AKSIOMA: Jurnal Program Studi Pendidikan Matematika
Volume 11, No. 4, 2022, 3160-3167

ISSN 2089-8703 (Print)
ISSN 2442-5419 (Online)

DOI: https://doi.org/10.24127/ajpm.v11i4.5825

STUDENTS' ERRORS IN SOLVING REASONING-BASED CONGRUENCE PROOF PROBLEMS

Scristia[1*], Siti Aisyah[2], Meryansumayeka[3], Erna Retna Safitri[4], Haris Kurniawan[5]

[1,2,3,4] Department of Mathematics Education, Universitas Sriwijaya, Palembang, Indonesia
[5]Department of Mathematics Education, Universitas Taman Siswa, Palembang, Indonesia

**Corresponding author. Le Grand 3, 30131, Palembang, Indonesia*

E-mail: *scristia@fkip.unsri.ac.id[1*)]*
siti_aisyah@gmail.com[2)]
meryansumayeka@fkip.unsri.ac.id[3)]
ernasafitri@fkip.unsri.ac.id[4)]
haris_kurniawan@unitaspalembang.ac.id[5)]

Received 27 July 2022; Received in revised form 30 November 2022; Accepted 28 December 2022

Abstract

This study aims to describe the student's errors in solving reasoning problems in the form of proving triangular congruence. This research method is descriptive qualitative. The subjects of this study were 25 class IX students at a junior high school in Palembang in an odd semester of the 2021/2022 academic year. The data was obtained from a written test which was conducted in November 2021. The results showed that the errors made by students when solving proof questions were due to, among other things; errors in writing mathematical symbols, conceptual errors in the form of errors in understanding the meaning of questions, errors in mathematical principles, and carelessness

Keywords: Congruence proof; Mathematical reasoning; Newman's error analysis; Students' errors

Abstrak

Penelitian ini bertujuan untuk mendeskripsikan kesalahan yang dihadapi siswa dalam menyelesaikan masalah penalaran berupa pembuktian kesebangunan segitiga. Metode penelitian ini adalah deskriptif kualitatif. Subjek penelitian ini adalah 25 siswa kelas IX di salah satu SMP di Palembang pada semester gasal tahun pelajaran 2021/2022. Data diperoleh dari tes tertulis yang dilakukan pada bulan November 2021. Hasil penelitian menunjukkan bahwa kesalahan yang dilakukan siswa pada saat menyelesaikan soal pembuktian disebabkan antara lain; kesalahan menulis simbol-simbol matematika, kesalahan konsep berupa kesalahan menangkap makna soal, kesalahan prinsip matematika, dan tidak teliti.

***Kata kunci**: Analisis kesalahan Newman; bukti kekongruenan; penalaran matematis; kesalahan siswa*

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INTRODUCTION

One of the essential competencies that must be mastered by students is reasoning ability (NCTM, 2000). Mathematical reasoning is defined as the ability to link statements into an idea so that they can solve mathematical problems (Salmina, M. & Nisa, S.K., 2018). It is an ability to connect mathematical premises that exist and is believed to be true to build mathematical conjectures to make logical-mathematical conclusions (Akuba, S.F., Purnamasari, D., & firdaus, R., 2020); (Permata, H.K., Meryansumayeka, Scristia, Muhammad

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Yusup, 2022). Reasoning abilitys are useful when solving problems that occur both in the personal sphere, society, and other broader social institutions (Adamura, F., Susanti, V. D., 2018). The development of mathematical reasoning abilitys has a positive influence on the development of problem-solving abilitys, especially in geometry problems (Salmina, M. & Nisa, S.K., 2018). In mathematical reasoning, students can learn more meaningfully, not only by remembering facts, concepts, and procedures, or by imitating examples, but also by being able to understand mathematical concepts in an integrated manner (Maesaroh, S., Sumarmo, U., & Hidayat, W., 2020); (Santosa, F.H., Negara, H.R.P., & Bahri, S., 2020); (Scristia, Hapizah, Sumarni, Jeri Araiku, 2020); (Syarifuddin, S., Fauzi, A., & Ariswoyo, S., 2020). Geometry is one of the topics in mathematics that is important for students' reasoning. Geometry standard includes a strong focus on developing reasoning and rigorous proofs, using definitions and undeniable facts (NCTM, 2000). One of the geometry materials contained in the 2013 Curriculum is the material of congruence and similarity, which is taught in grade IX SMP. Congruence and similarity can train students' mindset to be structured in learning mathematics because it contains components that are interconnected with each other (Nainggolan, J., & Pasaribu, H.B., 2021). Studying congruence can also develop advanced mathematical thinking abilitys (Otalora, 2016).

