

## STUDENTS' MATHEMATICAL REASONING ABILITY ON THE CIRCLE CONCEPT

Anderson Leonardo Palinussa<sup>1\*</sup>, Thobita Dias<sup>2</sup>

<sup>1\*</sup> Mathematics Education, Universitas Pattimura, Ambon, Indonesia

<sup>2</sup> SMA Negeri 57 Maluku Tengah, Indonesia

\*Corresponding author

E-mail: [palinussanderson1@gmail.com](mailto:palinussanderson1@gmail.com)<sup>1\*)</sup>  
[litadias05@gmail.com](mailto:litadias05@gmail.com)<sup>2)</sup>

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### Abstract

The condition of the Covid-19 pandemic that hit Indonesia, requires students and teachers to study from home in order to maintain their safety and health. However, entering the new normal era, people and even schools have started to carry out their usual activities while still paying attention to health protocols. Even though the learning process is still carried out online and offline, it is a challenge in learning mathematics. The purpose of this study is to analyze the mathematical reasoning abilities of students who take part in online and offline learning and to describe the attitudes of students who take online and offline learning towards learning mathematics in circle material. This research is an experimental research with the research instruments used are tests of mathematical reasoning abilities and questionnaires for implementing online and offline learning. This research was conducted on students of grade 11 science at SMA Negeri 14 Central Maluku. Based on the results of the study it was found that the mathematical reasoning abilities of students who received online and offline learning were more effective in the experimental class with the Inquiry-Discovery model compared to the control class. Thus learning using online and offline is suitable to be applied in learning mathematics, because students seem more active in solving the questions given by the teacher.

**Keywords:** circle concept; offline; online; mathematical reasoning

### Abstrak

Kondisi pandemi covid-19 yang melanda Indonesia, mengharuskan siswa maupun guru belajar dari rumah demi menjaga keselamatan dan kesehatan mereka. Namun memasuki era new normal, masyarakat bahkan di sekolah-sekolah sudah mulai melakukan aktivitas seperti biasa dengan tetap memperhatikan protokol kesehatan. Walaupun dalam proses pembelajaran masih dilakukan dengan daring dan luring, sehingga merupakan tantangan dalam pembelajaran matematika. Adapun tujuan dari penelitian ini adalah untuk menganalisis kemampuan penalaran matematis siswa yang mengikuti pembelajaran daring dan luring dan untuk mendeskripsikan sikap siswa yang mengikuti pembelajaran daring dan luring terhadap pembelajaran matematika pada materi lingkaran. Penelitian ini merupakan penelitian eksperimen dengan instrumen penelitian yang digunakan adalah tes kemampuan penalaran matematis dan angket pelaksanaan pembelajaran daring dan luring. Penelitian ini dilakukan pada siswa kelas 11 IPA pada SMA Negeri 14 Maluku Tengah. Berdasarkan hasil penelitian ditemukan bahwa kemampuan penalaran matematis siswa yang memperoleh pembelajaran dengan daring dan luring lebih efektif pada kelas eksperimen dengan model Inquiry-Discovery dibandingkan dengan kelas kontrol. Dengan demikian pembelajaran dengan menggunakan daring dan luring cocok diterapkan dalam pembelajaran matematika, karena siswa terlihat lebih aktif dalam menyelesaikan soal-soal yang diberikan oleh guru.

**Keywords:** Daring; luring; konsep lingkaran; penalaran matematis



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## INTRODUCTION

Mathematics is a very important subject in life in the 4.0 revolution era (Layco, 2022). The condition of the Covid-19 pandemic that has hit the world and even Indonesia requires students to study from home. Indonesian people are starting to carry out daily activities as usual in the new normal era. However, to maintain students' safety and health, many schools are still implementing a 30% online and 70% offline system (Yeung & Yau, 2022). However, it does not rule out the possibility of learning better so that students' knowledge, skills, abilities, and mindset will continue to develop and improve, especially their reasoning abilities in mathematics. (Rutherford et al., 2022).

Reasoning ability is one of the main competencies students must master to understand mathematics well. Mathematical reasoning is also the main tool for solving problems (Sary et al., 2022). The reasoning is a higher-order thinking skill done in one way to conclude (Park & Cho, 2022). The process of concluding can occur from individual problems to general problems, or vice versa, from general problems to something special. (Gürbüz & Erdem, 2016).

Mathematical reasoning is a mental process, and like other habits, mathematical reasoning ability must be built continuously through various contexts. If students understand, their knowledge of the material will stay longer in their thinking and can apply it in various situations, so that their ability is not only to do what is instructed but also follow the algorithm. (Tee et al., 2018).

