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Research Paper

Surface Wave Topography using The 4 Point FDM Simulator

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Abstract

The 2D topography proffers a new challenge of modeling surface waves with a 4-point finite difference (FDM) model. Topographic representation of wave propagation over a certain area will result in loss of accuracy of the numerical model. Then from this the need for appropriate modifications to reduce calculation errors. The existing approach requires value representation as an internal extrapolation solution for temporary exterior conditions. It is finally by providing boundary conditions and initial conditions in the system. However, the scheme sometimes becomes unstable for very irregular topography. 1D extrapolation along the parallel path is known to produce a simple and efficient scheme. During extrapolation, the stability of the 1D hyperbolic Schema improved by disregarding the nearest interior boundary point, which is less than half the lattice distance. Given the limited difference so that the stencils from both sides of the central evaluation point can be used as a 2D form modification if there are not enough inside points. So that in propagation space, waves that move and change according to changes in time. It will be following the wave nature of one source that travels in the x and y fields whose amplitude will change exponentially against propagation time. It is by the nature of surface wave motion.

Keywords

Two-Dimensional Waves, Mathematical Models, Finite Difference Method, Partial Differential Equations, Hyperbolic Schemes.

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1. INTRODUCTION

PDP continues to grow, from science and engineering problems and involves mathematical models (Feng et al., 2018). One of the uses of PDP is wave modeling. See (Liu et al. (2016); Raissi et al. (2019)). The two-dimensional wave phenomenon can be formulated with a mathematical model approach using Partial Differential Equations (PDP) (Seadawy (2017); Raissi and Karniadakis (2018)). The nonlinear evolution equation often used to describe some physical aspects that appear in various fields of nonlinear science (Yokus et al., 2017). where the value parameters change with time and distance (Stynes et al. (2017); Jin et al. (2014); Zheng et al. (2015); Le Vot et al. (2017)) and recognize numerical integration (Rogov, 2019). One method used is the finite difference method (FDM).

The FDM has a simple form, flexibility (Yan et al. (2016); Mori and Romao (2015)). To build numerical solutions, it is necessary to carry out the proper discretization as initial conditions and boundary conditions that can provide approximate solutions for surface wave motion (Gerdt and Robertz, 2019). The main advantage of FDM is that it is an asymptotic algorithm that can simulate an entire wavefield without losing accuracy. In frequency-domain modeling, attenuation can be substituted directly with the equation of system characteristics or imaginary coefficients. But in its application, the frequency domain will be hampered by a linear system depending on the size of the given equation (Yuan et al., 2018). From this, much of the focus of publications in simulating wave propagation as an attenuative medium with the time-domain method of finite difference (Xing and Zhu, 2018). A mechanical model can simulate quality factors that are almost constant over a specified frequency range. Variable storage entered into the convolution stress and strain relationship corresponding to numerical implementation.

The domain of a function partitioned to obtain several positions, and the approximation results show the derivative of the Taylor series expansion at one or more partition points (Li and Zeng, 2015). A reasonable discretization must provide a convergence of numerical solutions in the boundary when initial conditions tend to be zero. Except for certain limitations so, convergence cannot be built directly. Consistency and stability analyzed as conditions needed for convergence. In contrast, consistency indicates that when the grid distance tends to zero and stability provides error limits in numerical data with relatively small interference solutions. Based on this, the purpose of this study is to find out the numerical solutions of the two-dimensional wave equation using the 4 point FDM approach as a surface wave simulator.