However, the results of several studies indicate that there are still many high school students who have low mathematical reasoning abilities on geometry topics, including congruence and similarity materials (L. Ayunigrum, A. P. Kusuma, N. K. Rahmawati, 2019); (OECD, 2019); (Aisyah, S., Scristia, Meryansumayeka, Safitri, E., 2022); (IEA, 2016); (Rahayu, 2016). There are still many students who have difficulty solving congruence problems (Fadilah, R., & Bernard, M., 2021).

Students' errors in solving problems are related to learning disabilities or imperfect learning abilities (Utami, D.N., Kusmanto, N., & Widodo, S.A., 2019). (Jihan Della Safegi, 2021); (Utami, D.N., Kusmanto, N., & Widodo, S.A., 2019) states: the types of errors that students make in solving math problems include: (1) strategy errors, (2) errors, (3) translations, (4) concept errors, (5) systematic errors, (5) miscalculations. It is natural that in solving math problems, students make errors. However, if the errors that arise do not immediately get attention and follow-up, it will be bad for students. Considering that in mathematics lessons, the materials that have been given will be interrelated to support various subsequent materials such as geometry subjects. The results of the study by (Utami, D.N., Kusmanto, N., & Widodo, S.A., 2019) showed that the percentage of students' errors in solving geometry problems is very high. 89,84% of students make errors in terms of calculations, 85,16% of students make errors in terms of procedures, and 68,98% of students make errors in terms of concepts.

Based on the facts mentioned above, the question arises about what student's errors to experience in solving reasoning based congruence proof problems. Therefore, it is necessary to do further analysis about students' errors in solving reasoning based congruence proof problems so that these errors are known by the teacher so that the teacher can take the right action to his students

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and the student's reasoning ability can be improved. Further analysis can provide a clear and detailed picture of what errors students faced when solving problems (Badriani, I., Wyrasti, A., Tanujaya, B., 2022). The results of this error analysis can serve as an evaluation for teachers when making lesson plans, help students to improve their mathematical reasoning ability and minimize students' errors in solving reasoning based congruence proof problems

METHODS

The method used in this research is descriptive qualitative that aims to analyze and describe students' errors in solving reasoning problems about proving the congruence of triangles based on mathematical reasoning abilities. The subjects were 25 students of grades IX SMP in Palembang in the odd semester of the 2021/2022 academic year. Data collection was carried out by giving written tests to students in proving triangular congruence. Then the students' answers were analyzed based on indicators of mathematical reasoning abilities. Previously, researchers first prepared test instruments and scoring guidelines. The test questions given consist of 3 essay questions about proving the congruence of triangles that are arranged based on indicators of mathematical reasoning. Each item contains 4 indicators of mathematical reasoning; 1) making mathematical statements, 2) compiling evidence and providing reasons or evidence for the correctness of the solution, 3) drawing conclusions, and 4) checking the validity of arguments. Then, the instrument was validated by lecturers and mathematics teachers. The results stated that the instrument was valid and could be used to collect data. Next, the researcher gave test questions to students. After that, the students' test results were checked and scores were given. The indicators of mathematical reasoning are presented in Table 1.

Table 1. Mathematical reasoning indicators

No	Indicator	Descriptor
1	Making mathematical statements	Students can make correct mathematical statements
2	Compiling evidence; providing reasons or evidence for the correctness of the solution.	Students can provide arguments against each step of the method or strategy that has been determined in the solution by using valid evidence.
3	Drawing conclusions from statements.	Students can make new correct statements based on several statements whose truth has been proven or assumed previously through mathematical manipulation
4	Checking the validity of an argument.	Students can re-examine or investigate the truth of the statements made or given.

To find out the errors made by students, the data obtained were then analyzed based on mathematical reasoning indicators as presented in Table 1.

RESULTS AND DISCUSSION

The data was obtained from written test results. The following are student's errors when answering congruence proof problems. These errors were identified based on mathematical reasoning indicators

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Errors in making mathematical statements

Errors in making mathematical statements were performed by AR when answering question number 2. The AR's answer of question number 2 can be seen in Figure 1.

