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The New Fugacity Calculation in Finite Nuclear Matter

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Abstract

The caloric curve of hot nuclei in equilibrium states have been calculated with a new prescription namely a new fugacity calculation. In this technique, the fugacity is directly proportional to the nucleon density; hence, the fugacity is obtained through thermal wavelength. In contrast with the constant fugacity, the choosing of thermal wavelength approximation gives a simpler way to calculate the density profile and the entropy of finite nuclear matter. Variation of thermal wavelength value does not affect to the density and entropy. The phase transition temperature is dependent concerning both of the thermal wavelength value and the potential deep.

Keywords : *thermal wavelength, constant fugacity, caloric curve*

1. Introduction

The phase transition of the hot nuclear system is the one of interesting problems. The phase transition converts a nuclear liquid Fermi system into fragmentation. The converting process assists to reveal the problems of breaking up unstable nuclei into many fragments that is the connection between fission and phase transition. Peaks of heat capacity at constant volume usually show this phase transition.

The peaks of heat capacity have been pointed out not only by micro canonical description[1-3)] but also by caloric curve for finite nuclei[4,5)]. The evidence of the peaks of heat capacity has also proved experimentally[6,7)]. Theoretically, the refined Thomas-Fermi calculation[8)] has showed the peaks of heat capacity. In this calculation, a nucleonic density is obtained via self-consistency density profile[8,9)] in which the fugacity calculation is an iteration element of self-consistency. The refined Thomas-Fermi method has solved successfully many problems of hot nuclei[9)], such as level density parameters and caloric curve of the hot nuclear system.

Our works is fueled by idea of the fugacity calculation at zero temperature that is the finite value of fugacity. In this calculation, occupation probability has the old one fashion[10)]. There are two advantages of the technique; the calculation is rather simpler than self-consistency of the refined Thomas-Fermi method and the fugacity has finite value at zero temperature. Actually, the new technique is determined by thermal wavelength, which the thermal wavelength is not longer inversely proportional to square root of temperature. The thermal wavelength is chosen as constant value. In this work, variations of thermal wavelengths do not give significantly of discrepancies results. The excitation energies per nucleon of ^{208}Pb and ^{91}Zr are about the experimental range results. Effective interaction potential use the simple one that is the wood-saxon potential form, which it substitute the seyler-blanchrad type[11,12)].

The paper is organized as follows. In section 2, it briefly reviews modeling of hot nucleus and present equations relevance for calculating the caloric curve, in which the calculation are based on the new technique of fugacity calculation. The results are presented in section 3 and the conclusions are given in section 4.

2. Theoretical Framework

Occupation probability is obtained by minimizing thermodynamic potential,

$$G = E - TS - \mu A - P\Omega \tag{1}$$

one then led the occupation probability

$$f_\tau(\vec{r}, T, \vec{p}) = \frac{z_\tau(r,T)}{1 + z_\tau(r,T)\exp\left(\dfrac{H_T(\vec{r},T,\vec{p}) - \mu_\tau}{T}\right)} \tag{2}$$

In this equation, z_τ, H_τ and μ_τ are fugacity, Hamiltonian and chemical potential of nuclear system, in which they have expression

$$z_\tau(r,T) = \lambda_\tau^2 \rho_\tau(r,T) \tag{3}$$

$$H_\tau(\vec{r},T,\vec{p}) = \frac{p^2}{2m_\tau} + V_\tau(\vec{r},T,\vec{p}) \tag{4}$$

$$\mu_\tau = \int z_\tau(r,T)\log\big(z_\tau(r,T)\big)\, d^3\vec{r} \tag{5}$$

where τ stand as isospin and T is temperature.

In our works[13,14)], thermal wave length λ_τ is chosen as a constant value. Both of proton density and neutron density $\rho_\tau(r,T)$ have been calculating by using relation

$$\rho_\tau(\vec{r},T) = \frac{2}{h^3}\int f_\tau(\vec{r},T,\vec{p})\, d^3\vec{p} \tag{6}$$

Effective single particle potential $V_\tau(\vec{r}, T, \vec{p})$ experienced by nucleons has form

$$V_\tau(\vec{r},T,\vec{p}) = V_\tau^{(0)}(\vec{r},T) + p^2 V_\tau^{(1)}(r) + \delta_{\tau p} V_C(r) + \frac{P}{\rho_\tau(\vec{r},T)} \tag{7}$$

