

Mathematical Reasoning of Prospective Mathematics Teachers in Solving Numeracy Problems

Agustin Fatmawati¹, Uke Ralmugiz^{2*}, Nur Hasanah Syarief³

^{1,2,3}Pendidikan Matematika, Universitas Muhammadiyah Kupang, Indonesia
^{*}ukeralmugiz@unmuhkupang.ac.id

Abstrak

Penalaran matematis merupakan salah satu hal mendasar dalam memecahkan masalah matematis. Penelitian ini dilakukan bertujuan untuk mendeskripsikan penalaran matematis calon guru matematika dalam memecahkan masalah numerasi pada level pemahaman, level penerapan, dan level penalaran. Soal numerasi diberikan kepada subjek penelitian kemudian dilakukan wawancara untuk menkonfirmasi pemecahan masalah yang diberikan. Keenam subjek penelitian diambil berdasarkan hasil jawaban benar, mendekati benar dan menggunakan pendekatan yang unik. Teknik analisis data yang digunakan mengikuti tahapan teknis analisis data meliputi kondensasi data, penyajian data, dan penarikan kesimpulan. Hasil penelitian menunjukkan penalaran matematis keenam subjek pada level pemahaman memenuhi tahapan merumuskan, mengerjakan dan menafsirkan. Pada level penerapan, penalaran matematis dua subjek memenuhi tahapan merumuskan, menerapkan dan mengevaluasi. Sedangkan keempat subjek lainnya memenuhi tahapan merumuskan. Pada level penalaran, penalaran matematis dua subjek memenuhi tahapan merumuskan, menerapkan dan mengevaluasi.

Kata kunci: Literasi Numerasi, Pemecahan Masalah, Penalaran Matematis.

Abstract

Mathematical reasoning is one of the fundamental things in solving mathematical problems. This study aims to describe the mathematical reasoning of prospective mathematics teachers in solving numeracy problems at the level of understanding, level of application, and level of reasoning. Numeracy questions are given to the research subjects and then an interview is conducted to confirm the problem solving given. The six subjects of the study were taken on the basis of the results of correct answers, close to correct and using a unique approach. The data analysis techniques used follow the technical stages of data analysis including data condensation, data presentation, and drawing conclusions. The results showed that the mathematical reasoning of the six subjects at the level of understanding met the stages of formulating, working on and interpreting. At the level of application, the mathematical reasoning of two subjects meets the stages of formulating, applying and evaluating. While the other four subjects meet the stages of formulating. At the level of reasoning, the mathematical reasoning of two subjects meets the stages of formulating, applying and evaluating.

Keywords: Numeracy Literacy, Problem Solving, Mathematical Reasoning.

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INTRODUCTION

Mathematical reasoning is a thing of great consequence in solving mathematical problems, including solving numeracy literacy problems (Kusumawardani et al., [2018](#)). Efforts that can be made to improve mathematical reasoning are applying a problem-based learning model (Sumartini, [2015](#)), using a realistic mathematical approach (Saleh et al., [2018](#)), and doing mathematical tasks that can stimulate mathematical reasoning (Kusaeri et al., [2022](#)). Research on mathematical reasoning has been carried out and shows that a person's mathematical reasoning tends to be imitative reasoning, specifically using routine procedures in solving problems (Sukirwan et al., [2018](#)). Zuhri & Purwosetiyono ([2019](#)) examines the mathematical reasoning of prospective mathematics teachers in solving problems based on Polya's solving stages. Researchers have an interest in the study of mathematical reasoning that uses non-routine problems, namely using numeracy problems. Compared to previous research, the focus of this research is on mathematical reasoning starting from the level of understanding, the level of application, and the level of reasoning itself. It is important to identify how mathematical reasoning is in solving numeracy problems at each level. Based on the explanation above, the formulation of the problem in this study is how is the mathematical reasoning of prospective mathematics teachers in solving numeracy problems at the level of understanding, level of application, and level of reasoning?

