai (1996, p.523), there are three different types of mathematical cognitive activities namely: (1) pre-solution posing, which occurs when a student designs test items from the given situations and the student later is asked to propose the test items by linking the information to the knowledge that he or she has internalized; (2) within-solution posing, which occurs when a student is able to re-formulate the test items into several new sub-questions which problem-solving sequence has been completed previously; and (3) post-solution posing, which occurs when a student modifies the objectives or the conditions of the test items that have been completed in order to design similar test items.

Furthermore, Abu & El-Sayed (2000, pp.59-61) state that based on the questions that the students might ask the Problem-Posing Approach might be divided into three categories namely: (1) free Problem-Posing, in which the students design the test items freely based on the daily life situations; (2) semi-structured Problem-Posing, in which the students are provided with free or open situations and are asked to explore these situations by using the knowledge, the skills or the concepts that they have internalized; and (3) structured Problem-Posing, in which the students are asked to design other test items based on the test items that have been identified by changing the data or the information that has been under possession. Then, specific to the context of the present study, the stages of the Problem-Posing Approach that will be implemented in the study are as follows: (1) Selection of preliminary point: the students understand the problems and the situations that the teacher has presented; (2) Information discovery based on the given situations: the students elaborate the matters that they have identified from the problems or the situations that have been presented; (3) Inquiry based on the information that has been attained from the results of the observation toward the given situations: the students design questions that might appear from the information that they have attained from the second stage; (4) Prediction on the solution for the inquiry that has been proposed: the students define the solutions for the inquiry that has been designed; and (5) Discussion on the solution for the inquiry: the students are assisted by the teachers in discussing their work results.

According to Brown & Walter (Guvercin, Cilavdaroglu & Savas, 2014, p.130), Problem-Posing does not only generate new problems from the given situations but also re-formulates the problems and generalizes the solutions. This statement is in accordance to the stages in the process of the students' mathematical problem-solving activities. The re-formulation might only be performed by the students if the students are able to identify the presence of such problems. In addition, Silver (Guvercin, Cilavdaroglu & Savas, 2014, p.131) states that Problem-Posing is an alternative method for the students who are not good at Mathematics. The reason is that when the students raise questions they are aware the main and significant structure of a problem and identifies the clues within the process of attaining solutions.

In relation to the students' Mathematical Learning Achievement, Guvercin, Cilavdaroglu & Savas (2014, p.130) state that the activities within the Problem-Posing Approach provide more opportunities for the students to be responsible upon the learning process that the students have been attending to. Similarly, Pallak (Guvercin, Cilavdaroglu & Savas, 2014, p.130) states that the students sometimes encounter difficult test items and re-design the difficult test items during the problem-solving activities because the re-designing activities facilitate better understanding for the students. These activities are interesting because the difficult test items might improve the Mathematical Learning Process of the students. In other words, the students might re-construct the difficult test items by identifying the new ways for solving the test items.

In relation to the students' motivation, according to Freire (Guvercin, Cilavdaroglu & Savas, 2014, p.130) the Problem-Posing Approach might the students' motivation to participate in the learning process since the students are provided with the freedom within the learning process. This is the reason why the students will have higher order thinking skills with regards to their environment after the implementation of the Problem-Posing Approach. The main element in the Problem-Posing Approach is the critical thinking skills. the critical thinking skills provide more opportunities for the students to solve open-ended and non-routine test items. Thus, the understanding of the mathematical concept and process during the learning process by means of Problem-Posing Approach is a positive factor that might influence the students' Mathematical Learning Achievement.

On the other hand, the Investigation Approach refers to the learning approach that might encourage the activities of experiment, data collection, observation, pattern identification, conjecture and generalization. It is apparent that the Investigation Approach is compatible for the learning process that puts emphasis on the problem-solving activities. Bailey (2007, p.103) defines the term investigation as open-ended problems or questions that are compatible for enabling the activities of exploration, which lead to numerous mathematical ideas and/or solutions. According to Edmond & Knight (Grimison & Dawe, 2000, p.6), mathematical Investigation Approach refers to the open-ended activities of identifying pattern independently, decreasing teacher role, performing useful independent activities, not completing the real-life mathematical problems, using own method and not being confined by the teacher experience. Thereby, the students able to establish their own knowledge based on the activities that they perform independently.

With regards to the teacher as learning facilitator within the implementation of Investigation Approach, Haylock & Thangata (2007, p.97) state that the teachers should provide opportunities for the students to: (1) take participation in the assignments that make them challenged, interest and enthusiastic; (2) raise questions about mathematical situations; (3) plan their own learning process; (4) use mathematical knowledge and capacities that they have internalized; (5) attain satisfaction and experience in discovering something by means of their own efforts; (6) communicate their findings to other people; and (7) develop their own understanding toward the mathematical concepts. Specific to the context of the present study, the stages of the Investigation Approach implementation consist of: (1) Specialization: the students learn or comprehend the situations that have been presented; (2) Conjecturing: the students formulate hypotheses; (3) Justification: the students gather information, answer questions and elaborated the answers that they have found; and (4) Generalization: the students draw their own conclusions based on the answers that they have found. Indeed, it is apparent that the stages in the Investigation Approach have been in accordance to the heuristic process within the mathematical problem-solving. According to Ponte (2001, p.5), investigation might improve the students' mathematical problem-solving capacities because this approach provides opportunities for the students to benefit concepts, representations, ideas and procedures that they have internalized.

