UNP JOURNAL MANAGEMENT CENTER

math.ppj.unp.ac.id
p-ISSN 2716-0726
e–ISSN 2716-0734

Indexed

Mathematical Model of the Number of Smokers Influenced by Migration Factors with Quadratic Root Dynamics in Relapse Conditions

Tria Agus Krisan[1], Media Rosha[2]

[1] Jalan Andalas 1 no 72 C Padang, Indonesia
[2] Universitas Negeri Padang, Indonesia

Rangkiang Mathematics Journal
Volume 1, No. 1, 2022

Subject Areas:
Mathematics

Keywords:
Mathematical Model, Smoker Population, Asymptotically Stable, Equilibrium Point.

Author for correspondence:
Tria Agus Krisan
e-mail: triaaguskrisan16@gmail.com

RANGKIANG MATHEMATICS JOURNAL
Journal of Mathematics and Mathematics Education

Abstract- Smoking is a habit that some people like, but it causes health, economic, social, and environmental burdens not only for smokers but also for others. This study describes a mathematical model of the number of smokers which is influenced by the distribution factor of smokers using the dynamics of the square root in the relapse condition. The population was divided into three subpopulations, namely potential smokers, light smokers and heavy smokers. Based on the results of model analysis, it was found that one endemic equilibrium point of smokers was stable. Environmental influences make there always interactions between potential smokers and light smokers so that there are always smokers. The smaller the interaction between potential smokers and light smokers, the smaller the number of light smokers and heavy smokers.

1. Introduction

Smoking causes health, economic, social and environmental burdens not only for smokers but also for others. Cigarettes are tobacco products that are intended to be burned, smoked and inhaled, including white cigarettes, clove cigarettes, cigars or other forms of smoke containing nicotine, tar and carbon monoxide, with or without additives (Anwar et al, 2019).

The number of smokers has now reached 1.2 billion people worldwide and 800 million of them are in developing countries. Based on Atlas Tobacco's research, in 2016 the number of smokers reached nearly 55 million people in Indonesia and is on an increasing trend. This number makes the World Health Organization (WHO) say Indonesia is the third country with the highest number of smokers in the world after China and India (Infodatin, 2015).

The increase in cigarette consumption has an impact on the higher burden of disease caused by smoking and an increase in the death rate from smoking. The increase in cigarette consumption also results in the surrounding air being hotter than usual, this indicates that the air is polluted by smoking (Wahyuni et al, 2018).

The smoking habit is very difficult to break and is rarely recognized as a bad habit. According to a WHO report in 2018, it shows that 30.4 percent of Indonesian smokers have tried to quit, but only 9.5 percent of them are successful (Fernandez, 2020). Judging from the above percentage, there are many Indonesian smokers who try to quit smoking, but only a few are successful in being free and maintaining a smoking free period. Smokers who quit smoking have the potential to return to smokers, most of which will eventually return to smoking, which is known as a relapse.

math.ppj.unp.ac.id Rangkiang Mathematics Journal Vol. 1, No. 1, 2022

Many factors cause relapse in smokers. One of them is the addiction to nicotine in the smoker's body (Aswan, 2018). Smokers who quit smoking will have lower levels of nicotine in their blood. This will cause the smoker to experience withdrawal symptoms. Smokers who quit smoking are very susceptible to smoking again. To prevent withdrawal symptoms, generally smokers who quit smoking will smoke again (relapse) to get the comfort effect of nicotine.

Several researchers have developed mathematical models about increasing the number of smokers, one of which is a journal entitled Mathematical Model of the Number of Smokers with Quadratic Root Dynamics and Migration Factors (Soleh & Sazmita, 2017). Where in the journal uses square root dynamics in the process of spreading potential smokers to light smokers and adds the migration factor of each compartment.