Reason etymologically means sight or thinking. Linguistically, reason is defined as an activity that allows a person to think logically; a range of thought; the power of thought. According to the Indonesian Dictionary, the reasoning is how to use reason, how to think logically; mental process in developing the mind from some fact or principle. Based on this understanding, it can be interpreted that reasoning is an activity of thinking using logic/common sense from several facts or principles. In this paper, mathematical reasoning is limited to problems that have never been done by prospective mathematics teachers and are not routine questions. The reasoning described is mathematical reasoning that occurs at the level of understanding, level of application and level of reasoning. While the reasoning process studied is in the activities of formulating, applying and evaluating problems (OECD, [2022](#)).

The activity of formulating, applying and evaluating problems is needed when solving problems. This includes solving numeracy literacy problems. In Indonesia, the AKM (Asesmen Kompetensi Minimum) has been implemented. AKM is an assessment of the essential competencies needed by all students to develop their capacity and participate in

society (Pusmendik, [2022](#)). This assessment measures students' reading literacy and mathematical literacy (numbering). Not only students, teachers, and prospective teachers also need to understand reading literacy and numeracy literacy.

Numeration is a person's ability to formulate, use, and interpret mathematics in various contexts (OECD, [2022](#)). There are three numeration contexts: personal, socio-cultural, and scientific. Personal context is connected with activities of oneself, family, or peer groups. Socio-cultural context appropriated to community and culture. The scientific context is about the application of mathematics to the natural world, science, and technology.

There are four domains in numeracy: numbers, geometry, and measurement, algebra, data, and uncertainty (Burhan et al., [2022](#)). Cognitive complexity in Numeracy has three levels: understanding, application, and reasoning. Table 1 overview of the three levels.

Table 1. Cognitive Complexity in Solving Numerical Problems

Cognitive Level	Description
Understanding	Ability regarding facts, processes, concepts, procedures, applying mathematical concepts in everyday life and routine situations.
Applying	Ability to apply knowledge and conceptual understanding of facts, relations, processes, concepts, procedures, and methods in real-life contexts.
Reasoning	Ability to reason in analyzing data and information, making inferences, and expanding understanding in new situations, including previously unknown situations or more complex contexts. Questions can cover more than one approach or strategy

Sumber: (Susanto et al., [2021](#))

The process of solving problems in numeracy is emphasized the ability to reason as a core aspect of numeracy and is described as follows (OECD, [2022](#)):

Formulating

The term formulating in numeration is defined as an individual's ability to recognize and identify opportunities using mathematics and structure mathematics for problems presented in the contextual form. This stage starts by recognizing aspects of contextual problems that can be represented in the form of mathematics. The process of formulating includes the following activities:

- 1) Identify the mathematical aspects of a problem in a real-life context and identify significant variables.
- 2) Recognize the mathematical structure of a problem

- 3) Simplify the situation or problem so that it is easier to solve
- 4) Identify the assumptions and constraints behind problem-solving and simplification obtained from the context
- 5) Representing mathematical situations using symbols, diagrams, models, and variables appropriate
- 6) Represent the problem in different ways, including arranging it accordingly with mathematical concepts and making appropriate assumptions
- 7) Understand and explain the relationship between contextual language and symbolic language needed to represent it mathematically
- 8) Translating problems into language or mathematical representations
- 9) Recognizing aspects of the problem that correspond to mathematical concepts, facts, or procedures
- 10) Creating an orderly series of steps to solve the problem

Applying

The term applying in numeracy refers to an individual's ability to apply concepts, facts, procedures, and mathematical reasoning to solve problems formulated mathematically to obtain conclusions. Applying process includes activities:

- 1) Perform simple calculations
- 2) Draw simple conclusions
- 3) Choose the right strategy
- 4) Design and implement strategies to find mathematical solutions
- 5) Use mathematical tools including technology to help find solutions
- 6) Apply mathematical facts, rules, and structures
- 7) Processing numbers, data, graphic information, statistics, algebraic equations, and representations of geometric
- 8) Draw information from mathematical diagrams and graphs created
- 9) Making generalizations from mathematical procedures
- 10) Using and changing one representation to another

Interpreting and evaluating

The term interpretation (and evaluation) in numeracy focus on individual abilities to reflect mathematical solutions, results, or conclusions and interpret them in the context of real-life problems. The process of interpreting and evaluating includes the following activities:

- 1) Interpreting information presented in the form of graphs or diagrams
- 2) Evaluating mathematical results in context