Still according to Ponte (2001, p.6), and in addition according to Greenes (Diezmann, Watters & English, 2001, p.2), in relation to Mathematical Learning Motivation the Investigation Approach is able to improve the students' Mathematical Learning Motivation because the students are provided with the opportunity to be independent and responsible with their learning process. The problems that have been presented through this approach triggers the students' attention and curiosity. In the same time, the students are habituated to make assumptions and prove the assumption by looking for relevant information from the book or from the discussion. These activities increase the students' curiosity and involvement within the learning process. Furthermore, the Investigation Approach is able to improve the students' learning achievement because during the learning process the students are provided with the opportunities to think originally and freely so that the students might be brought into deeper concept understanding.

Departing from the above elaboration on the Problem-Posing Approach and the Investigation Approach, both approaches a single similarity namely that the students are able to develop their own mathematical knowledge from the learning process that they perform through the implementation of both approaches. Therefore, there should be a comparison in order to identify which learning approach that might be more effective from the perspective of Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement. The expectation is that the teachers will have a matter of comparison in selecting the relevant learning approach for the conduct of the Mathematical Learning Process in order to optimize the students' Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Motivation within the Mathematical Learning Process. In the same time, through the conduct of the present study it is also expected that the implementation of both the Problem-Posing Approach and the Investigation Approach is able to improve the learning quality from the perspective of problem-solving skills, learning motivation and learning achievement.

## METHOD

The type of study that had been adopted in the study was the quasi-experimental research. The data for the study were gathered from the State 2 Junior High School Gamping, Sleman, from April 1$^{st}$, 2014 until April 29$^{th}$, 2014. The population within the conduct of the study was all of the Grade VII students in the given junior high school from the Academic Year 2013/2014; in total, the number of the population was 206 students. Then, the sample was selected randomly from the given population and the respondents that had been selected as the sample for the study were 32 students from Grade VII A and 34 students from Grade VII B. The students from Grade VII A were provided with the Problem-Posing Approach as the treatment, while the students from Grade VII B were provided with the Investigation Approach as the treatment. The data were gathered from the students by implementing the Pre-Test and Post-Test Design.

The data within the study consisted of the preliminary data and the final data of the students' Mathematical Problem-Solving Skills, the preliminary data and the final data of the students' Mathematical Learning Motivation and also the preliminary data and the final data of the students' learning achievements. In order to gather the necessary data, the instruments that had been administered were the Mathematical Problem-Solving Skills Test instrument, the Mathematical Learning Motivation Questionnaire and the Mathematical Learning Achievement Test instrument.

Within the data gathering activities, the data gathering techniques that had been implemented were the Mathematical Problem-Solving Skills pre-test items, the Mathematical Learning Motivation Preliminary Questionnaire guideline and the Mathematical Learning Achievement pre-test items. Then, the treatment, namely the Problem-Posing Approach and the Investigation Approach, was provided to each of the experimental group. At the end of the treatment, the Mathematical Problem-Solving Skills post-test, the Mathematical Learning Motivation Final Questionnaire and the Mathematical Learning Achievement post-test were administered to each experiment group in order to identify the effectiveness of each learning approach.

Data analysis was conducted during and after the data gathering activities. Within the study, the data analysis techniques that had been adopted were the descriptive analysis and the inferential analysis. The descriptive analysis for the Mathematical Problem-Solving Skills, the Mathematical Learning Motivation and the Mathematical Learning Achievement consisted of mean score, lowest score, highest score and standard deviation. The data that had been attained were interpreted into the criteria which percentage had been defined.

The data for the Mathematical Problem-Solving Skills were attained by administering the essay-type test instrument. Then. The data for the Mathematical Learning Achievement were attained by administering the multiple choice-type test instrument. The score that had been attained from both instruments were converted into the score that ranged between 0 (zero) and 100 (one hundred). Afterward, these scores were classified into the criteria that had been based on the passing grade of Mathematics assigned by the school namely 75. The passing grade was used for determining the students' success or failure within the Mathematical Learning Process.

On the other hand, the data for the students' Mathematical Learning Motivation were attained by using the checklist-form non-test instrument with Likert scale. The data that had been gathered for the students' Mathematical Learning Motivation were categorized into several criteria based on the Mathematical Learning Motivation table. The scoring for the students' Mathematical Learning Motivation within the study was assigned to the range between 35 and 175. In order to identify the criteria of the measurement results, a classification based on the ideal mean score (Mi) and the standard deviation (Si) should be adopted. The ideal mean score was calculated by attaining half of the addition between the maximum score and minimum score, whereas standard deviation was calculated by attaining one-sixth of the subtraction between the maximum score and the minimum score. The total actual score that had been attained then was converted into the qualitative data and the results of the conversion might be consulted in Table 1 below.

**Table 1.** Criteria of the Students' Mathematical Learning Motivation

| Interval | Score (X) | Criteria |
|---|---|---|
| Mi+1,5Si < X≤Mi+3Si | 140<X≤175 | Very High |
| Mi+0,5Si<X≤Mi+1.5Si | 117<X≤140 | High |
| Mi-0,5Si<X≤Mi+0.5Si | 93<X≤117 | Moderate |
| Mi-1,5Si<X≤Mi-0.5Si | 70<X≤93 | Low |
| Mi-3Si≤X≤Mi-1.5Si | 35≤X≤70 | Very Low |

Source: (Azwar, 2010, p.163)

After the data on the measurement of the students' Mathematical Learning Motivation had been attained, the total score of each unit was categorized based on the criteria that had been assigned in Table 1. Then, the percentage of the total score that had been gathered from each unit was calculated for each category namely: (1) Very Low; (2) Low; (3) High; and (4) Very High. The score was used for defining the effectiveness of the learning approach adoption within the study.