Here P and ρ are constant external pressure and nuclear density. In this work, the nucleon effective mass is gotten close by nucleon rest mass[15]; with the result of that, potential $V_\tau^{(1)}$ is negligible and $V_\tau^{(0)}$ is nuclear potential which it is written as[16].

$$V_\tau^{(0)}(\vec{r},T) = -\frac{V_o}{1+\exp\left(\frac{r-R}{d}\right)}\rho_\tau(\vec{r},T) \tag{8}$$

Coulomb energy density per proton at temperature T, which it is suitable numerically for the calculation, is given by

$$E_C(\rho_p) = \frac{1.2}{Z}\rho_p(\vec{r}',T) \left(\int \rho_p(\vec{r},T)V_C(\vec{r},T)d^3\vec{r} - 0.73856\left(\rho_p(\vec{r}',T)\right)^{1/3}\right) \tag{9}$$

where $V_C(\vec{r},T)$ is obtained by adding exchange and direct coulomb potential, and nuclear matter energy density per nucleon at temperature T is expressed by

$$E^{(0)}(\rho_m) = \sum_\tau -\frac{0.833}{A}\frac{V_o\left(1+\exp\left(-\frac{R}{d}\right)\right)}{\left(1+\exp\left(\frac{r-R}{d}\right)\right)}\rho_\tau(\vec{r},T) \tag{10}$$

The total energy density per nucleon at temperature T is then written as[17],

$$E(\rho) = 2.3925\left(\rho_m(\vec{r},T)\right)^{5/3} + E_C(\rho_p) + E^{(0)}(\rho_m) \tag{11}$$

The heat capacity[14] is then obtained by

$$C_v = \left(\frac{dE^*}{dT}\right) \tag{12}$$

3. Numerical Results

The proton density and the neutron density of ^{208}Pb and ^{91}Zr that calculated by the new technique, which it use thermal wavelength arbitrarily, are showed by figure 1.

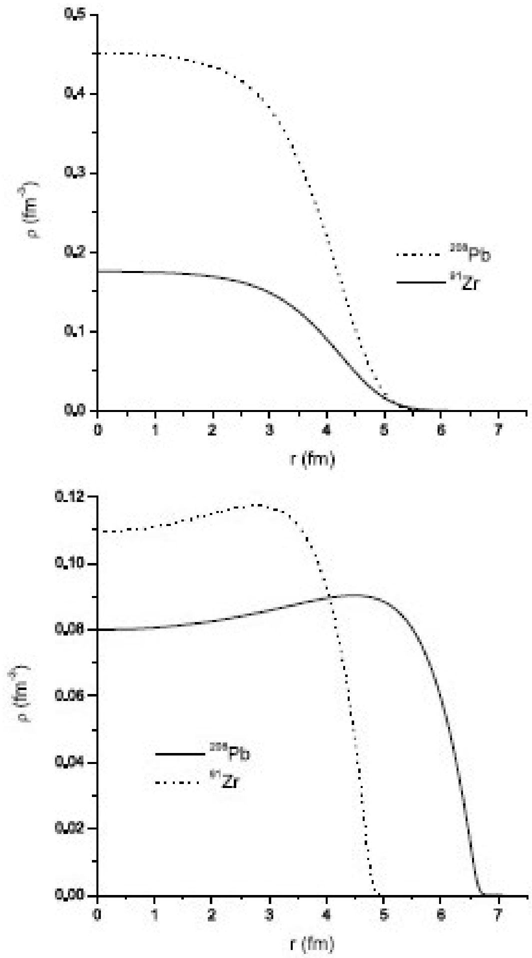

Figure 1. The neutron (left) and the proton (right) density profiles of ^{208}Pb and ^{91}Zr at T = 0 MeV and λ = 1 fm

As the early works both of theoretically and experimentally, the proton density that calculated at r = 0 has not curve peak. The peak is shifted approximately up to 2.9 fm for ^{91}Zr and 4.5 fm for ^{208}Pb. higher nuclear mass gives longer peak shift.

As refined Thomas Fermi calculated[18], Figure 1 shows that nuclear density is independent from the thermal wavelength. The curve of nuclear density of a nucleus is shaped by temperature only. Nuclear density is obtained statistically; hence, the thermal wavelength does not influence both of nucleonic densities. Base on this condition, the fugacity approximation that is determined by thermal wavelength is suitable in order to calculate nucleonic densities.