- 3) Interpreting mathematical results back into real-life contexts
- 4) Evaluating the reasonableness of mathematical solutions in real-world contexts
- 5) Explaining why mathematical results or conclusions make sense or don't make sense according to the context of the problem

Criticize and identify the limitations of the model used to solve the problem

This research is a qualitative descriptive study. Qualitative descriptive research focuses on research related to how something happens and then it is studied in more depth. It uses an inductive approach (Yuliani, [2018](#)). The purpose of this study is to describe how the mathematical reasoning of prospective mathematics teachers in solving numeracy problems. Techniques test and non-test in the form of an interview. The research instrument used was a numeration test validation sheet, a test sheet containing adapted numeration questions consisting of three description questions consisting of one understanding level question, one application level question, and one reasoning level question. The question instrument was given to 32 research subjects who were randomly selected from a population of 120 people. All the answers obtained were then taken by six subjects with the consideration that the answers given had a unique way of solving the problem. These results were then confirmed using interviews.

Analysis of qualitative data according to the stages according to Miles ([2014](#)), namely data condensation, data presentation, and concluding. The activities carried out in each stage are:

- 1) At the data condensation stage, all research subjects' numeracy test results were compiled and analyzed according to the steps of problem-solving. After obtaining the data analysis, the results are grouped based on the answers given by subjects. Six data were selected because of having the correct answer or close to correct and a unique approach. It was considered based on mathematical reasoning performed by the research subjects. The results were then confirmed with the results of the interviews.
- 2) At the data presentation stage, research results were displayed in answers to research subjects, data interviews, and an analysis of the two results presented descriptively.
- 3) At the conclusion drawing stage, the results of the analysis and discussion are briefly concluded based on the research objectives

Applying

At this stage, the six subjects perform calculations and choose the right strategy to solve the problem. A simple calculation performed by the MRD subject when determining the area of the baking sheet is shown in the following

$$\begin{aligned} \text{Jawab :} \\ L &= P \times l = 20 \text{ cm} \times 20 \text{ cm} = 400 \text{ cm} \\ &= 10 \text{ cm} \times 20 \text{ cm} = 200 \text{ cm} \end{aligned}$$

Picture 2. Simple calculation of subject MRD

The subject chooses the right strategy to solve the problem. The simple conclusion obtained that each 20 cm x 10 cm baking pan needs x .

$$\begin{aligned} (*) \quad 400 &= 8 \text{ butir telur} \\ 200 &= x \\ x &= \frac{200 \times 8}{400} = 4 \text{ butir telur} \end{aligned}$$

Picture 3. Comparing the number of ingredients on subject MRD

The subjects chose the right strategy by comparing the area of the pan with the materials needed. **Interpreting and Evaluating**

The six subjects interpret and conclude appropriately. They return the calculation results to the context in which presented. The MRD subject gets an initial conclusion and then calculates the amount of material needed by multiplying each number of ingredients x by three as shown in the following figure:

Untuk membuat 1 bolu ukuran 10 x 20 membutuhkan
4 butir telur, 125 gram gula pasir, 125 gram tepung terigu, dan
100 gram Margarin

Jika ingin membuat 3 bolu ukuran 10 x 20 maka dibutuhkan

$$\begin{aligned} 4 \times 3 &= 12 \text{ butir telur} \\ 125 \times 3 &= 375 \text{ gram gulapasir} \\ 125 \times 3 &= 375 \text{ gram tepung terigu} \\ 100 \times 3 &= 300 \text{ gram Margarin} \end{aligned}$$

Picture 4. Interpreting and Evaluation Stage on Subject MRD

Subjects MRD interpreting and evaluating the results of the multiplication are the number of ingredients needed for three 20 cm x 10 cm baking pans.