Based on the explanation above, the researcher is interested in discussing the increase in the number of smokers by modeling it into a mathematical model. By reconstructing the M. Soleh model again by dividing the population into 3 groups, namely the potential smoker population, the light smoker population, and the heavy smoker population. This is because in reality it will be difficult for a smoker to completely quit smoking (stop permanently), and eventually smoke again (relapse).

2. Methods

This research is basic research. The method used by researchers is by studying literature, namely studying books or journals related to problems in the mathematical model of the number of smokers influenced by migration factors with the dynamics of square roots in relapsing conditions. The steps taken in the research are:

1. See and study the problems of the development of the number of smokers.
2. Determine what factors must be considered in the development of the number of smokers.
3. From these factors we can determine the assumptions, variables, parameters that can assist in the preparation of a mathematical model of the number of smokers.
4. Forming a mathematical model of the number of smokers given the migration factor with the dynamics of the square root in the relapse condition.
5. Conduct an analysis of the model conducting formed by determining the point of equilibrium and stability and simulations.
6. Interpret the results of the analysis from the mathematical model.
7. Make conclusions.

3. Results and Discussion

(a) Mathematical Model of Number of Smokers

Based on the steps in forming a mathematical model, the first step taken is to express problems in the real world into the form of mathematical problems. This step is done by determining the factors that are considered important or in accordance with the problem. Includes identification of variables, parameters and establishing relationships between these variables and parameters.

The variable used to form a mathematical model of the number of smokers that is influenced by migration factors with square root dynamics in the relapse condition is the group of potential smokers (P), namely individuals who have never smoked and have the potential to become smokers, or individuals who have ever smoked, who are again vulnerable to smoking. Individual groups of light smokers (L), namely individuals who do not smoke habits or activities every day or who smoke every day (≤10 cigarettes/day). Group of heavy smoking individuals (S), namely individuals who smoke habits or activities every day (≥10 cigarettes/day). The parameters used are:

μ_i	:	Population immigration rate
μ_e	:	Population emigration rate
δ	:	Natural death rate
θ	:	individuals aged ≥10 years
γ	:	Reduction rate of light smokers
β	:	The rate of reduction in potential smokers
α	:	Smoking cessation rates in heavy smokers
ρ	:	Smoking cessation rates in light smokers

The next step is to determine the assumptions that will be used in forming a mathematical model of the number of smokers influenced by migration factors with the dynamics of the square root under relapse conditions. Based on the observed problems, the assumptions used are as follows:

1. The population is open, that is, the migration process occurs in the population.
2. Recruitment for potential smokers is individuals aged ≥10 years.
3. There is a natural death process in the smoking population with a constant mortality rate.
4. Deaths due to smoking are ignored, because they exist in every compartment, because it is the same as natural death.
5. Individuals who are potential smokers will become smokers, because of the interaction with light smokers, while individuals who are light smokers will become heavy smokers if there is an intense interaction between the two.
6. There is always an interaction between potential smokers and light smokers, namely $\sqrt{PL} \neq 0$ meaning $P \neq 0$ and $L \neq 0$.
7. The light smoker category is people who smoke not every day or smoke every day (≤10 cigarettes/day), while heavy smokers are people who smoke every day (> 10 cigarettes/day).
8. Individuals who have quit smoking, both light smokers and heavy smokers do not have permanent immunity to not smoke, they are very susceptible to smoking again so that they can return to being individuals who have the potential to smoke, and may experience relapse.

With the variables, parameters, and assumptions that have been described, can be drawn a diagram of the mathematical model of the number of smokers influenced by migration factors with the dynamics of the square root under relapse conditions, as shown in Figure 1.