According to NCTM (2000), indicators of mathematical reasoning ability, namely: (1) Drawing logical

conclusions, (2) Providing explanations with models, facts, properties, and relationships, (3) Estimating answers and solution processes, (4) Using patterns and relationships to analyze situations or make analogies and generalizations, (5) Compile and test conjectures, (6) Create counter-examples, (7) Follow the rules of inference and check the validity of arguments, (8) Compose valid arguments, (9) Arranging direct and indirect proofs and using mathematical induction (Sriraman & Pizzulli, 2005). Meanwhile, according to Napitupulu et al. (2016), there are four indicators to measure students' mathematical reasoning abilities, namely (1) Make logical conclusions, (2) Give explanations about existing models, facts, properties, relationships, or patterns, (3) Make conjectures and evidence (4) Use of relationship patterns to analyze situations, make analogies, or generalize.

Mathematical reasoning abilities based on the main indicators above must be trained and implemented with a process to make conclusions correctly so that students find solutions to every mathematical problem they face. The findings in the field stated that students' mathematical reasoning abilities were still low, namely 17% (Sugandi & Bernard, 2020). The factor causing the low ability of students' mathematical reasoning is the selection of teaching methods that need to be more student-centered (Capone, 2022).

The characteristics of learning mathematics which are more focused on procedural abilities, one-way communication, monotonous classes, always relying on textbooks, and being more dominant in working on routine questions than non-routine questions in this day and age must be abandoned

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(Yazgan et al. al., 2021). The teacher gives more problems that contain high-level questions so that students are more creative and use their high-level thinking skills in solving the problems given (Lee & Kim, 2018).

Therefore, a learning model is needed to activate students in learning to improve their understanding and mathematical reasoning. The Inquiry-Discovery problem-based learning model for growing mathematical reasoning is an effective learning model. The model has its uniqueness in the learning process, so it does not present learning in its final form, but students are expected to be able to organize the learning by themselves. The Inquiry-Discovery learning model emphasizes students to find concepts through a series of data or information obtained through observation or experimentation (Wartono et al., 2018). The results of previous research stated that the average ability of students who used the inquiry-discovery learning model increased student learning outcomes to 82.08% (Wartono et al., 2018). Besides, it could also increase HOTS students (Wartono et al., 2018a )

The Inquiry-Discovery learning model means a series of learning activities that optimally involve all students' abilities to search and investigate systematically, critically, logically, and analytically. Therefore, they can confidently formulate their findings. The main targets of Inquiry-Discovery learning activities are (1) maximum involvement of students in the learning process; (2) the direction of activities logically and systematically towards learning objectives; and (3) developing students' self-confidence about what is found in the learning process (Jaworski & Potari, 2021).

Research on Inquiry-Discovery that focuses on students' mathematical reasoning abilities has yet to be found in circles. Therefore, this study aims to analyze students' mathematical reasoning abilities using online and offline learning and to describe the attitudes of students who take online and offline learning towards learning mathematics in circle material..

## METHODS

This research is a quasi-experimental design with a pretest-posttest control group. In this study, two classes were used: the experimental and control classes. The early stage of this research was to determine the research sample. Then, they were randomly taken to two classes, one for the experimental and one for the control classes. The research design used in this study is as follows.

$$\begin{array}{ccc} O & X & O \\ \hline O & & O \end{array}$$

The experimental group was given treatment (x), namely the Discovery learning model with an online learning approach, and the control group was given an inquiry learning model with an offline learning approach. Each research class was carried out pre-test and post-test.

The population in this study were all 11th-grade science students at Senior High School 14 Central Maluku. The sample in this study was a saturated sample or population sample. The variables of this study consist of: (1) independent variables, including the Discovery Learning model and the Inquiry learning model with Online and Offline learning approaches; (2) the dependent variable, including students'

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mathematical reasoning abilities. The instrument used in this study was a test instrument. The test given was a mathematical reasoning ability test in the form of essay questions consisting of 10 questions.

The data collection technique in this study was a test technique. The test is a mathematical reasoning ability test that was carried out after learning given. This test was given to improve students' reasoning abilities after being given the Discovery Learning and Inquiry learning models with Online and Offline learning approaches.

There are two types of data in this study, namely quantitative data and qualitative data. Quantitative data were obtained through an analysis of student answers on students' reasoning ability tests, then grouped based on the learning used, namely the Discovery Learning model and the Inquiry learning model with Online and Offline learning approaches. Qualitative data was obtained through a questionnaire given to students. This data was analyzed descriptively to support the completeness of quantitative data in answering research questions. The processing of quantitative data was done through several stages, namely:

#### *Stage one*

Performing a descriptive analysis of the data and calculating the values of the Pretest, Posttest, Gain and Normalized Gain. In this stage, the amount of achievement and increase in students' reasoning abilities in online and offline learning classes can be seen. Then, the normalized gain was calculated  $\langle g \rangle$  (normalized gain score).

