

Figure 6. Antiprism Graph A_n

Theorem 2 For arbitrary positive integer $n \geq 3$, the cartesian product $P_2 \times A_n$ is edge odd gracefulful.

Proof. Let the vertices and the edges of $P_2 \times A_n$ be

$$V(P_2 \times A_n) = \{a_i, b_i, A_i, B_i | i = 1, 2, \dots, n\}$$

and

$$\begin{aligned} E(P_2 \times A_n) = & \{a_i a_{(i+1) \bmod n} | i = 1, 2, \dots, n\} \cup \{b_i b_{(i+1) \bmod n} | i = 1, 2, \dots, n\} \cup \\ & \{a_i b_i, a_i b_{(i+1) \bmod n} | i = 1, 2, \dots, n\} \cup \{A_i A_{(i+1) \bmod n} | i = 1, 2, \dots, n\} \cup \\ & \{B_i B_{(i+1) \bmod n} | i = 1, 2, \dots, n\} \cup \{A_i B_i, A_i B_{(i+1) \bmod n} | i = 1, 2, \dots, n\} \cup \\ & \{a_i A_i, b_i B_i | i = 1, 2, \dots, n\}. \end{aligned}$$

It is clear that $|V(P_2 \times A_n)| = 4n$ and $|E(P_2 \times A_n)| = 10n$ and hence we have $k = \max\{|V(P_2 \times A_n)|, |E(P_2 \times A_n)|\} = 10n$. Now we construct an edge odd labeling

$$f_2 : E(P_2 \times A_n) \rightarrow \{1, 2, \dots, 20n - 1\}$$

by

$$\begin{aligned}
 f_2(a_i a_{(i+1) \bmod n}) &= 2n - 2i + 1 \\
 f_2(b_i b_{(i+1) \bmod n}) &= 8n - 2i + 1 \\
 f_2(a_i b_{(i+1) \bmod n}) &= 14n + 2i - 1 \\
 f_2(a_i b_i) &= 8n + 2i - 1 \\
 f_2(A_i A_{(i+1) \bmod n}) &= 4n + 2i - 1 \\
 f_2(B_i B_{(i+1) \bmod n}) &= 12n - 2i + 1 \\
 f_2(A_i B_{(i+1) \bmod n}) &= 18n + 2i - 1 \\
 f_2(A_i B_i) &= 14n - 2i + 1 \\
 f_2(a_i A_i) &= 4n - 2i + 1 \\
 f_2(b_i B_i) &= 16n + 2i - 1
 \end{aligned}$$

for all $i = 1, 2, \dots, n$. It is clear that

$$\begin{aligned}
 \{f_2(a_i a_{(i+1) \bmod n}) | i = 1, 2, \dots, n\} &= \{1, 3, \dots, 2n - 1\} \\
 \{f_2(b_i b_{(i+1) \bmod n}) | i = 1, 2, \dots, n\} &= \{6n + 1, 6n + 3, \dots, 8n - 1\} \\
 \{f_2(a_i b_{(i+1) \bmod n}) | i = 1, 2, \dots, n\} &= \{14n + 1, 14n + 3, \dots, 16n - 1\} \\
 \{f_2(a_i b_i) | i = 1, 2, \dots, n\} &= \{8n + 1, 8n + 3, \dots, 10n - 1\} \\
 \{f_2(A_i A_{(i+1) \bmod n}) | i = 1, 2, \dots, n\} &= \{4n + 1, 4n + 3, \dots, 6n - 1\} \\
 \{f_2(B_i B_{(i+1) \bmod n}) | i = 1, 2, \dots, n\} &= \{10n + 1, 10n + 3, \dots, 12n - 1\} \\
 \{f_2(A_i B_{(i+1) \bmod n}) | i = 1, 2, \dots, n\} &= \{18n + 1, 18n + 3, \dots, 20n - 1\} \\
 \{f_2(A_i B_i) | i = 1, 2, \dots, n\} &= \{12n + 1, 12n + 3, \dots, 14n - 1\} \\
 \{f_2(a_i A_i) | i = 1, 2, \dots, n\} &= \{2n + 1, 2n + 3, \dots, 4n - 1\} \\
 \{f_2(b_i B_i) | i = 1, 2, \dots, n\} &= \{16n + 1, 16n + 3, \dots, 18n - 1\}
 \end{aligned}$$