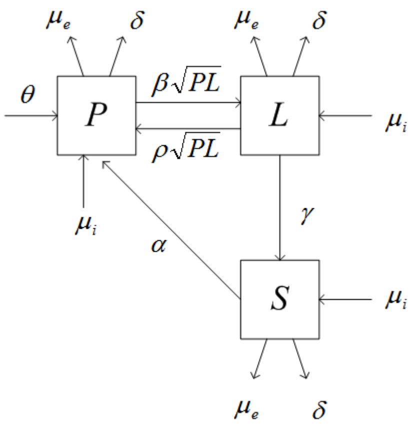


Figure 1. Compartment Condition Chart of Number of Smokers

Based on Figure 1, can be formulated a mathematical model of the number of smokers influenced by migration factors with the dynamics of the square root under relapse conditions as follows:

$$\frac{dP}{dt} = \theta + \alpha S + (\rho - \beta)\sqrt{PL} + (\mu_i - \mu_e - \delta)P$$
$$\frac{dL}{dt} = (\beta - \rho)\sqrt{PL} + (\mu_i - \mu_e - \delta - \gamma)L$$

$$\frac{dS}{dt} = \gamma L + (\mu_i - \mu_e - \delta - \alpha)S$$

To simplify the analysis, it will be as follows:

$$A_1 = \mu_i - \mu_e - \delta$$
$$A_2 = \mu_i - \mu_e - \delta - \gamma$$
$$A_3 = \mu_i - \mu_e - \delta - \alpha$$

So that it is obtained as follows:

$$\frac{dP}{dt} = \theta + \alpha S + (\rho - \beta)\sqrt{PL} + A_1 P$$
$$\frac{dL}{dt} = (\beta - \rho)\sqrt{PL} + A_2 L$$
$$\frac{dS}{dt} = \gamma L + A_3 S$$

(b) Analysis of The Mathematical Model of The Number of Smokers

In the analysis of the mathematical model the number of smokers who are influenced by migration factors with the dynamics of the square root in the relapse condition, a fixed point will be sought, a stability analysis of a fixed point, and simulations from the analysis of the mathematical model.

1. Fixed Points for Smokers Endemic

In the mathematical modeling of the number of smokers who are influenced by migration factors with the dynamics of the square root in the Relapse condition, it is found that one fixed point is endemic to smokers, this means that in a group of individuals there are always smokers due to the interaction between potential smokers and light smokers. Fixed point smoking endemic means that there are a number of individuals affected by smoking in the population. Mathematically it can be expressed as P>0, L>0 and S>0. So that the endemic fixed point of smokers is obtained:

$$P^* = \frac{-\theta\left((A_2)^2 A_3\right)}{\left((A_2)^2 A_3 A_1 - \gamma\alpha(\beta - \rho)^2 + (\beta - \rho)(\rho - \beta)A_3 A_2\right)}$$
$$L^* = \frac{-\theta A_3 (\beta - \rho)^2}{\left((A_2)^2 A_3 A_1 - \gamma\alpha(\beta - \rho)^2 + (\beta - \rho)(\rho - \beta)A_3 A_2\right)}$$
$$S^* = \frac{\gamma\theta(\beta - \rho)^2}{\left((A_2)^2 A_3 A_1 - \gamma\alpha(\beta - \rho)^2 + (\beta - \rho)(\rho - \beta)A_3 A_2\right)}$$

2. The Stability of Smoker's Endemic Fixed Point

Fixed point is said to be stable if all the eigenvalues of the Jacobian matrix at the endemic fixed point of smokers are negative. The fixed point of endemic to smokers is $e_* = (P^*, L^*, S^*)$. The Jacobian matrix of the fixed point $e_* = (P^*, L^*, S^*)$ is:

$$J = \begin{bmatrix} \frac{(\rho - \beta)(\beta - \rho)}{2A_2} + A_1 & \frac{A_2(\rho - \beta)}{2(\beta - \rho)} & \alpha \\ \frac{(\beta - \rho)^2}{2A_2} & \frac{3A_2}{2} & 0 \\ 0 & \gamma & A_3 \end{bmatrix}$$