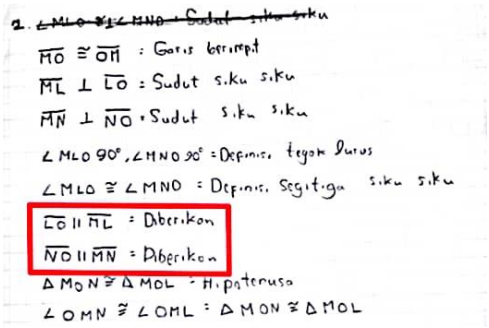

Figure 1. The answer to question number 2 by AR

AR students make mathematical statements that $\overline{LO} \parallel \overline{ML}$ and $\overline{NO} \parallel \overline{MN}$ since they are known. It means that AR make such statements referring to the information that given on the question. While in the question, the written information is "the LO line is perpendicular to the ML line and the NO line is perpendicular to the MN line". So the statement should be $\overline{LO} \perp \overline{ML}$ and $\overline{NO} \perp \overline{MN}$. This can happen because AR is errorn about perpendicular notation to parallel notation.

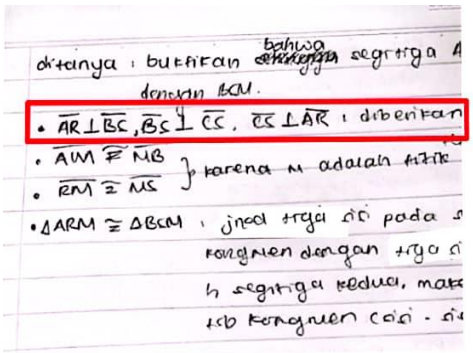

Figure 2. The answer to question number 3 by AS

Errors in making mathematical statements were also performed by AS when answering question number 3. In the question, there is information that "R and S are points on the CS line in such a way that AR and BS are perpendicular to the CS line". From the information, AS make statements that $\overline{AR} \perp \overline{BS}$, $\overline{BS} \perp \overline{CS}$, $\overline{CS} \perp \overline{AR}$. There is an incorrect mathematical statement, namely the statement $\overline{AR} \perp \overline{BS}$. Because based on the information provided on the question, the lines that are perpendicular to each other are only $\overline{AR}$ with $\overline{CS}$ and $\overline{BS}$ with $\overline{CS}$. This thing that AS experience can be caused by a miss in reading comprehension.

Errors in Compiling evidence; providing reasons or evidence for the correctness of the solution

Errors in compiling evidence was performed by AY when answering question number 1. The AY's answer of question number 2 can be seen in Figure 3.

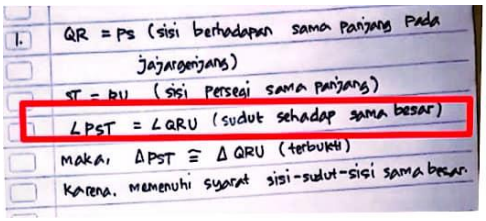

Figure 3. The answer to question number 1 by AY

Based on Figure 3, it can be seen that AY gives the argument that $\angle PST = \angle QRT$ with reasons that the angle is as large as it is. The argument is correct, but the reason is invalid, $\angle PST = \angle QRT$ because both angles are formed by two pairs of congruent sides, namely $\overline{PS}$ and $\overline{ST}$ which make up $\angle PST$ congruent with $\overline{QR}$ and $\overline{RU}$ which make up $\angle QRU$, not because the two angles are opposite. This error

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occurs since AY do not understand the concept of relationships between lines and angles.

Errors in drawing conclusions from statements

Errors in drawing conclusions from statements were performed by RP when answering question number 2. The RP's answer of question number 2 can be seen in Figure 4.