At low temperature, entropy of ^{91}Zr has been calculated by using fugacity approximation with λ = 1 fm. As early works[9,19] the calculating of entropy at zero temperature limits has form $S = bT$. Figure 2 refers the calculation, which is indicated by circle, has a linear curve fitting.

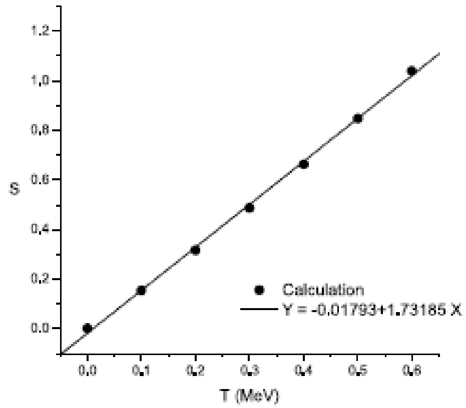

Figure 2. Entropy of ^{91}Zr at λ = 1 fm

Choosing of thermal wavelength does not give entropy differences significantly. Ten times of thermal wavelength value gives only ten scales of differences of entropy. Based on figure 3, variation of thermal of wavelength values do not contribute valuable on the entropy.

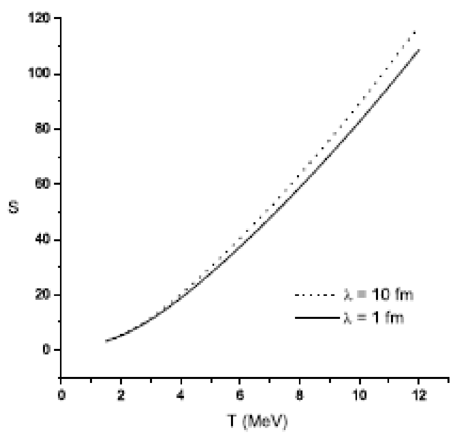

Figure 3. Entropy of ^{91}Zr at Vo = 15.699 MeV

A variation of potential deep also gives no contribution on entropy values. It is showed clearly by figure 4. The entropy is independent from not only potential deep but also the thermal wavelength values.

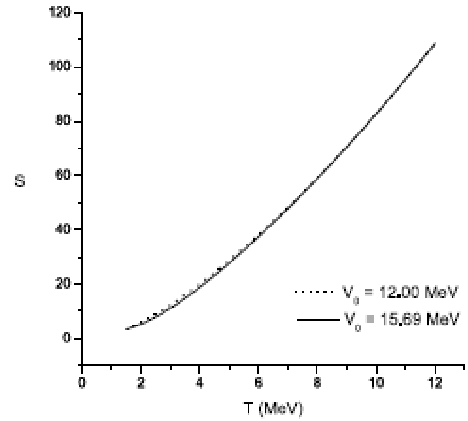

Figure 4. Entropy of ^{91}Zr at λ = 1 fm

The entropy is affected very significant by nuclear mass number. As shown by figure 5, the entropy of ^{208}Pb increases rapidly as increasing temperature. Higher nuclear mass tends to increase entropy, which it is a way to reach the equilibrium states. Even though the differences value is large, entropy shapes are similar. The entropy likes linear curve at temperature great than 5 MeV and less than 0.6 MeV other wise the curve has parabolic shape.

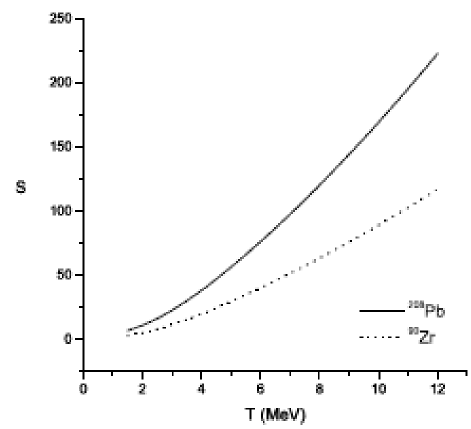

Figure 5. Entropy ^{208}Pb and ^{91}Zr at λ = 10 fm and Vo = 15.699 MeV

Although the differences of mass number between nuclei are significantly such as ^{208}Pb and ^{91}Zr, excitation energies per nucleon do not have varying values sharply. There are discrepancies at low energies excitation between the present work and the Pochodzalla experiment.