For the inferential analysis, the very first stage that should be conducted was the assumption test. The assumption test consisted of normality test and homogeneity test. Both tests were conducted by using the univariate and multivariate manner. The multivariate normality test was conducted by using the mahalanobis gap statistical test with the assistance from the Microsoft Excel 2010 through the following procedures: (1) defining $d_i^2 = (x_i - x)'S^{-1}(x_i - x)$ in which $x_i$ was the i-observation and $S^{-1}$ is the $S$ covariance matrix inverse; and (2) the $d_i^2$ value was ordered from the smallest to the greatest. The assumption of multivariate normality would be met if 50.00% of $d_i^2$ is lower than $x^2_{0.5(3)}$ $\left(d_i^2 < x^2_{0.5(3)}\right)$. On the contrary, the univariate normality test was conducted by using the Kolmogorov-Smirnov statistical test (Pearson, 2010, p.292). For the conduct of the multivariate homogeneity test, the statistical test that had been adopted was the Box's M test (Rencher, 1998, pp.139-140) whereas for the conduct of the univariate homogeneity test the statistical test that had been adopted was the Levene's test (Pearson, 2010, p.212).

The second stage of the study was the analysis on the effectiveness between the Problem-Posing Approach and the Investigation Approach toward each variable, namely the Mathematical Problem-Solving Skills, the Mathematical Learning Motivation and the Mathematical Learning Achievement. The test within the second stage was conducted by using the one-sample t-test formula as follows:

$$t = \frac{\bar{x}-\mu_0}{s\sqrt{n}}$$ (Walpole, 1997, p.305)

Note:
$\bar{x}$ = sample mean score
$\mu_0$ = assigned mean score
$s$ = sample variant
$n$ = number of samples

Within the study, $\mu_0$ that had been assigned for the students' Mathematical Problem-Solving Skills and the students' Mathematical Learning Motivation was 75 whereas $\mu_0$ that had been assigned for the students' Mathematical Learning Motivation was 117.

The third stage of the study was the MANOVA test. The MANOVA test was conducted toward the data that had been attained before and after the treatment. The MANOVA test before the provision of the treatment was intended to identify whether the students had preliminary capacity or not. Then, the MANOVA test after the provision of the treatment was intended to identify the effectiveness of the Problem-Posing Approach and of the Investigation Approach from the perspective of Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement. The data that had been attained were simultaneously analysed from the perspective of Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement. The conduct of the MANOVA test adopted the multivariate two-group test (Hotelling's Trace) with the assistance from the SPSS 16.00 for Windows. The formula of the multivariate two-group test (Hotelling's Trace) was as follows:

$$T^2 = \frac{n_1 n_2}{n_1+n_2}(\bar{y}_1 - \bar{y}_2)' S^{-1}(\bar{y}_1 - \bar{y}_2)$$

Note:
$T^2$ = $T^2$ Hotelling's
$n_1$ = Number of subjects in the Problem-Posing treatment
$n_2$ = Number of subjects in the investigation treatment
$\bar{y}_1$ = Mean vector for the Problem-Posing Group
$\bar{y}_2$ = Mean vector for the Investigation Group
$S^{-1}$ = S variance-covariance inverse matrix

After the $T^2$ Hotelling's value had been attained, the value was transformed in order to attain the F distribution value by using the following formula:

$$F = \frac{n_1+n_2-p-1}{(n_1+n_2-2)p} T^2$$ (Stevens, 2009, p.148)

Note:
$p$ = Number of dependent variables

The fourth or the last stage in the study was the identification toward the different rate of effectiveness between the Problem-Posing Approach and the Investigation Approach. The identification test was conducted in order to identify which learning approach (the Problem-Posing Approach and the Investigation Approach) that had been more effective from the perspective of Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement. The statistical test that had been adopted for the identification was the independent sample t-test. The formula for the conduct of the independent sample t-test was as follows:

$$t = \frac{\bar{y}_{1i} - \bar{y}_{2i}}{\sqrt{\frac{(n_1-1)s_{1i}^2+(n_2-1)s_{2i}^2}{n_1+n_2-2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$ (Stevens, 2009, p.147)

Note:
$\bar{y}_{1i}$ = Mean score for the Problem-Posing Group in the variable *i*
$\bar{y}_{2i}$ = Mean score for the Investigation Group in the variable *i*
$s_{1i}^2$ = Variance of the Problem-Posing Group in the variable *i*
$s_{2i}^2$ = Variance of the Investigation Group in the variable *i*
$n_1$ = Number of Problem-Posing Group members
$n_2$ = Number of Investigation Group members

## RESULTS AND DISCUSSIONS

### Results

The data in the study consist of the before-treatment data and after-treatment data from each experiment group. The before-treatment data consist of the pre-test data of the Mathematical Problem-Solving Skills, the pre-test data of the Mathematical Learning Achievement and the completion of the preliminary questionnaire on the Mathematical Learning Motivation. On the contrary, the after-treatment data consist of the post-test data of the Mathematical Problem-Solving Skills, the post-test data of the Mathematical Learning Achievement and the completion of the final questionnaire on the Mathematical Learning Motivation. After the before-treatment data and the after-treatment data have been attained, the analysis toward the overall data might be started.

The first stage in the data analysis is the descriptive analysis. The results of the descriptive analysis for the pre-test data and the post-test data of the Mathematical Problem-Solving Skills between the two experimental groups might be consulted in Table 2.