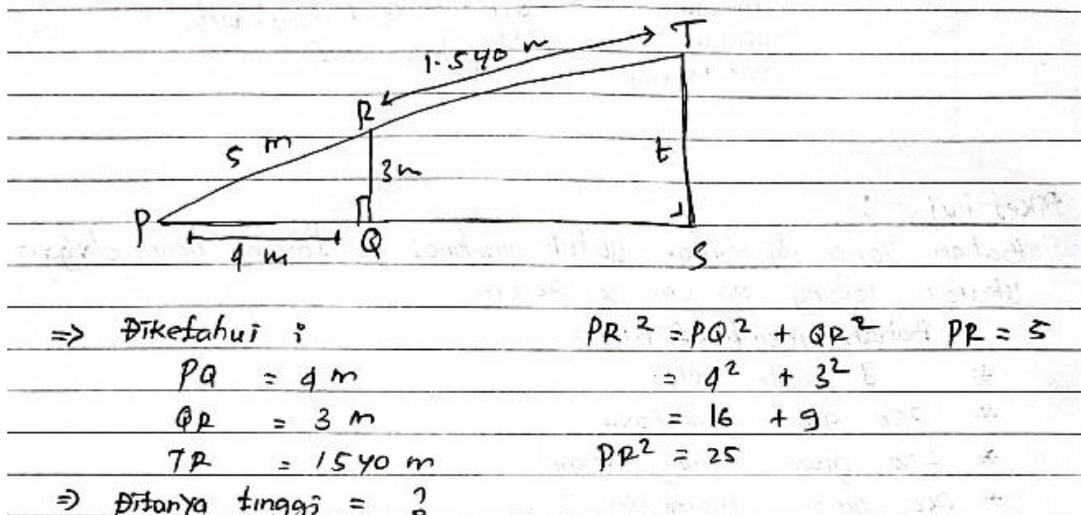
These three stages showed that the six subjects at the stage of formulating understood the questions asked and what they wrote. They explained the problem-solving process precisely. During interpretation and evaluation, they explained back the results obtained mathematically in the context of the problem presented.

Mathematical reasoning on literacy problems applying

At the application level, of the six subjects, two subjects answered correctly, namely the MIM subject and the AAN subject, and the other four subjects used the incorrect approach. The interviews showed that the four subjects understood the problems, performed calculations, and interpreted mathematical results, but the concepts used in solving the problems were still not quite right.

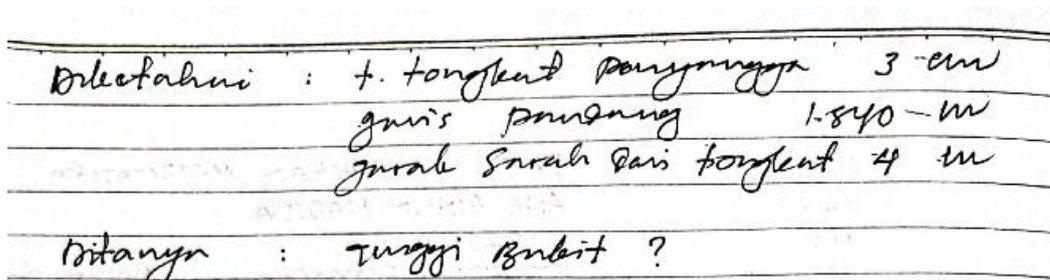
Formulating

At this stage, subject MIM makes assumptions by giving letters at each point where the two lines meet. The subject translates the problem into a mathematical representation and recognizes aspects of the problem that are by mathematical concepts such as the following:



Picture 5. Formulating Stage of Subject MIM

Meanwhile, for AAN, the formulating stage is indicated by identifying significant variables, namely the height of the crutches, line of sight, and Sarah's distance from the cane.



Picture 6. Formulating Stage of Subject AAN

Applying

MIM and AAN performed a simple calculation using the Pythagorean theorem to determine the distance between Sarah's eye position and the tip of the crutches.

$$\begin{aligned}
 PR^2 &= PQ^2 + QR^2 & PR &= 5 \\
 &= 4^2 + 3^2 \\
 &= 16 + 9 \\
 PR^2 &= 25
 \end{aligned}$$

Gunakan teorema Pythagoras
jarak pandang Sarah ke ujung tongkat
pengunyah adalah

$$\begin{aligned}
 \text{minim,} &= \sqrt{(4\text{ m})^2 + (3\text{ m})^2} \\
 &= \sqrt{16\text{ m}^2 + 9\text{ m}^2} \\
 &= \sqrt{25\text{ m}^2} \\
 &= 5\text{ m}
 \end{aligned}$$