confirming that f_2 is a bijection. Furthermore we then obtain the following vertex weights $\bmod 2k$ for $2k = 20n$ under the labeling f_2 :

$$\begin{aligned}
 wt_{f_3}(a_1) &= 8n + 1 \\
 wt_{f_3}(a_i) &= 10n - 2i + 3 && \text{for } i = 2, 3, \dots, n \\
 wt_{f_3}(b_1) &= 14n + 1 \\
 wt_{f_3}(b_i) &= 14n + 2i - 1 && \text{for } i = 2, 2, \dots, n \\
 wt_{f_3}(A_1) &= 6n - 1 \\
 wt_{f_3}(A_i) &= 4n + 2i - 3 && \text{for } i = 2, 3, \dots, n \\
 wt_{f_3}(B_1) &= 12n - 1 \\
 wt_{f_3}(B_i) &= 12n - 2i + 1 && \text{for } i = 2, 3, \dots, n.
 \end{aligned}$$

It is easy to check that

$$\begin{aligned}
 \{wt_{f_3}(a_1)\} &= \{8n + 1\} \\
 \{wt_{f_3}(a_i) | i = 2, 3, \dots, n\} &= \{8n + 3, 8n + 5, \dots, 10n - 1\} \\
 \{wt_{f_3}(b_1)\} &= \{14n + 1\} \\
 \{wt_{f_3}(b_i) | i = 2, 3, \dots, n\} &= \{14n + 3, 14n + 5, \dots, 16n - 1\} \\
 \{wt_{f_3}(A_1)\} &= 6n - 1 \\
 \{wt_{f_3}(A_i) | i = 2, 3, \dots, n\} &= \{4n + 1, 4n + 3, \dots, 6n - 3\} \\
 \{wt_{f_3}(B_1)\} &= 12n - 1 \\
 \{wt_{f_3}(B_i) | i = 2, 3, \dots, n\} &= \{10n + 1, 10n + 3, \dots, 12n - 3\}
 \end{aligned}$$

showing that the weights are all distinct. Hence f_2 is an edge odd graceful on $P_2 \times A_n$ and therefore $P_2 \times A_n$ is an edge odd graceful. \square

In Figure 7., we give an illustration of an edge odd labeling on $P_2 \times A_8$. We separate the graph into several part to make the labeling look clear.