**Table 2.** Description of the Students' Mathematical Problem-Solving Skills

| Statistics | Problem Posing | | Investigation | |
|---|---|---|---|---|
| | Pre-test | Post-test | Pre-test | Post-test |
| Number of Students | 32 | 32 | 34 | 34 |
| Mean Score | 38.54 | 82.40 | 44.02 | 80.20 |
| Standard Deviation | 13.33 | 10.27 | 15.61 | 11.37 |
| Lowest Score | 13.33 | 63.33 | 13.33 | 56.67 |
| Highest Score | 66.67 | 96.67 | 66.67 | 96.67 |
| Theoretical Minimum Score | 0 | 0 | 0 | 0 |
| Theoretical Maximum Score | 100 | 100 | 100 | 100 |
| Passing Grade Completion (%) | 0 | 25 (78.13) | 0 | 26 (76.47) |

From the results in Table 2, it is apparent that the mean score of the pre-test data for the students' Mathematical Problem-Solving Skills between the Problem-Posing Group and the Investigation Group is 38.54 and 44.02 respectively. On the contrary, it is also apparent that the mean score of the post-test data for the students' Mathematical Problem-Solving Skills between the Problem-Posing Group and the Investigation Group is 82.40 and 80.20 respectively. In the same time, it is also apparent that the percentage of the passing grade completion between the Problem-Posing Group and the Investigation Group within the Mathematical Problem-Solving Skills is 0.00%. The implication of this statement is that there has not been any student who has passed the minimum passing grade.

After the treatment in the form of Problem-Posing Approach and investigation-approach has been provided, the students' Mathematical Problem-Solving Skills in both experimental groups show improvement. The improvement is apparent from the post-test results in both experimental groups. 25 students from the Problem-Posing Group and 26 students from the Investigation Group have altogether passed the minimum passing grade. Thereby, it might be concluded that the percentage of the passing grade completion for the Problem-Posing Approach and the Investigation Approach within the post-test results of the Mathematical Problem-Solving Skills has been 78.13% and 76.47% respectively. Next, the description on the results of the Mathematical Learning Motivation completion in both experimental groups might be consulted in Table 3.

**Table 3.** Description of the Students' Mathematical Learning Motivation

| Statistics | Problem Posing | | Investigation | |
|---|---|---|---|---|
| | Before Treatment | After Treatment | Before Treatment | After Treatment |
| Number of Students | 32 | 32 | 34 | 34 |
| Mean Score | 113.06 | 131.91 | 119.09 | 124.35 |
| Standard Deviations | 13.65 | 11.12 | 14.16 | 12.55 |
| Lowest Score | 81 | 109 | 88 | 99 |
| Highest Score | 135 | 149 | 151 | 146 |
| Theoretical Minimum Score | 35 | 35 | 35 | 35 |
| Theoretical Maximum Score | 175 | 175 | 175 | 175 |

In Table 3, it is apparent that the mean score of the before-treatment data for the Mathematical Learning Motivation between the Problem-Posing Group and the Investigation Group has been 113.06 and 119.09 respectively with the category "Moderate." On the other hand, it is also apparent that the mean score of the after-treatment data for the Mathematical Learning Motivation between the Problem-Posing Group and the Investigation Group has been 131.91 and 124.35 respectively with the category "High."

The frequency and the percentage of the students in each criteria of Mathematical Learning Motivation have been calculated in accordance to the score range that has been defined. The distribution of the frequency and the percentage in the students' Mathematical Learning Motivation before and after the treatment between both groups might be consulted in Table 4.

**Table 4.** Distribution on the Frequency and the Percentage of the Students' Mathematical Learning Motivation Before and After the Treatment

| Criteria | Before Treatment | | | | After Treatment | | | |
|---|---|---|---|---|---|---|---|---|
| | Problem Posing | | Investigation | | Problem Posing | | Investigation | |
| | f | % | f | % | f | % | f | % |
| Very High | 0 | 0.00 | 1 | 2.94 | 7 | 21.88 | 4 | 11.76 |
| High | 14 | 43.75 | 21 | 61.76 | 21 | 65.63 | 22 | 64.71 |
| Moderate | 16 | 50.00 | 10 | 29.41 | 4 | 12.50 | 8 | 23.53 |
| Low | 2 | 6.25 | 2 | 5.88 | 0 | 0.00 | 0 | 0.00 |
| Very Low | 0 | 0.00 | 0 | 0.00 | 0 | 0.00 | 0 | 0.00 |
| Total | 32 | 100 | 34 | 100 | 32 | 100 | 34 | 100 |

Based on the results in Table 4, with regards to the Mathematical Learning Motivation before the treatment, the students do not display any "Highly Low" category in both the Problem-Posing Group and the Investigation Group. Within the Problem-Posing Group, 14 students (43.75%) belong to "High" category, 16 students (50.00%) belong to "Moderate" category and 2 students (6.25%) belong to "Low" category. On the contrary, in the Investigation Group 1 student (2.94%) belongs to "Very High" category, 21 students (61.76%) belong to "High" category, 10 students (29.41%) belong to "Moderate" category and 2 students (5.88%) belong to "Low" category. Then, after the provision of the treatment the students do not belong to both "Low" and "Very Low" category in both the Problem-Posing Group and the Investigation Group. Within the Problem-Posing Group, 7 students (21.88%) belong to "Very High" category, 21 students (65.63%) belong to "High" category and 4 students (12.50%) belong to "Moderate" category. In the meantime, within the Investigation Group 4 students (11.76%) belong to "Very High" category, 22 students (64.71%) belong to "High" category and 8 students (23.53%) belong to "Moderate" category.

Furthermore, the description the pre-test results and the post-test results of the Mathematical Learning Achievement between both experimental groups might be consulted in Table 5.