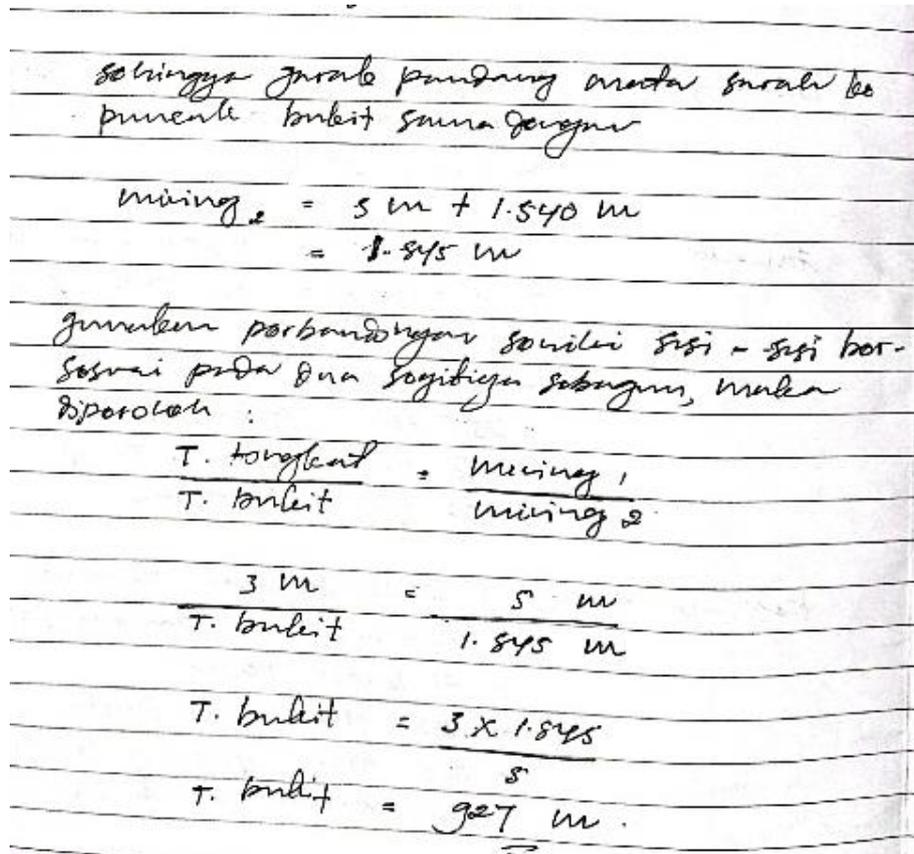
Picture 7. Calculation of Subject MIM and AAN

Both subjects also used the concept of worth comparison to estimate hill height. MIM subjects use assumptions according to the formulating stage.

$$\begin{aligned}
 \Rightarrow RQ &= PR \\
 TS &= PT \\
 3\text{ m} &= 5\text{ m} \\
 t &= 5 + 1540 \\
 5t &= 3(5 + 1540) \\
 5t &= 3 \times 1545 \\
 5t &= 4.635 \\
 t &= \frac{4.635}{5} \\
 t &= 927\text{ m}
 \end{aligned}$$

Picture 8. Applying Stage of Subject MIM

Subjects AAN calculate the distance between Sarah's eyes and the tip of the hill first and enter the calculation into an equivalent comparison.

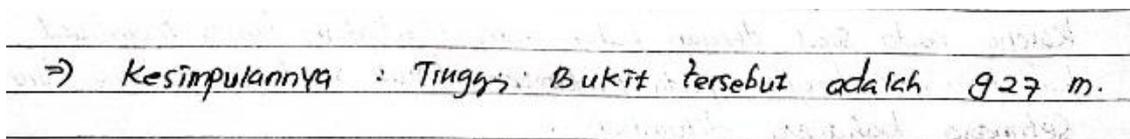


Picture 9. Applying Stage of Subject AAN

Both of them choose the right strategy to solve the problem. The subject processes the geometric representation of the similarity concept using an equivalent comparison. The small triangle is equal to the large triangle.