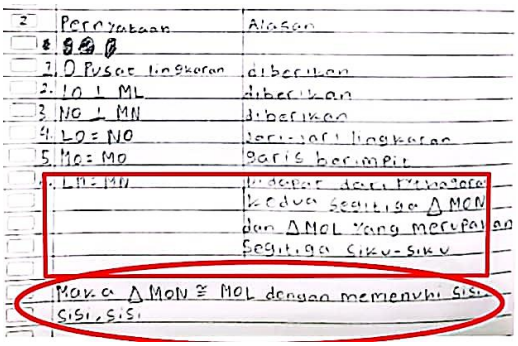

Figure 4. The answer to question number 2 by RP

RP experience errors caused by the argument $LM = MN$, which cannot be verified for truth, because the reasons given are also invalid. The argument led RP to think that the three pairs of sides on the triangle ΔMON and ΔMOL were congruent and concluded that $\Delta MON \cong \Delta MOL$ because it meets the conditions of the Side-Side Postulate. The wrong argument and evidence triggers the wrong conclusion.

Errors in checking the validity of an arguments

Errors in cecking the validity of an srguments were performed by KR when answering question number 2. The KR's answer of question number 2 can be seen in Figure 5.

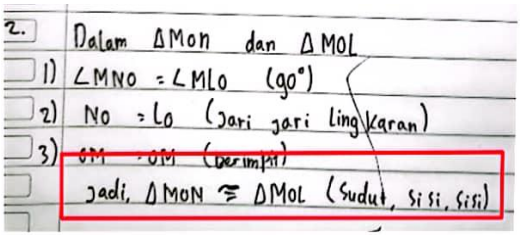

Figure 5. The answer to question number 2 by KR

The statements given by KR are all true, but the final answer shows that $\Delta MON \cong \Delta MOL$ because it corresponds to the Angle-Side-Side Postulate. In proving the congruence of the triangle there is no Angle-Side-Side Postulate, it should be $\Delta MON \cong \Delta MOL$ because it corresponds to the Hypotenuse-Leg Postulate. This shows that KR do not check the truth of the postulate it provides. The error occurs because students are careless or indeed do not know the applicable postulates.

This study shows 4 types of errors based on mathematical reasoning indicators. In the indicator of makes a mathematical statement, namely the ability to make a correct value mathematical statement by using the correct mathematical notation based on the information contained in the problem. Common errors made by students are error in using notation and misunderstood the meaning of the problem. Most of them have used the right notation, some are still not disciplined in using notation, and some of them have used it, but do not understand the meaning of each notation. This is in line with (Titin Sofitri & Yonita Roza, 2015); (Badriani, I., Wyrasti, A., Tanujaya, B., 2022) that the errors experienced by students in solving geometry problems are notation errors and students' failure to understand the problems to be converted into correct mathematical sentences.

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In the indicators of compiling evidence, providing reasons or evidence of the truth of the solution, it was found that the errors made by students occurred due to weak understanding of basic concepts of geometry such as lines and angles, definitions of congruence, and the characteristic of shape. (Siti Komariyah, Dian Septi Nur Afifah, Gaguk Resbiantoro, 2018) "Understanding concepts is very important in learning mathematics, because with a mature understanding, students can solve a problem and being able to apply this learning to the real world. Understanding concepts in mathematics is a thing that is mutually beneficial. So that if students cannot understand a concept then the student will have difficulty in continuing the material learned. If students have difficulty understanding the material, then students will also have difficulty in solving problems related to material." Therefore, if a student has not understood the concept of the material will then have difficulty in solving the problem related to the material. This is what triggers errors in students' answers.

For some students, errors in making conclusions are caused because students are wrong in giving reasons from written proof, error in reading information in the problem so as to provide inexact arguments. Furthermore, the error in using the principle, namely the error in using the appropriate postulates or theorems about triangles.

Then, student errors in analyzing the information contained in the problem, after the arguments were given students did not check the validity of the arguments they wrote, so that there was a difference between the final conclusions and the written proof process. The impact of these errors resulted in students' inappropriate reasoning.

Besides some of the findings given. Because this research only analyzes the errors of students' answers in writing based on the results of the proof they provide. Therefore, in further research, a learning process can be carried out that uses the proof method to support students' reasoning processes when proving.

CONCLUSION

Based on the data obtained from this study, it can be seen that students must be given a lot of practice in solving reasoning problems. Errors encountered by students when solving proof questions were caused, among others; errors in writing mathematical symbols, conceptual errors in the form of errors in understanding the meaning of questions, errors in mathematical principles, carelessness, and carelessness.

Based on these results, it is recommended for teachers to continue practicing questions and always ensure students' understanding of the prerequisite material before entering new material.

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ISSN 2089-8703 (Print)
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