**Table 5.** Description of the Students' Mathematical Learning Achievement

| *Statistik* | *Problem Posing* | | *Investigation* | |
|---|---|---|---|---|
| | *Pre-test* | *Post-test* | *Pre-test* | *Post-test* |
| Number of Students | 32 | 32 | 34 | 34 |
| Mean Score | 49.58 | 84.17 | 47.25 | 84.12 |
| Standard Deviation | 11.48 | 9.39 | 12.21 | 9.43 |
| Lowest Score | 33.33 | 66.67 | 66.67 | 66.67 |
| Highest Score | 73.33 | 100.00 | 26.67 | 100.00 |
| Theoretical Minimum Score | 0 | 0 | 0 | 0 |
| Theoretical Maximum Score | 100 | 100 | 100 | 100 |
| Ketuntasan (%) | 0 | 26 (81.25) | 0 | 27 (79.41) |

According to the results in Table 5, it is apparent that the mean score for the pre-test results of the students' Mathematical Learning Achievement between the Problem-Posing Group and the Investigation Group has been 49.58 and 47.25 respectively. On the other hand, the mean score for the post-test result of the students' Mathematical Learning Achievement between the Problem-Posing Group and the Investigation Group has been 84.17 and 84.12 respectively. In the same time, it is also apparent that the percentage of passing grade completion for the students in the Problem-Posing Group and the Investigation Group in the pre-test results of the Mathematical Learning Achievement has been 0.00%, which implies that none of the students have passed the grade.

After the treatment in the form of Problem-Posing Approach and Investigation Approach has been provided, the students' Mathematical Learning Achievement between the two experimental groups has shown improvement. The improvement might be traced back to the post-test results of the two experimental groups. The number of the students who have met the passing grade in the Problem-Posing Group is 26 people, whereas the number of students who have met the passing grade in the Investigation Group is 27 people. Therefore, the percentage of the passing grade completion between the Problem-Posing Group and the Investigation Group within the post-test results of the students' Mathematical Learning Achievement is 81.25% and 79.41%.

After the descriptive analysis has been conducted, the next procedure is conducting the inferential analysis. The first stage of the inferential analysis conduct is assumption test, which consists of normality test and homogeneity test. The normality test is conducted in order to define whether the data have come from the normally-distributed population, according to both univariate and multivariate manner, or not. The results of the multivariate normality test before and after the treatment might be consulted in Table 6.

**Table 6.** Results of Multivariate Normality Test

| Group | Number of Students | Percentage of $d_i^2 < \chi^2_{0,5(3)}$ | |
|---|---|---|---|
| | | Before Treatment | Before Treatment |
| Problem-Posing | 32 | 54,55% | 51,52% |
| Investigation | 34 | 48,49% | 45,46% |

From the results in Table 6, it is apparent that the percentage of $d_i^2 < x^2_{0,5(3)}$ for both groups has been 50.00% both before and after the treatment. Therefore, it might be inferred that the assumption of multivariate normality both before and after the treatment has been met.

On the other hand, the univariate normality test is conducted by using the Kolmogorov-Smirnov Test with the assistance from the SPSS 16.00 for Windows program. The criteria of decision is as follows: if the significance value > 0.05 then the data will be from the normally distributed populaion. The result of the univariate normality test both before and after the treatment might be consulted in briefly in Table 7.

From the results in Table 7, it is apparent that the significance value of the students' Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement for both experimental groups has been higher than 0.05. Therefore, it might be inferred that the assumption of univariate normality has been met.

**Table 7.** Results of Univariate Normality Test

| Variable | Group | Sign. | |
|---|---|---|---|
| | | Before Treatment | After Treatment |
| Mathematical Problem-Solving Skills | Problem Posing | 0.111 | 0.183 |
| | Investigation | 0.053 | 0.076 |
| Mathematical Learning Motivation | Problem Posing | 0.200 | 0.200 |
| | Investigation | 0.200 | 0.200 |
| Mathematical Learning Achievement | Problem Posing | 0.199 | 0.105 |
| | Investigation | 0.065 | 0.092 |

Another assumption that should be met is the assumption of variance-covariance matrix similarity and the dependent variable variance similarity, which is also known as the homogeneity test. The homogeneity test is conducted toward each dependent variable (variance homogeneity test) and all dependent variables simultaneously (variance-covariance matrix homogeneity test). The results of the homogeneity test both before and after the treatment might be consulted in Table 8.

**Table 8.** Results of Homogeneity Test

| Homogeneity Test | Variable | Sign. | |
|---|---|---|---|
| | | Before Treatment | After Treatment |
| Variance-Covariance Matrix | Mathematical Problem-Solving Skills, Mathema-tical Learning Motivation and Mathematical Learning Achievement | 0.967 | 0.292 |
| Variance | Mathematical Problem-Solving Skills | 0.182 | 0.767 |
| | Mathematical Learning Motivation | 0.962 | 0.514 |
| | Mathematical Learning Achievement | 0.741 | 0.913 |

From the results in Table 8, it is apparent that the significance value for the variance-covariance matrix homogeneity both before and after the treatment for both experimental groups has been higher than 0.05. Consequently, the assumption of variance-covariance matrix homogeneity has been met. In addition, it is also apparent that the variance for the students' Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement has been higher than 0.05. Therefore, it might also be inferred that the assumption of variance homogeneity has been met.

The second stage of the inferential analysis is conducted in order to identify the effectiveness of the Mathematical Learning Process by means of Problem-Posing Approach and of Investigation Approach from the perspective of each variable. In this regard, the data that have been tested are the post-test results of the students' Mathematical Problem-Solving Skills and Mathematical Learning Achievement and also the completion of the students' mathematical learning questionnaire completion after the provision of the treatment. The statistical test that has been adopted in the second stage is the one-sample t-test with the assistance from the SPSS 16.00 for Windows. Then, the criteria of decision will be as follows: if the significance value < 0.05 or if the $t_{\text{-count}}$ value > $t_{\text{-table}}$ value the Ho will be rejected. The results of the effectiveness test might be consulted in Table 9.