So we get the following characteristic equation:

$$\lambda^3+\lambda^2\left(-A_3-\frac{3A_2}{2}-\frac{(\rho-\beta)(\beta-\rho)}{2A_2}-A_1\right)+\lambda\left(\frac{3A_2A_3}{2}+\frac{(\rho-\beta)(\beta-\rho)A_3}{2A_2}+\frac{(\rho-\beta)(\beta-\rho)}{2}+A_1A_3+\frac{3A_1A_2}{2}\right)$$
$$+\left(\frac{-(\rho-\beta)(\beta-\rho)A_3}{2}-\frac{3A_1A_2A_3}{2}+\frac{(\beta-\rho)^2\gamma\alpha}{2A_2}\right)$$

This equation can be written in a λ-shaped equation, namely:

$$a_0\lambda^3+a_1\lambda^2+a_2\lambda+a_3=0$$

With

$$a_0=1$$

$$a_1=-A_3-\frac{3A_2}{2}-\frac{(\rho-\beta)(\beta-\rho)}{2A_2}-A_1$$

$$a_2=\frac{3A_2A_3}{2}+\frac{(\rho-\beta)(\beta-\rho)A_3}{2A_2}+\frac{(\rho-\beta)(\beta-\rho)}{2}+A_1A_3+\frac{3A_1A_2}{2}$$

$$a_3=\frac{-(\rho-\beta)(\beta-\rho)A_3}{2}-\frac{3A_1A_2A_3}{2}+\frac{(\beta-\rho)^2\gamma\alpha}{2A_2}$$

Furthermore, the stability analysis can be searched using the Routh-Hurwitz criteria. With the characteristic equation are:

$$\lambda^k+a_1\lambda^{k-1}+a_2\lambda^{k-2}+\ldots+a_k=0$$

For $k=3$ the following criteria are obtained:

$$a_1>0,\qquad a_3>0,\qquad a_1a_2>a_3$$

So for the characteristic equation $a_0\lambda^3+a_1\lambda^2+a_2\lambda+a_3=0$ can obtained as follows:
Assume A_1,A_2,A_3 are negative, because $\mu_i<\mu_e$. Then:

$$a_1=-A_3-\frac{3A_2}{2}-\frac{(\rho-\beta)(\beta-\rho)}{2A_2}-A_1>0$$

Because $-A_3>0$, $-\frac{3A_2}{2}>0$, $-\frac{(\rho-\beta)(\beta-\rho)}{2A_2}>0$, and $-A_1>0$.

then $a_1>0$.

$$a_3=\frac{-(\rho-\beta)(\beta-\rho)A_3}{2}-\frac{3A_1A_2A_3}{2}+\frac{(\beta-\rho)^2\gamma\alpha}{2A_2}>0$$

Because $\frac{-(\rho-\beta)(\beta-\rho)A_3}{2}>0$, $-\frac{3A_1A_2A_3}{2}>0$, $\frac{(\beta-\rho)^2\gamma\alpha}{2A_2}<0$

$$\frac{(\beta-\rho)^2\gamma\alpha}{2A_2}<\frac{-(\rho-\beta)(\beta-\rho)A_3}{2}-\frac{3A_1A_2A_3}{2}$$

Then $a_3>0$.

$$a_1a_2>a_3$$

Because a_1,a_2 all positive values, while a_3 has a negative value. Then multiplication $a_1a_2>a_3$.

From the results obtained that $a_1 > 0$, $a_3 > 0$, $a_1 a_2 > a_3$, then the Routh Hurwitz stability requirements have been met, where the coefficient is positive and in other words the eigenvalue of the characteristic equation above is negative or has a negative real part. It can be concluded that the point remains stable endemic to smokers.