2. EXPERIMENTAL SECTION

2.1 Methods

Studying 2D cases requires 1D literature, to learn some things related to consider second-order, constant density with the wave equation presented

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \nabla^2 u = f(x,t) \tag{1}$$

with

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2}, u(x,t)$$

is pressure, c is the speed of the wave, and $f(x,t)$ is a representation of the grid solution $x_t = x_0 + t\Delta x$, where $t = 0, 1, 2, \ldots, (x-1)$. The standards are spatially criticized even to the order M is

$$\frac{\partial^2 u}{\partial t^2}|_{x_t} \approx f\frac{(D_{xx}u)_t}{(\Delta x)^2}, \tag{2}$$

Where,

$$(D_{xx}u)_t = a_0 u_t + \sum_{k=1}^{M/2} a_m(u_{t+m}u_{t-m}), and \tag{3}$$

$$a_0 = \sum_{m=1}^{M/2}\frac{2}{m^2}, a_m = (-1)^m\sum_{m=n}^{M/2}\frac{2}{M}\frac{(m!)^2}{(m-n)!(m+n)!},$$

For $m = n = 1, 2, 3, \ldots, M/2$ (Martin et al., 2015). The second-order time-stepping scheme is

$$u_t^{k+1} - 2u_t^k + u_t^{k-1} = \left(\frac{c_i\Delta t}{\Delta x}\right)^2\left(D_{xx}u^k\right) + \Delta t^2 f_k^t \tag{4}$$

With sample time $t^k = t^0 + k\Delta t$ if the boundary of the right side of the domain is at the dot $x_b = x_{(x-1)} + \xi\Delta x, \xi\epsilon(0,1)$ with initial conditions $\frac{\partial^{2m}u(t,x_b)}{\partial t^{2m}} = 0$ with a limit $m\epsilon N$. if $c(x)$ is the closest constant from the initial condition. So, as a result of the extrapolation of polynomials to degrees M into shape $u(x) = \sum_{m=0}^{M} b_m(x-x_b)^m$. $1 + M/2$ is a result of initial conditions $b_m = 0$ with a positive m value $= 0, 2, 4, \ldots$ while the other coefficient is $m = 1, 3, 5, \ldots$ of additional value-solution solutions $M/2$ for each grid. For the case $M = 4$ then the extrapolation formula is

$$\begin{pmatrix} u_x \\ u_{x+1}\end{pmatrix} = E^I\begin{pmatrix} u_{x-2} \\ u_{x-1}\end{pmatrix} \tag{5}$$

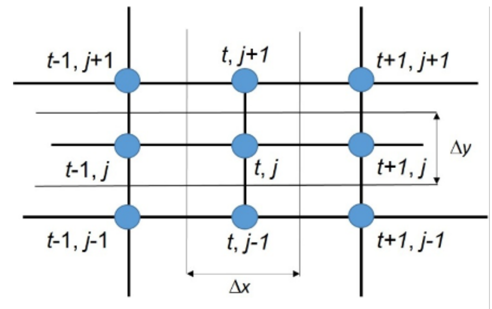

Figure 1. The Illustration of a two-dimensional hyperbolic scheme

with

$$E^I = \begin{pmatrix} -\frac{(1-\xi)(1-2\xi)}{(1+\xi)(1+2\xi)} & -\frac{4(1-\xi)}{1+2\xi} \\ -\frac{4(2-\xi)(1-\xi)}{(1+\xi)(1+2\xi)} & -\frac{3(2-\xi)(1-2\xi)}{\xi(1+2\xi)}\end{pmatrix} \tag{6}$$

In this case, the element $E^I_{2,2}$ with the denominator ξ, which is near zero, will cause instability. The scheme labeled I, becomes unstable when $\xi \downarrow 0$ at the last term of the mesh point. However, instability can be suppressed by reducing the time step by choosing $\xi_0 = 1/2$, Then extrapolation for the 4^{th} order becomes,

$$\begin{pmatrix} u_x \\ u_{x+1}\end{pmatrix} = E^{II}\begin{pmatrix} u_{x-3} \\ u_{x-2}\end{pmatrix} \tag{7}$$

$$E^I = \begin{pmatrix} -\frac{4\xi(1-\xi)}{(2+\xi)(3+2\xi)} & -\frac{3(1-\xi)(1+2\xi)}{(1+\xi)(3+2\xi)} \\ -\frac{3(2-\xi)(1-2\xi)}{(2+\xi)(3+2\xi)} & -\frac{8\xi(2-\xi)}{(1+\xi)(3+2\xi)}\end{pmatrix} \tag{8}$$

With the following illustration as shown in Figure 1 (Ghosh and Constantinescu, 2016).