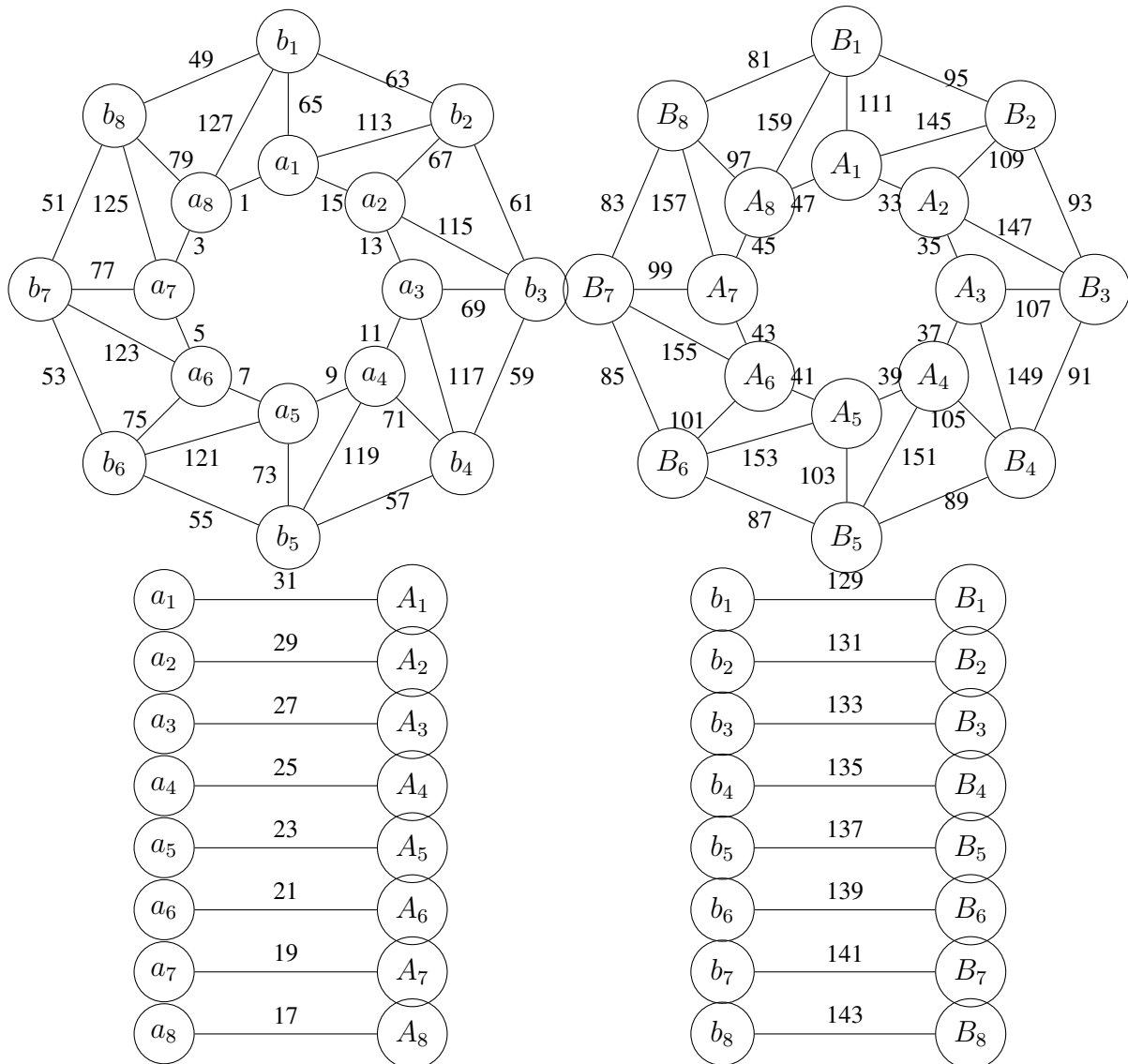


Figure 7. Edge Odd Graceful Labeling on Cartesian Product $P_2 \times A_8$

From the Figure 7., we obtain vertex weights *mod* 160 respect to labeling f_2 as follows:

$wt_{f_3}(a_1) = 65$	$wt_{f_3}(b_1) = 113$	$wt_{f_3}(A_1) = 47$	$wt_{f_3}(B_1) = 95$
$wt_{f_3}(a_2) = 79$	$wt_{f_3}(b_2) = 115$	$wt_{f_3}(A_2) = 33$	$wt_{f_3}(B_2) = 93$
$wt_{f_3}(a_3) = 77$	$wt_{f_3}(b_3) = 117$	$wt_{f_3}(A_3) = 35$	$wt_{f_3}(B_3) = 91$
$wt_{f_3}(a_4) = 75$	$wt_{f_3}(b_4) = 119$	$wt_{f_3}(A_4) = 37$	$wt_{f_3}(B_4) = 89$
$wt_{f_3}(a_5) = 73$	$wt_{f_3}(b_5) = 121$	$wt_{f_3}(A_5) = 39$	$wt_{f_3}(B_5) = 87$
$wt_{f_3}(a_6) = 71$	$wt_{f_3}(b_6) = 123$	$wt_{f_3}(A_6) = 41$	$wt_{f_3}(B_6) = 85$
$wt_{f_3}(a_7) = 69$	$wt_{f_3}(b_7) = 125$	$wt_{f_3}(A_7) = 43$	$wt_{f_3}(B_7) = 83$
$wt_{f_3}(a_8) = 67$	$wt_{f_3}(b_8) = 127$	$wt_{f_3}(A_8) = 45$	$wt_{f_3}(B_8) = 81.$

3.3. Cartesian Product of Path P_2 and Double Sun Flower DSF_n

Definition 1 A sun flower graph of order $2n$, denoted by SF_n , is a graph that is isomorphic to a graph obtained by deleting edges of the outer cycle from antiprism graph A_n (see FIGURE 8.).