3. Simulation of the Stability of the Mathematical Model of the Number of Smokers

Numerical simulation on the mathematical model of the number of smokers influenced by migration factors with the dynamics of the square root in the relapse condition provides a clearer picture. Simulations were carried out using Maple 17 software by providing values for each parameter.

a. Mathematical model simulation with a fixed point of endemic smokers

It will be simulated for the condition that there are individuals who are affected by smoking behavior so that the parameters used are:

Table 1. Parameters for smoking endemic fixed points

Parameters	Value
$\boldsymbol{\alpha}$	0,001
$\boldsymbol{\beta}$	0,006
$\boldsymbol{\delta}$	0,0021
$\boldsymbol{\gamma}$	0,06
$\boldsymbol{\rho}$	0,001
$\boldsymbol{\theta}$	2
$\boldsymbol{\mu_i}$	0,001
$\boldsymbol{\mu_e}$	0,002

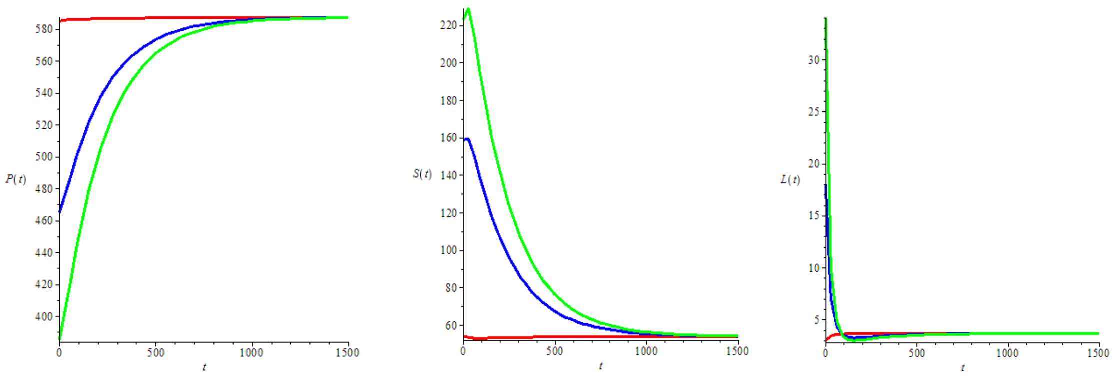
From the parameter values above, it is calculated that the value of the endemic fixed point of smokers which is influenced by migration factors with the dynamics of the square root in the relapse condition is$e_0(585, 3, 54)$. In the simulation of the endemic fixed point of smokers that is influenced by migration factors with the dynamics of the square root under relapse conditions, three initial values are used as follows:

$$P(0) = 585,\ L(0) = 3,\ S(0) = 54,$$

$$P(0) = 465,\ L(0) = 18,\ S(0) = 159$$

$$P(0) = 385,\ L(0) = 34,\ S(0) = 223$$

Based on the parameter values and initial values above, the graph of each group against time t is obtained as follows:



Graph *P(t)* Graph *S(t)* Graph *L(t)*

Figure 2. Trajectory around smoker endemic points

Based on Figure 2, the red curve represents the endemic fixed point of smokers which is influenced by migration factors with square root dynamics under relapse conditions, while the blue and green curves are curves with different initial values. The direction of motion of the blue and green curves to the red curves is what will determine whether or not it is stable at the fixed point of smokers which is influenced by migration factors with the dynamics of the square root under relapse conditions. Then it can be noticed that the fixed point $e_* = (P^*, L^*, S^*)$ is a stable fixed point because the trajectory (blue and green curves) of each graph moves closer to the endemic fixed point shown by the red curve.

b. Mathematical model simulation with a fixed point of endemic smokers changes the parameter β to $\beta = 0.003$