Similarly, if the second-order time step scheme is presented in two coordinates as a Laplacian function $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$. where the x-axis representation defined as,

$$\frac{\partial^2 u}{\partial x^2}|_{t,j} = \frac{u_{t,j+1}^k - 2u_{t,j}^k + u_{t,j-1}^k}{\Delta x^2} \tag{9}$$

The y-axis is defined as

$$\frac{\partial^2 u}{\partial y^2}|_{i,j} = \frac{u_{t+1,j}^k - 2u_{t,j}^k + u_{t-1,j}^k}{\Delta y^2} \tag{10}$$

As for the representative t according to equation (4) obtained

$$\frac{\partial^2 u}{\partial t^2}|_k = \frac{u_{t,j}^{k+1} - 2u_{t,j}^k + u_{t,j}^{k-1}}{\Delta t^2} \tag{11}$$

The next step is to determine the boundary conditions, $u(0,y,t) = 0$; $u(2,y,t) = 0$; $u(x,2,t) = 0$ while the initial system condition is

$$u(0,y,t) = 0,05cos(\pi x)sin(\pi y); \frac{\partial u(x,y,0)}{\partial t} = 0 \tag{12}$$

With a hose 0≤x≤2, 0≤y≤2, and 0≤t ≤2, With the parameters used as follows $c^2 = 0,025, B = [0404], T = 8, m_x = m_y = 100, n = 50$ where T is the periodization and n are many iterations.

3. RESULTS AND DISCUSSION

The FDM method is a topographic representation of the propagation of waves in a certain area, which in its application can be used to analyze partial differential equations that are difficult to solve analytically, such as the 4-point FDM method obtained from the equation,

$$\left(\frac{u^k_{t,j+1} - 2u^k_{t,j} + u^k_{t,j-1}}{\Delta x^2} + \frac{u^k_{t+1,j} - 2u^k_{t,j} + u^k_{t-1,j}}{\Delta y^2} \right) = \frac{1}{c^2} \frac{u^{k+1}_{t,j} - 2u^k_{t,j} + u^{k-1}_{t,j}}{\Delta t^2} \quad (13)$$

Equation 13 describes the discrete form of the 2-dimensional wave equation at the approximate point of calculation. The network of points on the x and y axes is assumed to be the proportion of a rectangular plane. So from equation (13) by algebraic manipulation, the iteration formula is obtained like equation (14)

$$u^k_{t,j+1} = r_x u^k_{t,j+1} + r_x u^k_{t,j11} + 2u^k_{t,j} - 2r_x u^k_{t,j} - 2r_y u^k_{t,j} + r_y u^k_{t+1,j} + r_y u^k_{t-1,j} - u^{k-1}_{t,j} \quad (14)$$

with
$r_x = c^2 \frac{\Delta t^2}{\Delta x^2}, r_y = c^2 \frac{\Delta t^2}{\Delta y^2}$ and $\Delta x = \frac{x_f}{m_x}, \Delta y = \frac{y_f}{m_y}, \Delta t = \frac{T}{n}$. x_f is largest x-axis limit and y_f is largest y-axis limit. Equation (14) only applies to $k > 0$ at the beginning where $k = 0$, then there is a term with a time index = -1. The last term in the iteration formula of equation (14) distributed through the first derivative formula. So obtained,

