NUMERICAL STUDY OF TSUNAMI GENERATION MECHANISM

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ABSTRACT

Tsunami can be gererated by seafloor deformation (fault), under water or sub-marine landslides, under water explosions, and asteroids impact. However, most of tsunami cases are generated by fault. Numerical study of tsunami generation mechanisms are investigated. Tsunami generation by faults are studied and focused on the effects of dispersion and nonlinearity on tsunami generation. Simulations included tsunami generation at deep, intermediate, and shallow water regions. Numerical simulation showed that at deep and intermediate water region the dispersion effect reduce the generated tsunami profile by shifting the profile into more than one wave while the nonlinearity effect does not important for this case. At shallow water, both dispersion and nonlinearity did not take effect to generated tsunami profile. Durations of seabed movement are also simulated to investigate the effect of fast and slow event generation of tsunami. For fast event tsunami propagation, generated tsunami profiles are similar to the profiles of bottom motion.

Keywords: tsunami, dispersion, nonlinearity

BACKGROUNDS

The main issues in tsunami modeling community are tsunami generation, propagation and runup. In order to increase the accuracy of numerical model, the effect of dispersive and nonlinearity on the tsunami generation are investigated in this study. The dispersive effect increases with the time and when the area of deformed bottom is decreases. During propagation of tsunami, an oscillating 'tail' is formed following the leading wave in which the leading wave amplitude is higher than followers. The water depth moving velocity of deformed bottom area also affects the magnitude of dispersive effect.

Using numerical model based on high order finite difference scheme, the dispersive and nonlinearity effects on tsunami generation are discussed here. The effects are investigated for tsunami generation at deep, intermediate and shallow water. The effects are also investigated for the case of 'fast' and 'slow' event of tsunami generation i.e.: the duration of seabed deformation. Tsunami propagation for long distance propagation on relatively deep and shallow water is presented to visualize the dispersive effect to the spatial profiles of tsunami.

MODEL EQUATIONS

Two sets of model equations are used here, i.e.: Nonlinear Shallow Water equations (NLSW) and a set of dispersive wave model which is referred here as Weakly Nonlinear Boussinesq equations (WNB). In one-dimensional form, the models equations are written as

$$(\bar{h}+\eta)_t + [(h+\eta)u]_x + \beta\left[a_1 h^3 u_{xx} + a_2 h^2\left((hu)_{xx} + \bar{h}_{xt}\right)\right]_x = 0 \qquad (1)$$

$$u_t + g\eta_x + uu_x + \beta\left[\tfrac{1}{2}\tilde{z}^2 u_{xx} + \tilde{z}\left((hu)_{xx} + \bar{h}_{xt}\right)\right]_t = 0 \qquad (2)$$

where u is depth-averaged velocity for NLSW equations or velocity at an arbitrary level, z_α , for WNB equations which is recommended to be evaluated at

z_α = -0.531h (Nwogu, 1993), g is gravitational acceleration, while h and η are still water depth and free surface displacement, respectively. Subscript x and t denote partial derivatives which respect to x and time, t, respectively. Time dependent water depth, $\bar{h}$, is included to the model equations. By including the time dependent water depth, the model equations are available to simulate tsunami generation by bottom motion, under water landslide and sub-marine landslide (Lynett and Liu, 2002).

In equations (1) and (2), β is a setting parameter. If β is set to be 1, the model equations will be WNB equations, while if β is set to be 0 it means the dispersion terms are removed from the model equations and the model equations will be NLSW equations. The NLSW model contains nonlinear effects but neglect the dependence fluid velocity by the argument that as long as the wavelength is much larger than the depth this will be realistic condition. By this reason, the seabed deformation area used in this study is taken much wider than the water depth.