From the results in Table 9, for the Problem-Posing Group the t-count is 4.072 for the Mathematical Problem-Solving Skills, 7.582 for the Mathematical Learning Motivation and 5.523 for the Mathematical Learning Achievement. These values show that the Mathematical Learning Process by means of Problem-Posing Approach has been effective from the perspective of Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement. On the other hand, for the Investigation Group the t-count is 2.665 for the Mathematical Problem-Solving Skills, 3.417 for the Mathematical Learning Motivation and 5.638 for the Mathematical Learning Achievement. These values show that the Mathematical Learning Process has been effective from the perspective of Mathematical Problem-Solving Skills, the Mathematical Learning Motivation and the Mathematical Learning Achievement.

**Table 9.** Results of Approach Effectiveness Test

| Group | Variable | $t_{count}$ | $t_{table}$ |
|---|---|---|---|
| Problem Posing | Mathematical Problem-Posing Skills | 4.072 | 2.04 |
| | Mathematical Learning Motivation | 7.582 | |
| | Mathematical Learning Achievement | 5.523 | |
| Investigation | Mathematical Problem-Posing Skills | 2.665 | 2.03 |
| | Mathematical Learning Motivation | 3.417 | |
| | Mathematical Learning Achievement | 5.638 | |

The conduct of the third stage in the inferential analysis has adopted the Manova test. The Manova test is conducted toward the data that have been attained both before and after the provision of the treatment. The analysis toward the data before the provision of the treatment is conducted in order to identify the differences on the preliminary skills and achievement and also preliminary achievement between the two experimental groups. Therefore, both experimental groups are provided with several pre-test treatments namely the Mathematical Problem-Solving Skills Test, the Mathematical Learning Achievement Test and the Mathematical Learning Motivation Preliminary Questionnaire. Then, the data that have been attained from these pre-test treatments are analysed in order to identify whether there have been differences or not in terms of the preliminary level of Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement between the two experimental groups prior to the provision of the treatment.

Since the assumption of both normality and homogeneity has been met, the multivariate test might be performed. Then, the multivarite test that has been conducted in order to test the mean score similarity within the study is the Hotelling's Trace with the assistance from the SPSS 16 for Windows. The results of the multivariate test might be consulted in Table 10.

**Table 10.** Results of Multivariate Test with the Hotelling's Trace

| | Before Treatment | After Treatment |
|---|---|---|
| F | 1.738 | 7.677 |
| Sig. | 0.168 | 0.000 |

From the results in Table 10, it is apparent that the significance value has been higher than 0.05 and $F_{-count}$ is equal to 1.738, which has been higher than the $F_{-table}$ $(F_{0.05,3.62}) = 2.75$ and thus there has not been any difference on the pre-test mean score between the Problem-Posing Group and the Investigation Group in terms of the Mathematical Problem-Solving Skills, the Mathematical Learning Motivation and the Mathematical Learning Achievement. In other words, it might be implied that the students in both experimental groups have equal Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement.

The assumption of normality and homogeneity for the after-treatment data has been met in the third stage. Based on the results of the one-sample t-test, the Mathematical Learning Process by means of Problem-Posing Approach and of Investigation Approach has been effective from the perspective of Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement. Therefore, a further test in the form of multivariate test should be conducted in order to identify whether there have been differences between the Problem-Posing Approach and the Investigation Approach or not.

With regards to the effectiveness between the two learning approaches, from the results in Table 10 above it is apparent that the significance value has been higher than 0.05 and the $F_{-count}$ is equal to 7.677, which has been higher than the $F_{-table}$ $(F_{0.05,3.62}) = 2.75$. Therefore, it might be implied that there have been differences between the Problem-Posing Approach and the Investigation Approach with regards to Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement.

The results from the previous analysis show that there have been differences between the Problem-Posing Approach and the Investigation Approach from the simultaneous perspective of Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical

Learning Achievement. Consequently, the four stage of the test should be conducted in order to identify which learning approach that might be more effective from the perspective of each variable. The four stage of the test is conducted by using the independent sample t-test with the assistance from the SPSS 16.00 for Windows. The results of the effectiveness difference test between both learning approaches from the perspective of each variable might be consulted in Table 11.

**Table 11.** Results of Effectiveness Comparison between the Problem-Posing Approach and the Investigation Approach from the Perspective of Each Variable

| Variable | Sig. | Rate of Significance | $t_{count}$ | $t_{table}$ |
|---|---|---|---|---|
| Mathematical Problem-Solving Skills | 0.414 | | 0.823 | |
| Mathematical Learning Achievement | 0.983 | 0.017 | 0.021 | 2.46 |
| Mathematical Learning Motivation | 0.012 | | 2.582 | |

From the results in Table 11, the $t_{-count}$ of Mathematical Problem-Solving Skills has been 0.823 with the significance value 0.414. Therefore, it might be concluded that the Mathematical Learning Process by means of Investigation Approach is equally or more effective in comparison to Problem-Posing Approach from the perspective of Mathematical Problem-Solving Skills. Then, the $t_{-count}$ of Mathematical Learning Achievement has been 0.021 with the significance value 0.983. Automatically, it might be concluded that the Mathematical Learning Process by means of Investigation Approach is equally or more effective in comparison to Problem-Posing Approach from the perspective of Mathematical Learning Achievement. Last but not the least, the $t_{-count}$ of Mathematical Learning Motivation has been 2.58 with the significance value 0.012. Consequently, it might be concluded that the Mathematical Learning Process by means of Problem-Posing Approach has been more effective in comparison to Investigation Approach from the perspective of Mathematical Learning Motivation.