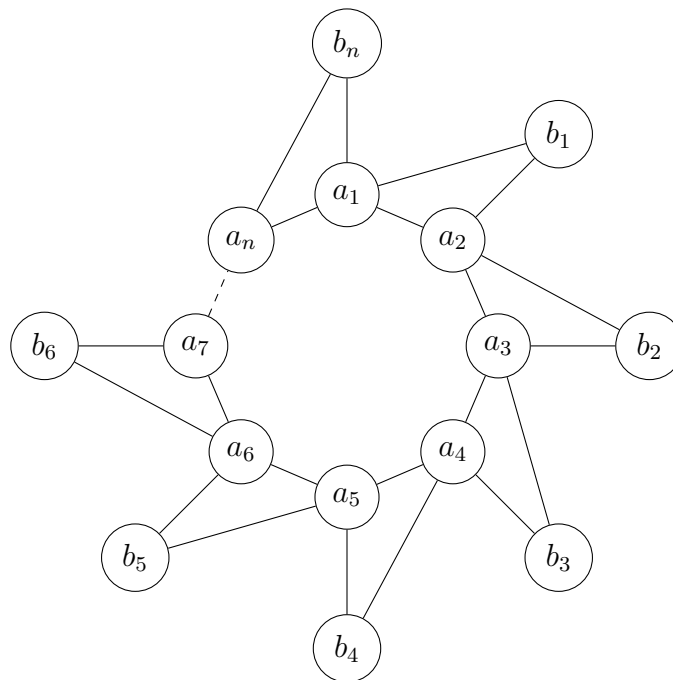


Figure 8. Sun Flower Graph SF_n

The vertex set and the edge set of SF_n of order $2n$, are, respectively

$$V(SF_n) = \{a_i, b_i | i = 1, 2, \dots, n\}$$

and

$$E(SF_n) = \{a_i a_{(i+1) \bmod n}, a_i b_i, a_{(i+1) \bmod n} b_i | i = 1, 2, \dots, n\}.$$

Thus we have $|V(SF_n)| = 2n$ and $|E(SF_n)| = 3n$.

Definition 2 By a double sun flower graph of order $3n$, denoted by DSF_n , is a graph obtained from the graph SF_n (see FIGURE 8.) by inserting a new vertex c_i on each edges $a_i a_{i+1}$ and adding edges $b_i c_i$ for each i . (See FIGURE 9.)

