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Author for correspondence:
Khairul Azmi
e-mail: khairul_azmi@stkipadzkia.ac.id

The Classical Ramsey Number R(3,4)>8 Using the Weighted Graph

Khairul Azmi[1], Elva[2] and Widia[3]

[1, 2, 3] Universitas Adzkia, Padang, Indonesia

Abstract- At present, research on Ramsey Numbers has expanded to a wider scope, not only between 2 complete graphs that are complementary to each other but also a combination of complete graphs, circle graphs, star graphs, wheel graphs, and others. While the classic Ramsey number still leaves problems that need to be solved. Ramsey number R(3,4) > 8. This means that m=8 is the largest integer such that K (8) which contains components of a red graph G and a complete blue graph G which is still possible not to get K_3 in graph G and not get a blue K_4 in graph G. The graph K_(8) has a total of 28 edges. There are as many as the combination (28,3) red edge pairs that need to be avoided so as not to get any red K_3 edge pairs. And there are as many as the combination (28,4) edge pairs blue. That needs to be avoided in order not to get a pair of blue K_4 edges. Determining the coloring of the graph directly is certainly very difficult, especially if the Ramsey number is getting bigger. It's like looking for a needle in a haystack. Need to use a special method in order to solve this problem. The weighting graph method, where each edge is given weight with a certain value, is able to solve this problem. The weighting graph method is able to display the graph K_8 in the form of a G matrix with the order of 8×8.

1. Introduction

The Ramsey number was first developed by Frank Ramsey in 1928. The concept of the Ramsey number was inspired by the question "How many people are at least invited to a party so there may be 3 people who know each other or 3 people who don't know each other?". Then the question is solved in the graph. What is the n smallest natural number of K_n such that K_3 or graph must be formed? complement (K_3) which is isomorphic.

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To make it easier to determine the value of n, there is another question that needs to be solved "How many people (n_1) are invited guests at a party at most, so it is possible that there are no 3 people who know each other and no 3 people who don't know each other? " To answer this question, the value of the largest natural number n_1 is determined, so that a graph K_(n_1) is formed which can be further divided into 2 graphs, call them graphs G and H. Graph G does not contain K_3 and isomorphic K_3. The graph H which is a complement of G also does not contain K_3 and isomorphic K_3. After the value of n_1 is obtained, in the end, the value of n=n_1+1 is obtained.
After going through a series of studies, it turns out that the value of n_1=5. This means that it takes a maximum of 5 guests so that there may be no 3 guests who know each other and no 3 people who do not know each other. After the value of n_1 = 5, the value of n = 6 is obtained, which means it takes 6 people at least guests so that it can be ascertained that 3 people know each other or 3 people who don't know each other.

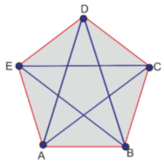


Figure 1. Graph of K_5 (Picture source: wikipedia.com)

In Figure 1 is a graph K_5 which consists of a red graph G and a blue graph H. The graph G obtained is a star graph S_5 and the graph H formed is a circle graph C_5. Both Graph S_5 and Graph C_5 do not form components of graph K_3 and isomorphic K_3. In the end, we get the Ramsey number r(3,3)=6.
Subsequent developments Ramsey numbers continue to be developed to solve other cases that are larger. Research continues to be fast to determine the value of the Ramsey number of 2 complete graphs that are complementary to each other.
At present, research on Ramsey numbers has expanded to a wider scope, not only between two complementary complete graphs, but also a combination of complete graphs, circle graphs, star graphs, wheel graphs, and so on. A Ramsey Number consisting of 2 complete graphs that are complementary to each other is called a classical Ramsey Number.