## Discussions

The Problem-Posing Approach has been implemented in the Mathematical Learning Process of Grade VII A. Based on the results of and the discussion within the study with regards to the after-treatment data, it might be inferred that the Problem-Posing Approach has been more effective from the perspective of Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement. Through the Problem-Posing Approach the students are provided with the freedom to devise their own inquiry and discovery both independently and collaboratively. In this process, the students will mutually assist each other and will perform mutual communication from one to another within the group. The students who have higher level of capacities will assist the students who have moderate level and lower level of capacities in devising the inquiry and the discovery. Thereby, the students who have moderate or lower level of capacities might improve their performance in the Mathematical Problem-Solving Skills so that they will be more motivated in learning Mathematics. In the stage of Solution Discovery and Presentation, the students might take active participation in order to solve the problems that have been assigned by the other groups so that they will gain the best score. In addition, through the implementation of Problem-Posing Approach the students might feel challenged to deliver the presentation of problem-resolution that might be understood by the other students.

In the meantime, the Investigation Approach is implemented within the Mathematical Learning Process of Grade VII B. Based on the results of and the discussion within the study with regards to the after-treatment data, it might be inferred that the Investigation Approach has been effective from the perspective of Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement. Through the Investigation Approach the students take active participation within the learning process by having discussions with their fellow group members. With the presence of the Students' Worksheets, the students have the opportunity to develop their thinking skills through the specialization, conjecturing, justification and generalization process. This statement is in line with the argument proposed by Jaworski (2003, p.6), who states that the idea of investigation is fundamental both to the study of Mathematics itself and also to an understanding of the ways in which Mathematics can be used to extend knowledge and to solve problems in very many fields. Thereby, it might be inferred that both of the Problem-Posing Approach and the Investigation Approach have been effective from each aspect namely Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement.

The results of the multivariate test toward the before-treatment data conclude that the preliminary condition of both experimental groups that will be provided with the treatment has been equal. After both experimental groups have been provided with the treatment and test, the results of the one-sample t-test show that both of the Problem-Posing Approach and the Investigation Approach have been effective from each aspect namely Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Motivation. As a result, a further investigation on the difference of the effectiveness between the two learning approaches should be conducted.

Based on the results of the two-group MANOVA statistical test, it might be inferred that there have been differences between the Problem-Posing Group and the Investigation Group. These differences lie in all aspects within the study namely Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement. Since there have been differences in terms of multivariate manner, a further analysis by means of t-test should be conducted in order to identify whether there have been significant differences or not in terms of univariate manner within the Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement.

Based on the results of the t-test, it is found that the Problem-Posing Approach has been more effective in comparison to the Investigation Approach from the perspective of Mathematical Learning Motivation but has not been more effective from the perspective of Mathematical Problem-Solving Skills and of Mathematical Learning Achievement. The reason that the Problem-Posing Approach has not been more effective in comparison to the Investigation Approach from the perspective of Mathematical Problem-Solving Skills and of Mathematical Learning Achievement might be caused by several factors. For instance, the questions that the students devised through the Problem-Posing Approach tend to be conceptual, meaning that these questions only demand the concepts from the materials that have been studied. As a result, the students have less opportunity to stimulate and improve their Mathematical Problem-Solving Skills. Thus, when the students complete the Mathematical Problem-Solving Skills Test Item and the Mathematical Learning Achievement Test Item that have been dealing with the Mathematical Problem-Solving Aspects, they have difficulties.

## CONCLUSIONS

As having been found, the Mathematical Learning Process by means of Problem-Posing Approach and Investigation Approach has been effective from the perspective of Mathematical Problem-Solving Skills, Mathematical Learning Motivation and Mathematical Learning Achievement for the students in State 2 Junior High School Gamping, Sleman. However, the Mathematical Learning Process by means of Problem-Posing Approach has not been more effective in comparison to Investigation Approach from the perspective of Mathematical Problem-Solving Skills and Mathematical Learning Achievement but has been more effective in comparison to Investigation Approach from the perspective of Mathematical Learning Achievement. Therefore, it is suggested that all of the teachers in the grade of Junior High School should implement the Problem-Posing Approach and the Investigation Approach within the Mathematical Learning Process especially for the materials of Triangle and Rectangle. In the same time, it is also suggested that the future researchers should expand the materials of the study so that a wider generalization might be achieved.

**REFERENCES**


Abu, R., & El Sayed, E. (2000). Effectiveness of problem posing strategies on perspective mathematics teachers' problem-solving performance. *Journal of Science and Mathematics Education in S.E. Asia,* 1, 56-69.

Adams, D., & Hamm, M. (2010). *Demystify math, science, and technology: Creativity, innovation, and problem solving.* Plymouth: Rowman & Littlefield Publisher, Inc.

Alderman, M. K. (2004). *Motivation for achievement: Possibilities for teaching and learning.* Mahwah, NJ: Lawrence Erlbaum Associates.

Ambrose, S. A., Bridges, M. W., & DiPietro, M. (2010). *How learning work.* San Fransisco: John Wiley & Sons, Inc.

Arends, R.I., & Kilcher, A. (2010). *Teaching for student learning: Becoming an accomplished teacher*. New York: Routledge.

Azwar, S. (2010). *Sikap manusia: Teori dan pengukurannya.* Yogyakarta: Pustaka Pelajar.

Bailey, J. (2007). Mathematical investigations: A primary teacher educator's narrative journey of professional awareness. Dalam J. Watson & K. Beswick (Eds.), *Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia (MERGA): Mathematics: Essential research, essential practice* (Vol. 1, pp.103-112).

Bonotto, C. (2010). Engaging student in mathematical modeling and problem posing activities. *Journal of Mathematical Modelling and Application*, 3, 18-32.

Brophy, J. (2010). *Motivating students to learn*. New York, NY: Routledge.