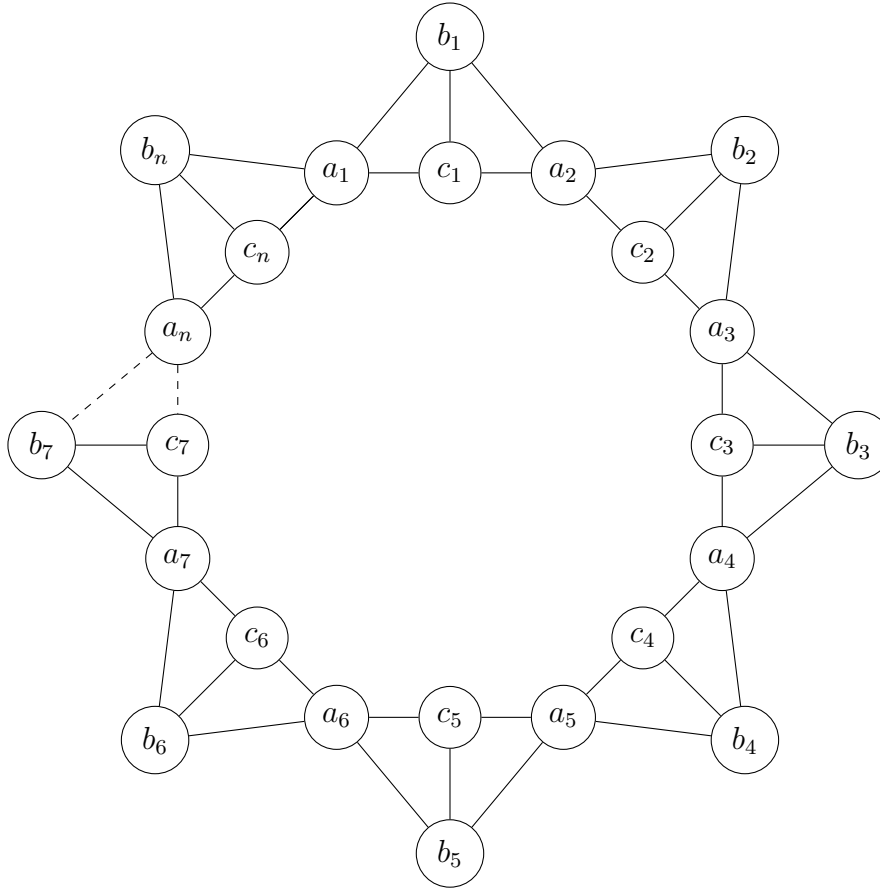


Figure 9. Double Sun Flower Graph DSF_n

Theorem 3 Let $n \geq 3$ be an arbitrary positive integer. Then the graph $P_2 \times DSF_n$ is edge odd graceful.

Proof. Let the vertex set and the edge set of $P_2 \times DSF_n$ be

$$V(P_2 \times DSF_n) = \{a_i, b_i, c_i, A_i, B_i, C_i | i = 1, 2, \dots, n\}$$

and

$$E(P_2 \times DSF_n) = \{a_i b_i, a_i c_i, b_i c_i, b_i a_{(i+1) \bmod n}, c_i a_{(i+1) \bmod n} | i = 1, 2, \dots, n\} \cup \\ \{A_i B_i, A_i C_i, B_i C_i | i = 1, 2, \dots, n\} \cup \\ \{B_i A_{(i+1) \bmod n}, C_i A_{(i+1) \bmod n}, a_i A_i, b_i B_i, c_i C_i | i = 1, 2, \dots, n\}.$$

It is clear that

$$|V(P_2 \times DSF_n)| = 6n$$

and

$$|E(P_2 \times DSF_n)| = 13n.$$

We construct an edge labeling $f_3 : E(P_2 \times DSF_n) \rightarrow \{1, 3, \dots, 26n - 1\}$ for two cases, whenever n is odd and n is even.

Case for n odd

We define the labeling as follows:

$$\begin{aligned} f_3(a_i b_i) &= 20n - 2i + 1 \\ f_3(a_i c_i) &= 16n - 2i + 1 \\ f_3(b_i c_i) &= 12n + 2i - 1 \\ f_3(b_i a_{(i+1) \bmod n}) &= 10n + 2i - 1 \\ f_3(c_i a_{(i+1) \bmod n}) &= 18n - 2i + 1 \\ f_3(A_i B_i) &= 10n - 2i + 1 \\ f_3(A_i C_i) &= 6n - 2i + 1 \\ f_3(B_i C_i) &= 22n + 2i - 1 \\ f_3(B_i A_{(i+1) \bmod n}) &= 2i - 1 \\ f_3(C_i A_{(i+1) \bmod n}) &= 8n - 2i + 1 \\ f_3(a_i A_i) &= 20n + 2i - 1 \\ f_3(b_i B_i) &= 2n + 2i - 1 \\ f_3(c_i C_i) &= 26n - 2i + 1 \end{aligned}$$