The initial phase of tsunami generation is investigated by taking bottom motions which represent the motion of tectonic plate. One of problems that are encountered of studying tsunami generation is the lack of seismic models or seismic process data itself. Usually, the tsunami wavelength is approximated by $\lambda = c_0 T_{event}$, where $c_0 = \sqrt{gh_0}$ is the velocity of long waves at the water depth h_0, and T_{event} is the duration of seismic event, but in fact this duration is largely unknown. The above relation is often be used to find the duration from the observed data or inversely calculated from wavelength of tsunami. Van Groesen and Klopman (2006) adjusted the expression of wavelength by including the width of moving region (W) as substantial parameter which is illustrated in **Fig. 1**. The $h(x,t)$ of linearized shallow water equation (1) leading to this equation

$$u_{tt} + (c^2 u)_{xx} = gh_{xt} \tag{3}$$

where $c = \sqrt{gh}$. Assuming that the bottom displacement is small, the change in velocity is negligible and speed in the left hand side can be replaced by its value at the event depth. Then (3) can be solved implicitly, leading to a corresponding surface elevation given by

$$\begin{aligned}\eta(x,t) = &-\frac{1}{2}\int_0^t \frac{\partial}{\partial t}\left(h(x - c_0 t + c_o\tau, \tau\right)d\tau \\ &-\frac{1}{2}\int_0^t \frac{\partial}{\partial t}\left(h(x + c_0 t - c_o\tau, \tau\right)d\tau\end{aligned} \tag{4}$$

Equation (4) has a clear interpretation in the (x, t)-plane (d'Alembert formula) as shown in left panel of **Fig. 1**. At a fix point (x, t) the contribution to the integrals are found on the characteristics through this point. The above formula immediately leads to an expression for wavelength

$$L = 2W + c_0 T_{rise} \tag{5}$$

as shown in the right panel of **Fig. 1**. This formula incorporates the spatial extent of the bottom motion through the half-width, W, and the rise time of bottom motion, T_{rise}. The 'fast' or 'slow' event of tsunami generation is characterized by an 'overflow' time as $t^* = W / c_0$ which define as the time that water needs to flow from the center of the region to the boundary. Then a 'fast' event of tsunami generation can be characterized by $T_{rise} < t^*$ and 'slow' event by $T_{rise} > t^*$. In case of very fast event, the resulting sea level will resemble the bottom motion event (Mansinha and Smyle, 1972).

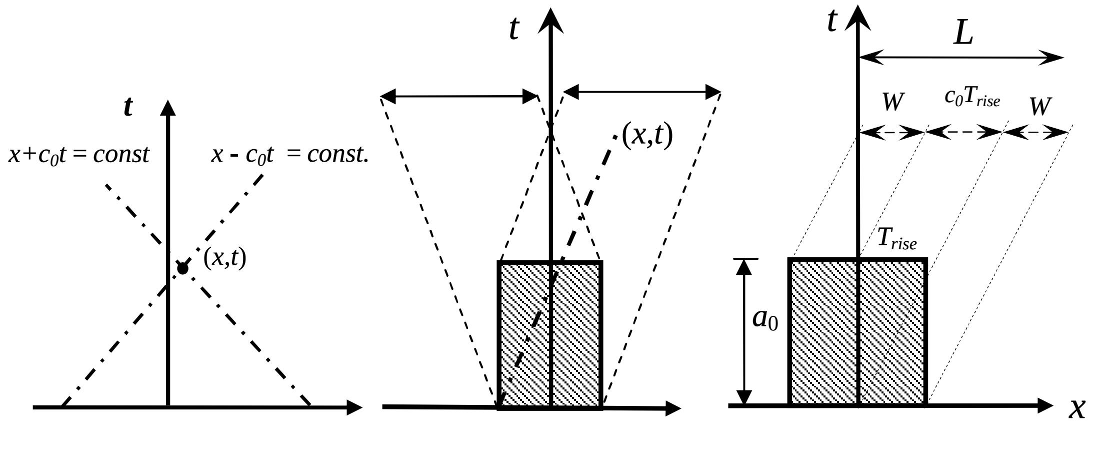
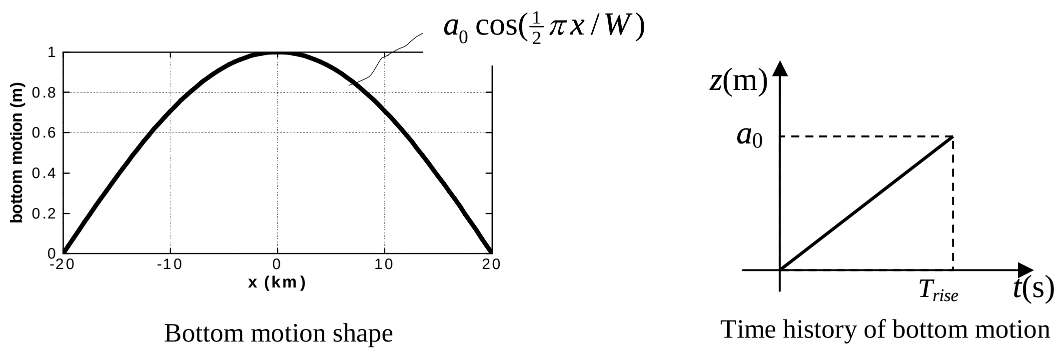