Brown, S.I., & Walter, M.I. (2005). *The art of problem posing (3rd ed.)*. Mahwah: Lawrence Erlbaum Associates, Publishers.

Byrnes, J. P. (2008). *Cognitive development and learning in instructional contexts (3rd ed.).* New York: Pearson Education, Inc.

Carson, J. (2007). A problem with problem solving: Teaching thinking without teaching knowledge. *Journal of The Mathematics Educator 2007, 2, 7-15.*

*Clark, A. (2009). Problem solving in Singapore Math. Diambil pada tanggal 18 Januari 2012, dari* http://www.hmheducation.com/singaporemath/ pdf/MIFProbSolving. pdf.

Cohen, R.J., & Swedlik, M.E. (2009). *Psychological testing and assesment: An introduction to tests and measurement (7th ed.)*. New York: McGraw Hill Published.

Cotton, T. (2010). *Understanding and teaching primary mathematics*. Harlow: Pearson Education.

Dembo, M. H. (2004). *Motivation and learning strategies for college success: A self-management approach.* Marwah, NJ: Lawrence Erlbaum Associates.

Depdiknas. (2006). *Peraturan Menteri Pendidikan Nasional RI Nomor 22, Tahun 2006, tentang Standar Isi untuk Satuan Pendidikan Dasar Dan Menengah.*

Diezmann, C. M., Watters, J. J., & English, L. D. (2001). Implementing mathematical investigations with young children. *Proceedings 24th Annual Conference of the Mathematics Education Research Group of Australasia*, pp 170-177, Sydney.

Elliot, S. N., et al. (2002). *Educational psychology: Effective teaching, effective learning (3rd ed.)*. Boston: McGraw Hill.

Gorman, R.M. (1974). *The psychology of classroom learning: An inductive approach*. Columbus: Meril Publishing Company.

Grimison, L. & Dawe, L. (2000). Report Supporting for the Advanced and Intermediate Courses of the NSW Mathematics Years 9–10 Syllabus. Dalam *Literature Review: Report on Investigational Tasks in Mathematics in Years 9–10 for Advanced and Intermediate Students*. New South Wales: University of New South Wales.

Guvercin, S., Cilavdaroglu, A. K., & Savas, A. C. (2014). The effect of problem posing instruction on 9th grade students' mathematics academic achievement and retention. *International Journal of Contemporary and Applied Study of Man*, 17, 129-136.

Haylock, D., & Thangata, F. (2007). *Key concepts in teaching primary mathematics*. London: SAGE Publications.

Hawkins, K. B., Florian, L., & Rouse, M. (2007). *Achievement and inclusion in schools.* New York, NY: Routledge.

Jaworski, B. (2003). *Investigating mathematics teaching: A constructivist enquiry.* London: Falmer Press.

Johnson, R. A., & Wichern, D. W. (2007). *Applied multivariate statistical analysis (6th ed.)*. New Jersey: Pearson Prentice Hall.

Kirkley, J. (2003). *Principles for teaching problem solving*. Indiana University. Downloaded on March 20th, 2012, from www.d55.k12.id.us/plato/I.../principlesforteachingproblemsolving.pdf.

Lavy, I., & Shriki, A. (2007*). Problem posing as a model means for developing mathematical knowledge of prospective teachers*. Emek Yezreel Academic College. PME-31.

NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Ormrod, J. E. (2003). *Educational psychology: Developing learners (4th ed.).* New Jersey, NJ: Merril Prentice Hall.

Pearson, R. W. (2010). *Statistical persuasion*. Thousand Oaks: SAGE.

Polya, G. (1973). *How to solve it: A new aspect of mathematical method.* Princeton: Princeton University Press.

Ponte, J. (2001). Investigating in mathematics and in learning to teach mathematics. In F. L. Lin & T. J. Cooney (Eds.), *Making sense of mathematics teacher education* (pp.53-72). Dordrecht: Kluwer.

Rencher, A. C. (1998). *Multivariate statistical inference and applications.* New York: John Wiley & Sons, Inc.

Royer, J. M. (Ed.). (2003). *Mathematical cognition.* Greenwich, Connecticut: Age Publishing, Inc.

Santrock, J. W. (2011). *Educational psychology (5th ed.)*. New York: McGrow-Hill Companies.

Shumway, R. J. (Eds.). (1980). *Research in mathematics education*. Reston: NCTM.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students [Versi Elektronik]. *Journal for Research in Mathematics Education*, 5, 521-539.

Silver, E. A., et al. (1996). Posing mathematical problems: An exploratory study [Versi Elektronik]. *Journal for Research in Mathematics Education,* 27(3), 293-309.

Slavin, R. E. (2006). *Educational psychology: Theory and practice (8th ed.)*. Boston, MA: Pearson Education.

Stevens, J. P. (2009). *Applied multivariate statistics for the social sciences (5th ed.)*. New York: Routledge.

Van de Walle, J. A., Karp, K. S., Bay-Williams, J. M. (2010). *Elementary and middle school mathematics: Teaching developmentally (7th ed.)*. Boston: Allyn & Bacon.

Walpole, R. E. (1997). *Pengantar Statistika (Edisi ke-3).* Jakarta: Gramedia Pustaka Utama.

Williams, K. C., & Williams, C. C. (2010). Five key ingredients for improving student motivation*. Research in Higher Education Journal.* Downloaded on November 20th, 2013, from http://www.aabari.com/manuscripts/11834.pdf

Woody, R. H., Lavoie, J. C., Epps, S. (1992). *School psychology: A development and social system approach.* New York: Allyn & Bacon.

Woolfolk, A. E. (2005). *Educational psychology (9th ed.).* Boston: Allyn and Bacon.