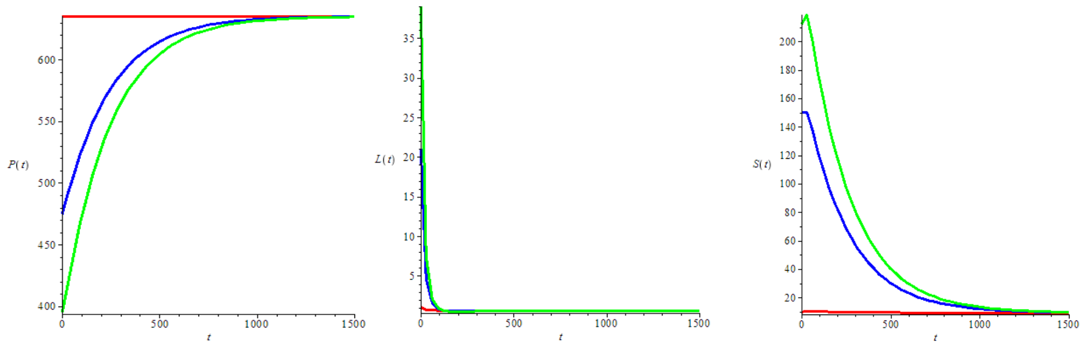
It will be simulated for the condition that there are individuals who are affected by smoking behavior by using table 1 but changing the parameter value β to $\beta = 0{,}003$ in order to obtain the endemic point value of smokers, namely $e_0(635, 1, 10)$. In the simulation of the endemic fixed point of smokers that is influenced by migration factors with the dynamics of the square root under relapse conditions, three initial values are used as follows:

$$P(0) = 635,\ L(0) = 1,\ S(0) = 10$$

$$P(0) = 475,\ L(0) = 21,\ S(0) = 150$$

$$P(0) = 395,\ L(0) = 39,\ S(0) = 212$$

Based on the parameter values and the initial values above, the graph of each group against time t is obtained:



Graph *P(t)* Graph *L(t)* Graph *S(t)*

Figure 3. Trajectories Around Smoker Endemic Points With $\beta = 0.003$

Based on Figure 3, the red curve represents the endemic fixed point of smokers which is influenced by migration factors with square root dynamics under relapse conditions, while the blue and green curves are curves with different initial values. Note that the fixed point $e_* = (P^*, L^*, S^*)$ is a fixed point which is asymptotically stable because the trajectory (blue and green curves) of each graph moves closer to the endemic fixed point indicated by the red curve.

c. Simulation of a mathematical model with an endemic fixed point of smokers changes the parameter γ to $\gamma = 0.03$

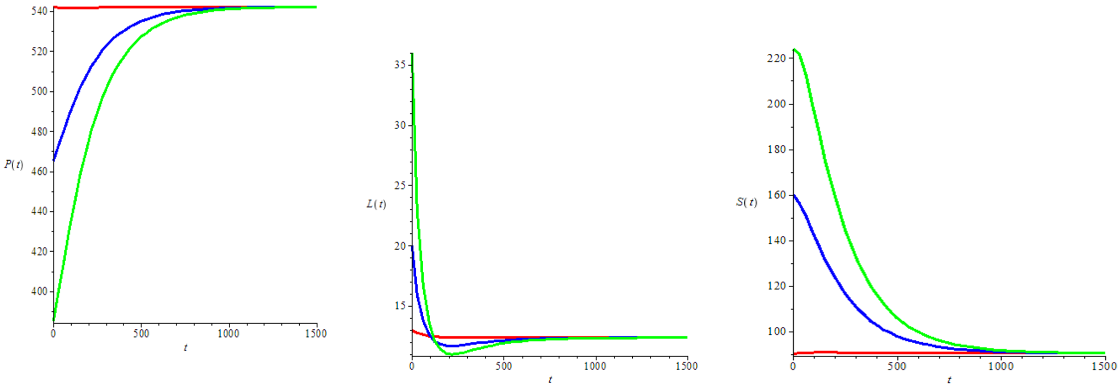
It will be simulated for the condition that there are individuals who are affected by smoking behavior by using table 1 but changing the parameter value γ to $\gamma = 0{,}03$ in order to obtain the endemic point value of smokers, namely $e_0(542, 13, 90)$. In the simulation of the endemic fixed point of smokers that is influenced by migration factors with the dynamics of the square root under relapse conditions, three initial values are used as follows:

$$P(0) = 542,\ L(0) = 13,\ S(0) = 90$$

$$P(0) = 465,\ L(0) = 20,\ S(0) = 160$$

$$P(0) = 385,\ L(0) = 36,\ S(0) = 224$$

Based on the parameter values and the initial values above, the graph of each group against time t is ob