Fig. 1 Illustration of (x, t) plane of generation region and interpreting wavelength.



Bottom motion shape

Time history of bottom motion

Fig. 2 Bottom motion shape and time history of bottom motion.

NUMERICAL SOLUTION

The model equations are solved using finite difference algorithm in a Cartesian grid as proposed by Wei et al., 1995. Numerical solution of the model equations is primarily a recreation of the Wei and Kirby, (1995), the difference exists in the additional term due to a time-dependent water depth caused by seafloor deformation and treatment of runup modeling by using slot method in which sponge layer is added at the slot region.

A high order predictor-corrector scheme is used, employing a third order in time explicit Adam-Bashforth as predictor step and a fourth order in time Adam-Moulton implicit scheme as corrector step. The implicit corrector step must be iterated until a convergence criterion is satisfied. Fourth-order finite difference scheme is used to all spatial derivatives, yielding a model that is numerically accurate to order $(\Delta x)^4$ in space and order $(\Delta t)^4$ in time. Detail of the numerical scheme is referred to Wei and Kirby, 1995 or Alwafi, 2008.

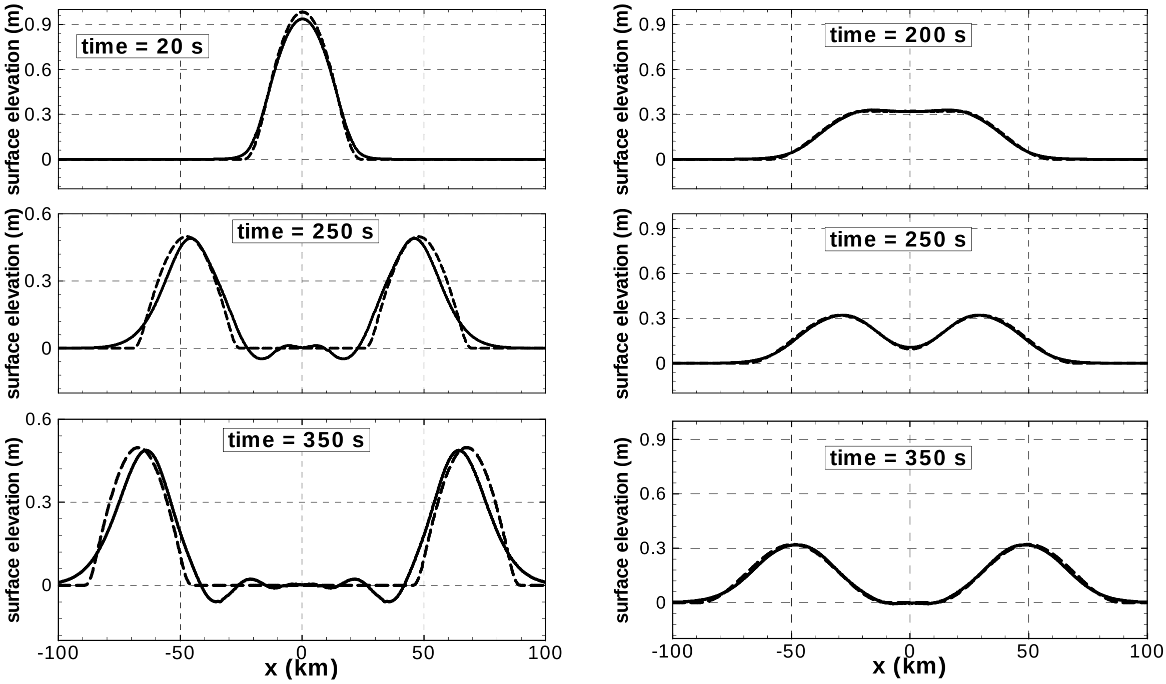


Fig. 3 Generated tsunami profiles for Case 1. Left panel shows results of bottom motions with T_{rise} = 20 s and the right panel shows results of bottom motion with T_{rise} = 200 s simulated by: NLSW model (dash lines) and WNB model (solid lines).

SIMULATION RESULTS AND DISSCUSSION

In this study, tsunami is generated by vertical bottom motion. Monotone bottom motions with amplitude a_0 = 1.0 m and width W = 20 km are considered. The final shape of bottom motion is defined by $a\cos(\frac{1}{2}\pi x/W)$ as shown in **Fig. 2**. The vertical motions are given by linear dynamic changes during the rising time, T_{rise}. The bottom changes are symmetric at the center x = 0. Two cases are considered here which corresponds to the bottom movement at a constant water depth h_0 = 4000 m and h_0 = 1000 m, respectively. The bottom motions with T_{rise} = 2t* and T_{rise} = t*/5 are considered for both two cases. Numerical simulations are performed by using Δx = 500m and Δt = 2s.

Spatial profiles of sea level for Case 1 are shown in **Fig. 3**. The left panel shows the results for bottom motion with T_{rise} = t* /5= 20s (fast event tsunami generation) here then referred as Case 1a and the right panel represents results of bottom motion with T_{rise} = 2t* = 200s (slow event tsunami generation) and referred as Case1b. Using equation (5) Case 1a and Case 1b produced wave length about 44 km and 80 km, respectively. Numerical simulation yielded initial wavelength agree very well to the theoretical calculation of equation (5). The ratio between water depth and wavelength for Case 1a and Case 1b are $h/L \approx 0.0909$ and $h/L \approx 0.05$, respectively.