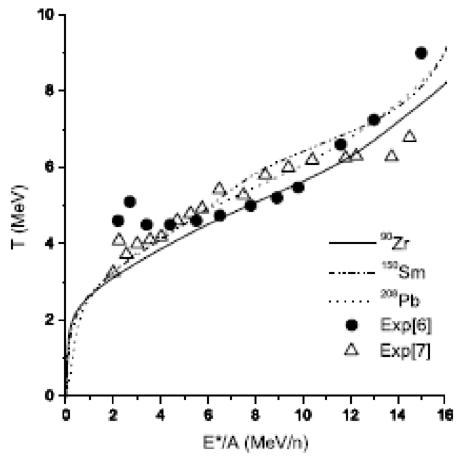

Figure 6. Caloric curve of ^{208}Pb, ^{150}Sm and ^{91}Zr at λ = 1 fm with Vo = 15.69 MeV

Figure 6 shows that mostly excitation energies per nucleon of ^{208}Pb, ^{150}Sm and ^{91}Zr are in the range of experimental results.

The thermal wavelength that it is used in calculation has value 1 fm; hence, the fugacity and nucleonic densities are exactly same. The fugacity value at beyond nuclear radii is smaller than in the nucleus, consequently an outer nucleon escapes easily than the inner one. By means of the potential deep that is chosen about 15.69 MeV ground state energy has values about 28 MeV, which is the value of Fermi energy.

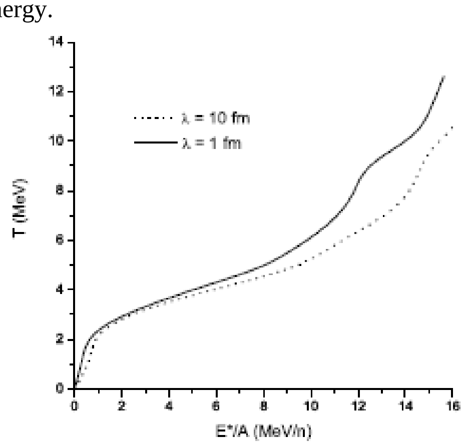

Figure 7. Caloric curve of ^{150}Sm at λ = 1 fm and λ = 10 fm with Vo = 12.0 MeV

The influence both of varying thermal wavelength and potential deep values are showed clearly by figure 7 and figure 8. Phase transition of the hot nuclear system with smaller thermal wavelength needs lower excitation energies to changes phase. According to these results, the heat capacity is smaller than the system that has longer thermal wavelength. Shorter thermal wavelength drives higher energy kinetically; hence, the liquid nucleon system tends to evaporate at lower excitation energies. Although the thermal wavelength extends influence on curve shapes, variation of curve shapes do not definitely different.

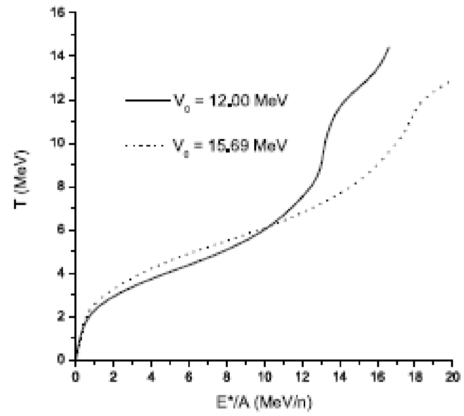

Figure 8. Caloric curve of ^{208}Pb at Vo = 12.0 MeV and Vo = 15.69 MeV with λ = 1 fm.

The deeper potential gives higher temperature of the heat capacity; on the contrary the changing phase is occurred at lower excitation energies, which it likes the varying thermal wavelength cases. It is usual that the deeper potential gives more room for nucleon to escape from nucleus. The liquid nucleon that is trapped in the deeper potential needs more energy to evaporate from the hot nuclear system.

4. Conclusions

A prescription method to calculate the self-consistently of the density profile by using the new technique calculation of fugacity has been proposed. The technique leads to the thermal wavelength to have influence on shifting of the phase transition temperature. Beside for the density profile and the entropy calculations, this method is applicable to determine the caloric curve of the hot nuclear system. The phase transition temperature is dependent concerning both of the thermal wavelength value and the potential deep.

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