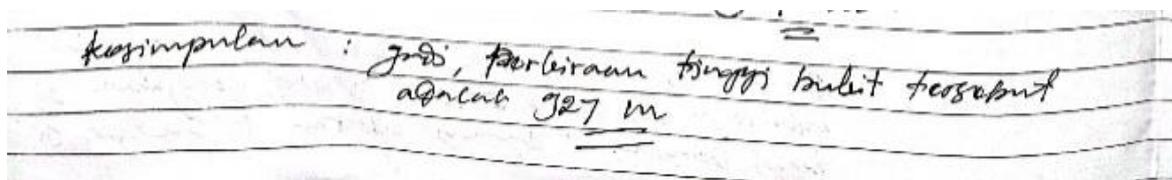
Interpretation and evaluation

At this stage, the two subjects interpret and evaluate the results of mathematical calculations in the context. MIM subjects interpret the calculation results as hill height.



Picture 10. Interpreting and Evaluation Stage of Subject MIM

The AAN subject interprets the results of the mathematical calculations as an estimate of hill height.



Picture 11. Interpreting and Evaluation Stage of Subject AAN

The interview shows that the two subjects at the stage of formulating the subject understand the problems presented and know what elements are needed to solve the problem. At the application, MIM and AAN subjects remember the concepts in the Pythagorean theorem to get the length of the slanted line in a right triangle. Meanwhile, at the stage of interpreting and evaluating, AAN were more confident when asked about their conclusions. The MIM subject corrected the conclusion that the results obtained were not the height of the hill but only an estimate from the calculation results. The other four subjects completed it. However, they realized that the approach used was not appropriate for the calculation and did not match the concept of congruence.

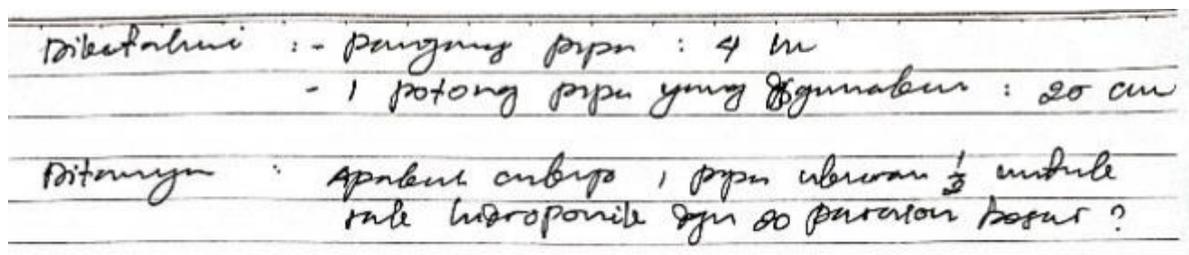
DISCUSSION

Mathematical reasoning on numeracy problems reasoning

At the reasoning level of the six subjects, AAN answered correctly, and MIM took the right approach. Both understand the problem presented and explain the results of the solution according to the design solution.

Formulating

At the stage of formulating, AAN identified significant variables, namely the length and the size of the pipe sold. The subject also understands and explains the relationship between the context language and the required symbolic language. The research subject determines the initial step needed to find the number of pipes for 20. The research subject represents the situation in a pipe circuit drawing model.



Picture 12. Formulating Stage of Subject AAN

Meanwhile, the MIM subject recognizes the variable, the number of pipes per 4 large pipes. The subject also determines the first step by finding the pipes needed for 20 large pipes

3. Diketahui :

Untuk membuat desain rak paralon diperlukan 3 jenis penghubung petalon yg berbentuk