We present data on finding the Ramsey number between 2 complete graphs that are complementary to each other.

r \ s	1	2	3	4	5	6	7	8	9	10
1	**1**	1	1	1	1	1	1	1	1	1
2		**2**	3	4	5	6	7	8	9	10
3			**6**	9	14	18	23	28	36	40–42
4				**18**	25[10]	36–41	49–61	59[14]–84	73–115	92–149
5					**43–48**	58–87	80–143	101–216	133–316	149[14]–442
6						**102–165**	115[14]–298	134[14]–495	183–780	204–1171
7							**205–540**	217–1031	252–1713	292–2826
8								**282–1870**	329–3583	343–6090
9									**565–6588**	581–12677
10										**798–23556**

Figure 2. the Ramsey number between 2 complete graphs (Picture source: wikipedia.com)

In Figure 2, it can be seen that there are still many classical Ramsey number values for which the certainty value is unknown. This is a challenge in itself to solve the case of the classic Ramsey number. Therefore, it is interesting to study determining the classical Ramsey number and making its graph.

2. Results and Discussion

Figure 2 shows the Ramsey number R(3,4)=9. This means that K_8 is the largest complete graph that can be made so that it is possible not to form a red K_(3) graph and not to form a blue K_4 graph. The graph of K_8 is presented as follows:

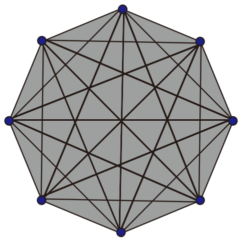


Figure 3. Graph of K_8 (Picture source: wikipedia.com)

In Figure 3, which edge is colored red and which edge is colored blue in such a way that it is possible that the graph of K_ (3) red is not formed and the graph of K_4 is not blue? If the staining is done. directly try and error, we will have trouble, and of course, it is not effective. Why is that?

Its graphs K_8 consists of 8 vertices, and each vertex has degree D(V)=7. So the total degree of all vertices is 56 vertices. Based on the handshake lemma where the sum of all the degrees of the vertices is equal to 2 times the number of edges, it is written:

$$\sum_{v\in V} d(v) = 2|E|$$

Then we get the number of edges on the graph E(K_8)=28. Since each edge is connected by 2 vertices, and there are 8 vertices, the total edge K_8 can also be calculated using the combination (8,3)=28.

If each edge is colored with 2 possible colors red and blue there are 2^28 possible colors. To get a pair of vertices to form a red graph K_(3) then the number of possibilities is (2^28¦3) pairs. Likewise, to get pairs of vertices to form a blue graph K_(4) then there are as many as (2^28¦4) pairs. This will certainly be more difficult to complete because of the many choices that will be combined. That's the reason coloring directly using the term try and error is not the right solution.

To make it easier to analyze the graph, K_8 can be analyzed more easily if you use a weighting method for each edge of the graph. For example, the weights for 2 adjacent vertices are 1, the two vertices bounded by one vertex are 2, the two vertices bounded by 2 adjacent vertices are 3, and so on, the data obtained are as follows:

Table 1. Graph Edge Weight

Bobot	Vertex 1st	Vertex 2nd	Vertex 3rd	Vertex 4rd	Vertex 5rd	Vertex 6rd	Vertex 7rd	Vertex 8rd
Vertex 1st	0	1	2	3	4	3	2	1
Vertex 2nd	1	0	1	2	3	4	3	2
Vertex 3rd	2	1	0	1	2	3	4	3
Vertex 4rd	3	2	1	0	1	2	3	4
Vertex 5rd	4	3	2	1	0	1	2	3
Vertex 6rd	3	4	3	2	1	0	1	2
Vertex 7rd	2	3	4	3	2	1	0	1
Vertex 8rd	1	2	3	4	3	2	1	0

The data can be transformed into a G matrix of order 8x8 as follows:

$$G = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 3 & 2 & 1 \\ 1 & 0 & 1 & 2 & 3 & 4 & 3 & 2 \\ 2 & 1 & 0 & 1 & 2 & 3 & 4 & 3 \\ 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\ 3 & 4 & 3 & 2 & 1 & 0 & 1 & 2 \\ 2 & 3 & 4 & 3 & 2 & 1 & 0 & 1 \\ 1 & 2 & 3 & 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

There is an interesting fact from matrix G that it turns out that the number of each row or column is 16. There are only 4 weight measures for Graph K_8, namely {1,2,3,4}. The value 0 is not counted because it is the value of the distance between the node and the node itself.