The calculation of the normalized Gain score was carried out because this study looked at student improvement and the quality of the increase. In

addition, calculating the normalized Gain score was carried out to eliminate the student's guess factor and the effect of the highest score to avoid biased conclusions (Hake, 1999). The  $\langle g \rangle$  value range was 0 to 1, then this  $\langle g \rangle$  value was processed, and the processing was adjusted to the problem.

#### *Stage two*

At this stage, a statistical prerequisite test is carried out which is used to test the hypothesis, namely the normality test for the distribution of sample subject data and the homogeneity test of variance for parts or the whole group.

#### *Stage three*

Determining the increase in students' mathematical reasoning abilities between the experimental and control classes and determining whether there was an interaction between the independent and dependent variables. The mean difference test was used using the SPSS 22.0 for Windows program to test these differences.

In addition to quantitative analysis, the researcher also conducted a qualitative analysis of the student questionnaire. It aimed to examine more about students' reasoning abilities and to determine whether the implementation of learning was following the learning provisions set for both.

## **RESULT AND DISCUSSION**

The results of the descriptive test of students' mathematical reasoning abilities with online and offline learning were known from the pretest, posttest and N-gain results from the experimental and control classes. The results of the descriptive test show an increase in the pretest, posttest, and N-gain scores. It can be seen in Table 1.

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Table 1. Descriptive statistics on pretest, posttest and N-gain of students' mathematical reasoning abilities with online and offline learning

	Experiment Class			Control Class		
	Minimum	Maximum	Average	Minimum	Maximum	Average
Pretest	65	80	74.11	68	88	72.78
Posttest	80	95	88.58	70	90	76.83
N-gain	39.39	75.00	56.9416	6.25	42.31	15.6307

Based on the results in table 1, there was a significant increase in students' mathematical reasoning abilities after receiving treatment. Students in the experimental class obtained a greater average than the control class. The average N-gain score for the experimental class was 56.9416 or 56.94% which was included in the quite effective category, with a minimum N-gain score of 39.39% and a maximum of 75%. Meanwhile, the average N-gain score for the control class was 15.6307 or 15.63%, which was included in the ineffective category, with a minimum value of 6.25% and a maximum value of 42.31%. It shows that students' mathematical reasoning abilities with online and offline learning in the experimental class are quite

effective compared to the ineffective control class.

Based on the statistical prerequisite test to determine the normality of the data obtained, the increase in pretest to posttest was calculated for the increase (gain). The mathematical reasoning abilities of students who receive online and offline learning increased by calculating the gains of the two groups using the normalized gain formula. The normalized gain calculation results can be seen in Table 2. Prerequisite test secondly, to see improvement students' mathematical reasoning abilities who get online learning and offline is by counting homogeneity of the two groups. Results from the homogeneity test presented in Table 3.

Table 2. The normality test results of N-gain score

		Tests of Normality					
		Kolmogorov-Smirnov <sup>a</sup>			Shapiro-Wilk		
NGain_Percent	Class	Statistic	Df	Sig.	Statistic	Df	Sig.
	Experiment	.112	19	.200*	.976	19	.889
	Control	.233	18	.011	.841	18	.006

\*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction

The normalized test results showed that a significant value in the experimental class was  $0.889 > 0.05$ ; in the control class, the significant value was  $0.006 > 0.05$ . Therefore,  $H_0$  is accepted, which implies that the N-gain scores for the mathematical reasoning

abilities of the experimental class and control class students are normally distributed. Then, the statistical prerequisite test for homogeneity of data was by calculating the homogeneity of the two groups.



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Table 3. N-gain score homogeneity test results

Test of Homogeneity of Variances			
Levene Statistic	df1	df2	Sig.
.400	1	35	.531

Based on the results of the homogeneity test seen in Table 3. It shows that an increase in students' mathematical reasoning abilities obtained a significant value of  $0.531 > 0.05$ . Then, the N-gain data scores of experimental and control class students are homogeneous. Improving students'

mathematical reasoning abilities between the experimental and control classes were carried out by the T-test, a follow-up test used to determine whether the data differed significantly between the two classes.