 7 buah
 7 buah
 3 buah

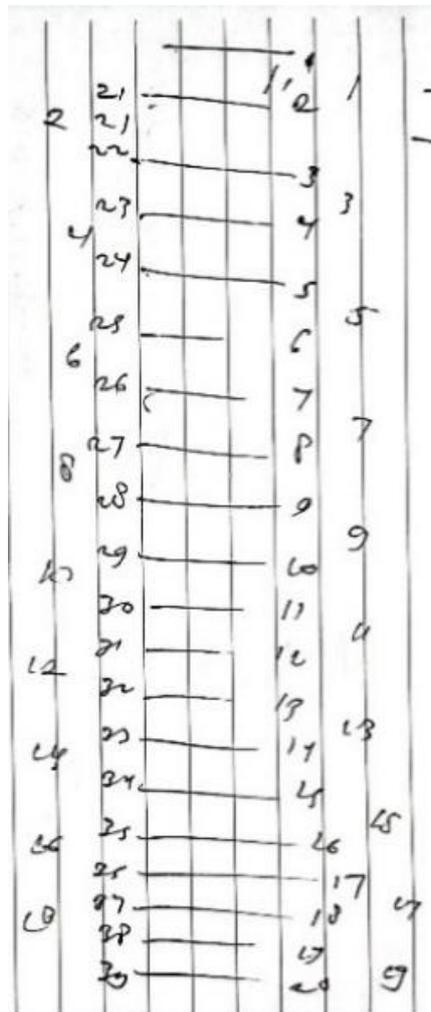
Ditanya :

Apakah cukup hanya membeli 1 pipa ukuran $\frac{1}{2}$ inch untuk rak hidroponik dengan 20 paralon besar?

Picture 13. Formulating Stage of Subject MIM

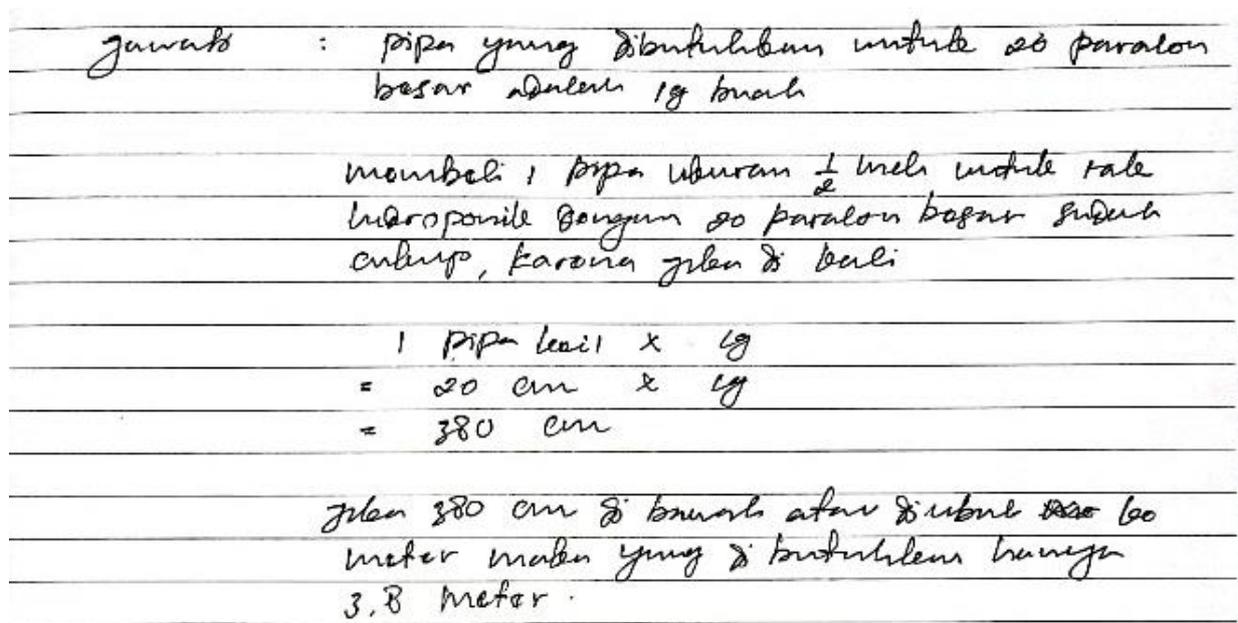
Applying

AAN and MIM subjects can process the information presented in the image to solve problems. AAN subjects choose the right strategy in determining the number of pipes needed. The method used is to describe 20 large paralon pipes and count the number of $\frac{1}{2}$ inch pipe.