for all $i = 1, 2, \dots, n$. It is easy to see that

$$\begin{aligned} \{f_3(a_i b_i) | i = 1, 2, \dots, n\} &= \{18n + 1, 18n + 3, \dots, 20n - 1\} \\ \{f_3(a_i c_i) | i = 1, 2, \dots, n\} &= \{14n + 1, 14n + 3, \dots, 16n - 1\} \\ \{f_3(b_i c_i) | i = 1, 2, \dots, n\} &= \{12n + 1, 12n + 3, \dots, 14n - 1\} \\ \{f_3(b_i a_{(i+1) \bmod n}) | i = 1, 2, \dots, n\} &= \{10n + 1, 10n + 3, \dots, 12n - 1\} \\ \{f_3(c_i a_{(i+1) \bmod n}) | i = 1, 2, \dots, n\} &= \{16n + 1, 16n + 3, \dots, 18n - 1\} \\ \{f_3(A_i B_i) | i = 1, 2, \dots, n\} &= \{8n + 1, 8n + 3, \dots, 10n - 1\} \\ \{f_3(A_i C_i) | i = 1, 2, \dots, n\} &= \{4n + 1, 4n + 3, \dots, 6n - 1\} \\ \{f_3(B_i C_i) | i = 1, 2, \dots, n\} &= \{22n + 1, 22n + 3, \dots, 24n - 1\} \\ \{f_3(B_i A_{(i+1) \bmod n}) | i = 1, 2, \dots, n\} &= \{1, 3, \dots, 2n - 1\} \\ \{f_3(C_i A_{(i+1) \bmod n}) | i = 1, 2, \dots, n\} &= \{6n + 1, 6n + 3, \dots, 8n - 1\} \\ \{f_3(a_i A_i) | i = 1, 2, \dots, n\} &= \{20n + 1, 20n + 3, \dots, 22n - 1\} \\ \{f_3(b_i B_i) | i = 1, 2, \dots, n\} &= \{2n + 1, 2n + 3, \dots, 4n - 1\} \\ \{f_3(c_i C_i) | i = 1, 2, \dots, n\} &= \{24n + 1, 24n + 3, \dots, 26n - 1\}. \end{aligned}$$

Thus f_3 is a bijection. Moreover, we then have vertex weights $\bmod 2k$, where $k = \max\{|V(P_2 \times DSF_n)|, |E(P_2 \times DSF_n)|\} = \max\{5n, 13n\} = 13n$ as follows:

$$\begin{array}{lll}
 wt_{f_3}(a_1) = 6n - 1 & wt_{f_3}(A_1) = 18n - 1 & \\
 wt_{f_3}(a_i) = 6n - 2i + 1 & wt_{f_3}(A_i) = 18n - 2i + 1 & \text{for } i = 2, 3, \dots, n \\
 wt_{f_3}(b_i) = 18n + 4i - 2 & wt_{f_3}(B_i) = 8n + 4i - 2 & \text{for } i = 1, 2, \dots, n \\
 wt_{f_3}(c_i) = 20n - 4i + 2 & wt_{f_3}(C_i) = 10n - 4i + 2 & \text{for } i = 1, 2, \dots, n
 \end{array}$$

so that

$$\begin{array}{ll}
 \{wt_{f_3}(a_1)\} & = \{6n - 1\} \\
 \{wt_{f_3}(a_i) | i = 2, 3, \dots, n\} & = \{4n + 1, 4n + 3, \dots, 6n - 3\} \\
 \{wt_{f_3}(b_i) | i = 1, 2, \dots, n\} & = \{18n + 2, 18n + 6, \dots, 22n - 1\} \\
 \{wt_{f_3}(c_i) | i = 1, 2, \dots, n\} & = \{16n + 2, 16n + 6, \dots, 20n - 2\} \\
 \{wt_{f_3}(A_1)\} & = \{18n - 1\} \\
 \{wt_{f_3}(A_i) | i = 2, 3, \dots, n\} & = \{16n + 1, 16n + 3, \dots, 18n - 3\} \\
 \{wt_{f_3}(B_i) | i = 1, 2, \dots, n\} & = \{8n + 2, 8n + 6, \dots, 12n - 2\} \\
 \{wt_{f_3}(C_i) | i = 1, 2, \dots, n\} & = \{6n + 2, n + 6, \dots, 10n - 2\}.
 \end{array}$$

It is easy to check that for arbitrary odd number $n \geq 3$, all the weights are different. Therefore f_3 is an edge odd graceful labeling. Thus, for $n \geq 3$ odd, $P_2 \times DSF_n$ is edge odd graceful.