Graph *P(t)* Graph *L(t)* Graph *S(t)*

Figure 4. Trajectories Around Smoker Endemic Points With β = 0.004

Based on Figure 4, the red curve represents the endemic fixed point of smokers which is influenced by migration factors with square root dynamics under relapse conditions, while the blue and green curves are curves with different initial values. Note that the fixed point $e_* = (P^*, L^*, S^*)$ is a fixed point that is asymptotically stable because the trajectory (blue and green curves) of each graph moves closer to the endemic fixed point shown by the red curve. Fixed point $e_* = (P^*, L^*, S^*)$ which is stable. It can be seen that the smaller the parameter value γ, the greater the number of smokers, this means that the smaller the rate of reduction of light smokers, the higher the number of smokers. Both light smokers and heavy smokers.

(c) Interpretation of the Mathematical Model of the Number of Smokers

Based on the results of the analysis that has been carried out, it can be seen that the interaction between potential smokers and light smokers can affect the number of light smokers and heavy smokers. The higher the rate of reduction in potential smokers, the higher the number of light smokers and heavy smokers. This is one thing that makes the number of smokers increase, because the interaction between potential smokers and light smokers is difficult to eliminate, so reducing the interaction between potential smokers and light smokers can reduce the number of individual smokers and the number of heavy smokers.

In addition to always having interactions between potential smokers and light smokers, the factor that makes the number of smokers increase is because the rate of reduction in light smokers causes the number of smokers to increase, seen in simulations when the value of γ is changed to be smaller, the value for smokers increases. This means that the more the rate of reduction of light smokers, the smaller the number smoking population. And another factor that makes the number of smokers increase due to immigration in each population.

4. Conclusion

From the discussion that has been done, it is obtained a mathematical model of the number of smokers affected by the migration factor with the dynamics of the square root under relapse conditions in the form of a system of differential equations. There is one point remains endemic to smokers. Based on the simulation, it can be seen that there is always an interaction between people who have the potential to smoke with light smokers and someone who has quit smoking has the potential to return to smoking so that they return to smoking (relapse) causing the individual to never disappear or in other words, individual smokers will always be in the population. Reducing the interaction between potential smokers and light smokers can reduce the number of light smokers as well as the number of heavy smokers.

References

1. Anwar, Andi Awaliya, dkk. 2019. *Kawasan Tanpa Rokok Di Fasilitasi Umum*. Ponorogo: Uwais Inspirasi Indonesia.
2. Aswan. 2018. *Pembatasan Dinamika Merokok dengan Menggunakan Pendekatan Model Matematika.* Skripsi. Fakultas Sains dan Teknologi, Matematika, UIN Alauddin Makassar.
3. Fernandez, Noviarizal. 2019. *Ternyata Hanya 9,5 Persen Perokok yang Sukses Berhenti Merokok*. Diakses online 3 Juni 2020. https://lifestyle.bisnis.com/read/20190318/220/901266/ternyata-hanya-95-persen-perokok-yang-sukses-berhenti-merokok.
4. Infodatin. 2015. *Perilaku Merokok Masyarakat Indonesia*. Kementrian Republik Indonesia. Diakses online28 Oktober 2019. https://www.kemkes.go.id/download.php?file=download/pusdatin/infodatin/infodatin-hari-tanpa-tembakau-sedunia.pdf.
5. Soleh, Mohammad & Delli Sazmita. 2017. *Model Matematika Jumlah Perokok dengan Dinamika Akar Kuadrat dan Faktor Migrasi.* UIN Sultan Syarif Kasim Riau.
6. Wahyuni, Esti, M. Subhan, & Media Rosha. 2018. *Model Matematika Penanggulangan Pencemaran Udara dengan Penanaman Pohon*. UNP Journal of Mathematics, vol 1.