To a K_3 graph, it is possible to choose 3 specific entries in the matrix such that the number in the largest entry is the sum of the other 2 entries in the matrix. Meanwhile, to form a graph K_4 requires at least 4 pairs of graphs K_3 that are interconnected.

The G matrix can be calculated to determine the K_3 and K_4 pairs formed. The large size of the matrix will be difficult to select a pair of matrices manually. Need computational assistance to calculate it.

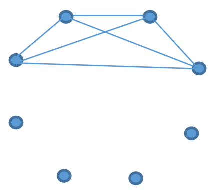
In order to be calculated manually, the matrix G can be transformed into a matrix G' whose entries are the magnitude of the edge weights in graph K_8.

$$G' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

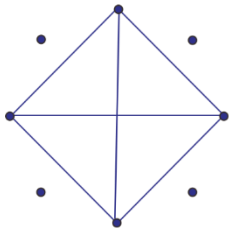
The red graph K_3 is obtained from the pairs of numbers (1,1,2), (1,2,3), and (2,2,4). In order not to get a red graph K_3 then the pair of numbers must be an edge weight pair for a blue graph K_4. Suppose M is the set for the pair of red edge weights and B is the set for the blue edge weight, so that Graph K_8 makes it possible not to get a red graph K_3 and not get a blue graph K_4 as follows:

1. $M = \{3,4\}\ and\ B = \{1,2\}$
2. $M = \{4\} and\ B = \{1,2,3\}$
3. $M = \{1,3\} and\ B = \{2,4\}$

At point 2, the set B = {1,2,3} where {1,2} ⊆ {1,2,3} is worth noting will obtain a blue graph of K_4 as in figure 4.a.



(a)



(b)

Figure 4 Graph of K_4

While point 3, the set B={2,4} allows the formation of a blue K_4 graph as shown in Figure 4.b. So the set pair that becomes the solution is point 1. At point 1 it is possible not to get red K_3 and not get blue K_4. The graph is as shown in Figure 5.

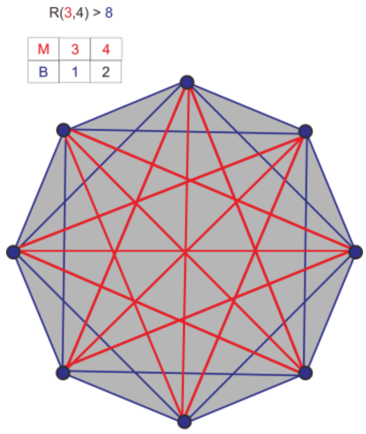


Figure 5 A graph K_8 which consists of a red graph G and a blue graph G

Figure 5 is a graph K_8 which consists of a red graph G and a blue graph G. It can be seen that the red graph G does not have a graph K_3 component and the blue graph G does not have a graph K_4 component. So it can be concluded that the largest integer value m is such that for K_m which has graph components G and G where graph G is still possible does not contain K_3 and graph G does not contain K_4 is 8. Write R(3,4)>8.

3. Conclusion

Here are some interesting facts found by using a weighting graph. The graph K_8 can be formed into an 8×8 matrix G. The number of each column or row is always the same, namely 16. The largest entry of the matrix is number 4. We get a new matrix G' with size 2×2. The entries in the matrix G' are combined into 2 sets M and B. Set M is the set for the pair of red edge weights and set B is the set for the blue edge weights. With the combination of 2 matching sets, it is obtained that the elements of the appropriate settings are M={3,4} and B={1,2}. The elements of this set can be re-created with a graph of Graph K_8 which consists of a red graph G and a blue complement of G. The red component of Graph G does not form K_3 and the blue component of G does not form K_(4).

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