Table 4. The T-test results of N-gain scores

		t-test for Equality of Means						
		T	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
							Lower	Upper
NGain_Persen	Equal variances assumed	13.4	35	.000	41.32	3.09	35.04	47.6
	Equal variances not assumed	13.4	34.9	.000	41.32	3.09	35.04	47.6

Based on the results of the T-test in Table 4, the value of  $\text{sig} = 0.000 < \alpha = 0.05$  is obtained so that it can be said that the two classes are in significantly different circumstances. It means that the experimental class and the control class have an increase in students' mathematical reasoning abilities with different online and offline learning. The description of students' mathematical reasoning abilities in solving mathematical problems is very good. These students were not in a hurry to analyze the solutions that could be used because they were concerned with the accuracy of the answers (Faradillah, 2018). Mathematical reasoning ability is very important to be developed for students. Many of these things are related to one's reasoning ability, which will be used in life in the future. In addition, a person will need

the ability to distinguish between appropriate and inappropriate, important or unimportant things when facing problems in everyday life. (Masfingat et al., 2020).

Some authentic evidence reveals that the application of the Inquiry-Discovery model can have a better effect on improving mathematical reasoning abilities than other learning. Students will learn better and gain superior conceptual understanding and good mathematical reasoning abilities (Evans & Dietrich, 2022). The Inquiry-Discovery model provides better learning achievement than the classical model, which produces high learning achievement and spatial intelligence (Aziz et al., 2017) and increases students' critical power (Pahrudin et al., 2021). Besides that, Inquiry-Discovery can improve students' science process

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skills (85.29%), make students enjoy learning (91.06), and increase learning outcomes (97.06%) (Adnyana & Citrawathi, 2017)

The results of other empirical studies also found that applying the Inquiry-Discovery model can have a positive effect on increasing mathematical reasoning abilities because each step of Inquiry-Discovery learning can hone and sharpen students' mathematical reasoning (Kartono & Shora, 2020). Şen et al. (2021) showed that the application of the Discovery Learning model was able to train students to fulfill the four indicators with good generalization skills in their research which analyzed the four indicators of mathematical reasoning. Students are stimulated and motivated to make some assumptions about problem solutions because they are stimulated and allowed to identify problems (Hjelte et al., 2020). Students also have no difficulty drawing conclusions and analyzing data until the generalization stage. Students can convey alternative arguments at the verification stage to prove whether the solution is right or wrong. In addition, students are encouraged by Inquiry-Discovery in the learning process so that they are independent and able to find patterns from a series of observations (Olsson & Granberg, 2022).

Lazonder & Harmsen (2016) also found the same effect, namely making assumptions, drawing conclusions, and finding patterns. The other indicator is the ability of students to perform calculations based on certain rules or formulas. Students can achieve this indicator through the stages of data processing.

Research by (Hidayat et al., 2020) gives different results. The results

showed that implementing Inquiry-Discovery in class could not encourage students to achieve indicators of mathematical reasoning (Wilkinson et al., 2018). The obstacle lay in the limited duration of learning, and the test questions given to students tend to require students to answer with evidence which is quite difficult for students to complete. The learning topic applied is the circle concept. Students needed quite a long time at the data collection and processing stages, while the duration of learning was limited, so students did not understand the concepts of sine and cosine. This finding is in line with Desouza (2017), which states that the drawback of the Inquiry-Discovery model is that the learning duration is quite long. Thus, teachers must consider the aspects of learning duration and material selection in designing learning if they want to use the Inquiry-Discovery model in class.

The application of Inquiry-Discovery received positive support from all students. The learning atmosphere in the classroom has become more flexible, where students do not feel pressured, and the teacher provides as many opportunities as possible to develop creativity and solve problems they face (Novitra et al., 2021). Even though students experienced difficulties at the initial meeting, students began to get used to it at the next meeting and found the right learning pattern. The initial difficulty was that students needed to be used to this learning model. However, they were encouraged to help each other because they had different levels of academic ability and intelligence in one group. The results of this study found that students also gain confidence from the teacher (Thuneberg et al., 2018).

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## CONCLUSION AND SUGGESTION

Based on the study's results, it was found that the mathematical reasoning abilities of students who received online and offline learning were more effective in the experimental class with the Inquiry-Discovery model compared to the control class. Therefore, learning online and offline is suitable to be applied in learning mathematics because students seem more active in solving the questions given by the teacher. The learning atmosphere with Inquiry-Discovery can encourage students to be more confident, increase high-level thinking, and make other students as teachers in learning. A humanist learning environment like this will support students in exploring mathematical ideas well.

The results of the Inquiry-Discovery research with mathematical reasoning can be a consideration for teachers, researchers, and practitioners in mathematics education in conducting explorations in research and learning activities in class. For further research, it is necessary to analyze the application of the Inquiry-Discovery model to several other important variables in improving and sharpening students' mathematics learning outcomes.

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