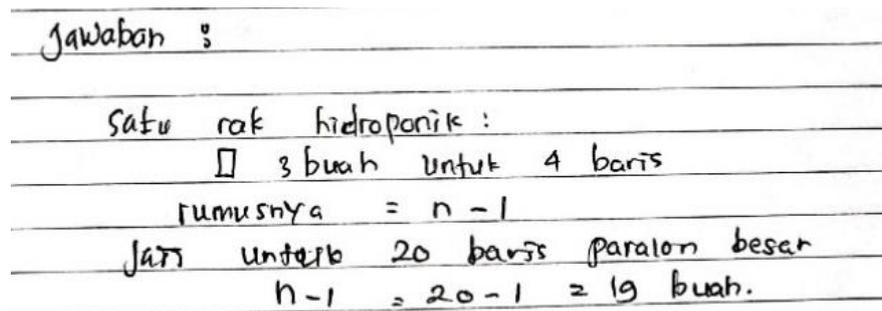
Picture 14. The Strategy Used by Subject AAN

Subjects perform simple calculations after knowing the number of pipes needed. They draw a simple conclusion from the results obtained, the overall length of the pipe is 380 cm.



Picture 15. Applying Stage of Subject AAN

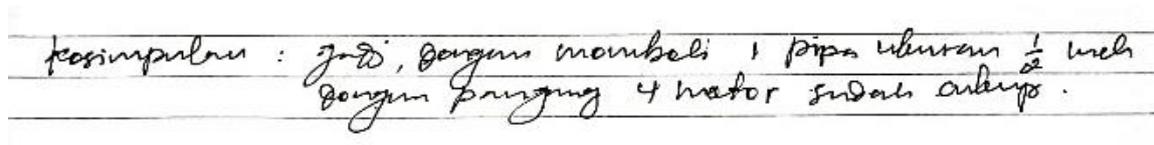
Subject MIM chooses the right strategy in determining the number of pipes needed. The subject makes generalizations using the equation $n - 1$ to determine the number of pipes.



Picture 16. Applying Stage of Subject MIM

Interpreting and evaluation

At this stage, the AAN subject interprets and evaluates the results of mathematical calculations in the context of hydroponic problems.



Picture 17. Interpreting and Evaluation of Subject AAN

While the MIM subject has not yet reached the stage of making conclusions, she can explain how the solution he provides will make sense to solve the problem. At the stage of

formulating, the interviews showed that the two subjects understood the problems presented. At applying, the MIM subject takes a long time to understand the problem. She read the questions repeatedly. She tried several strategies to solve the problem. She could not write conclusions on the answer sheet. Even so, at the stage of interpreting and evaluating the interview, the MIM subject showed that the subject understood the meaning of the mathematical answer and explained it in the hydroponic context. The others could not complete the level of reasoning due to difficulties in visualizing the picture of the hydroponic system.

The results of the explanation above can be a consideration when giving problems to prospective mathematics teachers. The problems given at least contain elements that can improve mathematical reasoning and are non-routine problems. It can also use HOTS so that they can practice the ability to use logic and reason (Burhan et al., [2022](#)). The interviews showed that four subjects had difficulty visualizing the problems given and could not predict the later pattern. It is in line with research showing that creativity in generating ideas and developing information obtained is still low (Aba et al., [2021](#)).

CONCLUSION

Mathematical reasoning of prospective teachers at the level of understanding, the level of application, and the level of reasoning includes formulating, applying, interpreting, and evaluating. At the level of understanding, all subjects can solve personal content numeracy literacy problems. Activities at the formulating stage include identifying significant variables and formulating mathematical situations using symbols. In the application of the subject's activities, namely doing calculations and drawing simple conclusions. Meanwhile, at the interpreting and evaluation stage, the subject's activities are evaluating mathematical results and interpreting mathematical results into the context of the problem.

At the application level, of the six subjects, two subjects can solve the problem correctly. Activities at the formulating stage include identifying significant variables, making appropriate assumptions, translating problems into mathematical representations, and recognizing aspects of the appropriate problems. Subject activities at the working stage include performing simple calculations, applying strategies to solve problems, and processing geometric representations. Subject activities at the interpreting and evaluation stage are evaluating mathematical results and interpreting the results into the context of the problem.

While at the level of reasoning, one subject completes correctly, and one subject uses the appropriate approach. The activities at the formulating stage are identifying significant variables, understanding and explaining the relationship between contextual language and symbolic language, and representing the situation into a model. Activities at the working stage include processing the information presented in the image to solve problems, perform calculations, draw simple conclusions, and make generalizations.

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