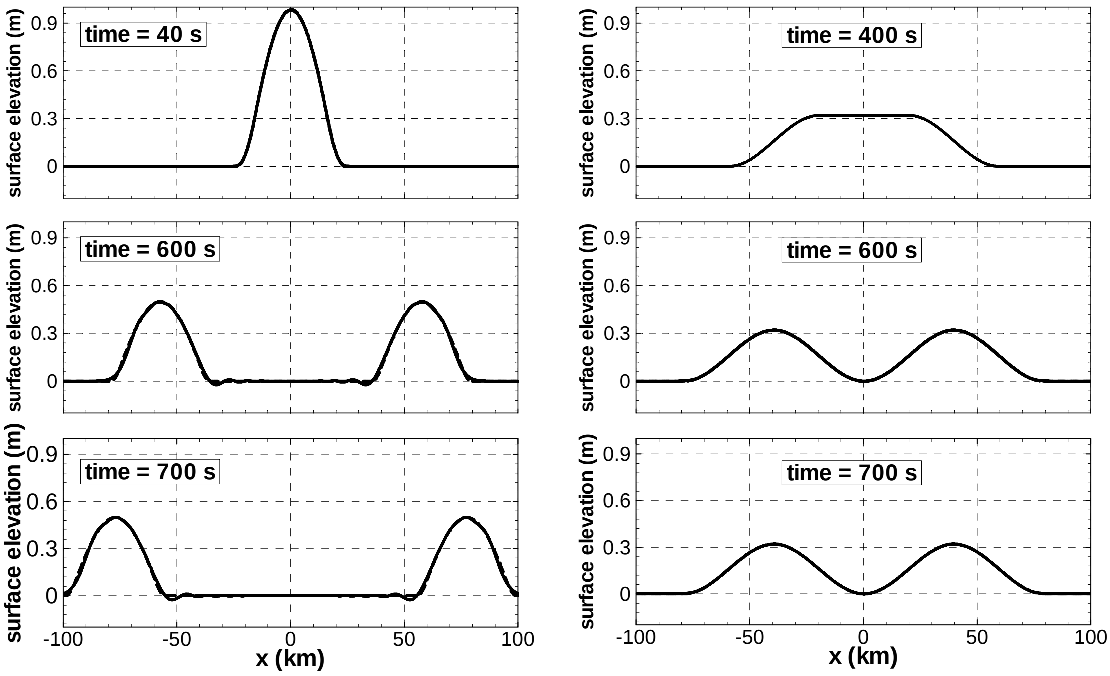


Fig. 4 Generated tsunami profiles for Case 2. Left panel shows results of bottom motions with T_{rise} = 40 s and the right panel shows results of bottom motion with T_{rise} = 400 s simulated by: NLSW model (dash lines) and WNB model (solid lines).

According to the value of h/L, Case 1a is categorized as tsunami generation at intermediate water and Case 1b as tsunami generation at the transition between intermediate and shallow water. The WNB model produces initial profiles of free surface elevation with the maximum amplitude a little bit lower compare to the NLSW model with larger wavelength in Case 1a. Case 1b produced initial tsunami wave with the maximum height is much lower compare to the bottom motion height. All of the three models produce similar initial free surface elevation in Case 1b. The initial waves propagate to he left and right direction from the source center. For Case 1a, the initial waves then immediately shifted into more than one wave caused by the dispersion effect for WNB model results, while the initial wave is unchanged for the NLSW model results. It is indicate that the dispersive effect is clearly visible already after a short propagation distance although immediately after generation the shapes are quite similar. Faster rise time will produce higher wave amplitude. In case of $T_{rise} > t*$ the amplitude of initial wave profiles are much lower compare to the bottom motion, while for $T_{rise} < t*$ the maximum amplitude nearly same to the maximum amplitude of bottom motion. Hence, for the case in which the bottom motion is very fast, the surface elevation mimics the final profile of bottom motion. The final profile of bottom motion could be used as initial condition of free surface elevation (Mansinha and Smyle, 1972).