$$\frac{u^1_{t,j} - u^{-1}_{t,j}}{2\Delta t} = i'_0(x_j, y_t) \quad (15)$$

$$u^1_t = \frac{r_x u^0_{t,j+1} + r_x u^0_{t,j-1} + r_y u^0_{t+1,j}}{2} \quad (16)$$
$$\frac{+r_y u^0_{t-1,} + 2u^0_{t,j} - 2r_x u^0_{t,j} - 2r_y u^0_{t,j} i'_0(x_j, y_t)\Delta t}{2}$$

from equation (10) the condition of the stability of the system is obtained

$$r = \frac{4c^2 \Delta t^2}{\Delta x^2 + \Delta} y^2 \leq 1 \quad (17)$$

The program defines PDP as follows

$$\frac{\partial^2 u(x,y,t)}{\partial x^2} + \frac{\partial^2 u(x,y,t)}{\partial y^2} - \frac{1}{0,025} \frac{\partial^2 u(x,y,t)}{\partial t^2} = 0 \quad (18)$$

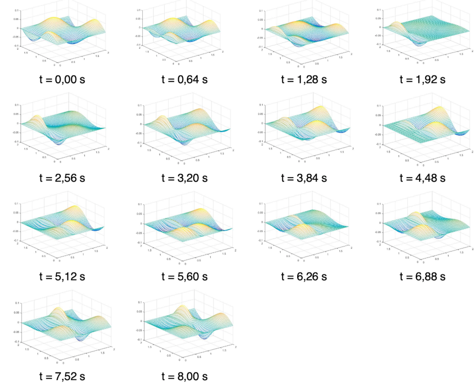

Figure 2. Visualization Results

Based on equation 18, solutions for real and imaginary systems can be obtained, which write in equations 19 and 20. If the real system is different, then the solution of equation (18) is

$$(x,y) = \begin{cases} e^k \\ e^{-k} \end{cases} \text{and} t = \begin{cases} e^{\frac{\sqrt{4(0,025k^2)}}{2}} i \\ e^{\frac{-\sqrt{4(0,025k^2)i}}{2}} i \end{cases} \quad (19)$$

If the system is imaginary, then the solution of equation (18) is

$$(x,y) = \begin{cases} e^{ik} \\ e^{-ik} \end{cases} \text{and} t = \begin{cases} e^{\frac{\sqrt{4(0,025k^2)}}{2}} i \\ e^{\frac{-\sqrt{4(0,025k^2)i}}{2}} i \end{cases} \quad (20)$$

with value , where l is the length of the path, the plane traversed. In this case, the system is considered imaginary so the general solution obtained is

$$u(x,y,t) = Csin\frac{n\pi}{l}xsin\frac{n\pi}{l}ycos\frac{\sqrt{4(0,025(\frac{n\pi}{l})^2)}}{2}t \quad (21)$$

Based on the scheme in Figure 1 and equation 18, the analysis results can be obtained in the form of wave visualization using the 4-point FDM method (Figure 2).

Based on Figure 2, when observed at each change in iteration time, it is known that the wave crests and troughs have changed. The visualization of changing waves due to an increase in iteration time of 0.64 s, where the formed waves change and shift to the right and continue continuously until the specified iteration limit. The shape of this visualization change because by the changing in the quantization of time that occurs exponentially if in real-time or imaginary it will form a sinusoidal state.

4. CONCLUSIONS

Modified FDM modification allows the merging of topography in 2D wave schemes whose density is constant for the second order. The existing extrapolation scheme was modified and simplified from the form of 1D operations. With additional extrapolation constraints that do not include the closest point of the specified limit in half grid spacing. It allows the method to run at the same stability as an interior scheme. Although the implementation of the 1D scheme for coordinate directions results in a simpler, more stable, and more robust scheme, it will lose its accuracy if the problem given more complexity. Because accuracy will also be reduced, but this interior model might be acceptable in practice. So based on the results of this study, it can be concluded that by involving initial conditions and boundary conditions, the stability of the system obtained in the propagation space. The simulation wave moves and changes with time. It is under the wave nature of one source that travels in the x and y plane, such as the nature of surface motion.

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