The study is continued to the Case 2 which is carried out by taking h_0 = 1000 m. Profile of seafloor movement of **Fig. 4** is generated with rising time T_{rise} = 40s and T_{rise} = 400s and referred as Case 2a and Case 2b, respectively in which Case 2a is fast event tsunami generation and Case 2b is slow event tsunami generation. **Fig. 4** shows the initial wave profiles for Case 2, the left panel shows results of fast event tsunami generation and the right is slow event. Analysis of wavelength using equation (5) for Case 2a and Case 2b are 40 km and 80 km, respectively which agree very well to the numerical results. The ratio between water depth and wavelength for Case 2a and Case 2b are $h/L \approx 0.0227$ and $h/L \approx 0.0125$, respectively. According to value of h/L, both Case 2a are categorized as tsunami generation at shallow water. All of he models give similar result of initial wave profile. It indicates that the dispersion effect is less important for tsunami generation at shallow water. However, the dispersion will affect the wave profile after long distance propagation (see lowest left panel of **Fig. 4**). Again the fast event tsunami generation produces the same profiles of free surface elevation to the bottom motion profiles (upper left panel of **Fig. 4**). Hence, the final profile of bottom movement can be used as initial profile of generated tsunami.

CONCLUSIONS

According to the numerical simulations, conclusions could be inferred here:

1. The empirical formula (5) can be used to predict tsunami wavelength, hence tsunami generation phase can be determined as shallow, intermediate or deep water wave.
2. The nonlinearity effect is less important for tsunami generation phase but it is important when tsunami entered the runup region.
3. The dispersion effect is less important for tsunami generation at shallow water region but it should to be counted for intermediate or deep water region.

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