Case for n even

For $n \geq 3$ even we define the labeling as follows:

$$\begin{array}{ll}
 f_3(a_i b_i) & = 16n - 2i + 1 \\
 f_3(a_i c_i) & = 20n - 2i + 1 \\
 f_3(b_i c_i) & = 2i - 1 \\
 f_3(b_i a_{(i+1) \bmod n}) & = 10n + 2i - 1 \\
 f_3(c_i a_{(i+1) \bmod n}) & = 16n - 2i + 1 \\
 f_3(A_i B_i) & = 8n - 2i + 1 \\
 f_3(A_i C_i) & = 10n - 2i + 1 \\
 f_3(B_i C_i) & = 12n + 2i - 1 \\
 f_3(B_i A_{(i+1) \bmod n}) & = 22n + 2i - 1 \\
 f_3(C_i A_{(i+1) \bmod n}) & = 18n - 2i + 1 \\
 f_3(a_i A_i) & = 20n + 2i - 1 \\
 f_3(b_i B_i) & = 2n + 2i - 1 \\
 f_3(c_i C_i) & = 26n - 2i + 1
 \end{array}$$

for all $i = 1, 2, \dots, n$. We obtain

$$\begin{aligned}
 \{f_3(a_i b_i) | i = 1, 2, \dots, n\} &= \{14n - 1, 14n + 1, \dots, 16n - 1\} \\
 \{f_3(a_i c_i) | i = 1, 2, \dots, n\} &= \{18n + 1, 18n + 3, \dots, 20n - 1\} \\
 \{f_3(b_i c_i) | i = 1, 2, \dots, n\} &= \{1, 3, \dots, 2n - 1\} \\
 \{f_3(b_i a_{(i+1) \bmod n}) | i = 1, 2, \dots, n\} &= \{10n + 1, 10n + 3, \dots, 12n - 1\} \\
 \{f_3(c_i a_{(i+1) \bmod n}) | i = 1, 2, \dots, n\} &= \{14n + 1, 14n + 3, \dots, 16n - 1\} \\
 \{f_3(A_i B_i) | i = 1, 2, \dots, n\} &= \{6n + 1, 6n + 3, \dots, 8n - 1\} \\
 \{f_3(A_i C_i) | i = 1, 2, \dots, n\} &= \{8n + 1, 8n + 3, \dots, 10n - 1\} \\
 \{f_3(B_i C_i) | i = 1, 2, \dots, n\} &= \{12n + 1, 12n + 3, \dots, 14n - 1\} \\
 \{f_3(B_i A_{(i+1) \bmod n}) | i = 1, 2, \dots, n\} &= \{22n + 1, 22n + 3, \dots, 24n - 1\} \\
 \{f_3(C_i A_{(i+1) \bmod n}) | i = 1, 2, \dots, n\} &= \{16n + 1, 16n + 3, \dots, 18n - 1\} \\
 \{f_3(a_i A_i) | i = 1, 2, \dots, n\} &= \{20n + 1, 20n + 3, \dots, 22n - 1\} \\
 \{f_3(b_i B_i) | i = 1, 2, \dots, n\} &= \{2n + 1, 2n + 3, \dots, 4n - 1\} \\
 \{f_3(c_i C_i) | i = 1, 2, \dots, n\} &= \{24n + 1, 24n + 3, \dots, 26n - 1\}.
 \end{aligned}$$

We then have vertex weights $\bmod 2k$, where $k = \max\{|V(P_2 \times DSF_n)|, |E(P_2 \times DSF_n)|\} = \max\{5n, 13n\} = 13n$ as follows:

$$\begin{aligned}
 wt_{f_3}(a_1) &= 20n - 1 \\
 wt_{f_3}(a_i) &= 20n - 2i + 1 && \text{for } i = 2, 3, \dots, n \\
 wt_{f_3}(b_i) &= 2n + 4i - 2 && \text{for } i = 1, 2, \dots, n \\
 wt_{f_3}(c_i) &= 26n - 4i + 2 && \text{for } i = 1, 2, \dots, n. \\
 wt_{f_3}(A_1) &= 26n - 1 \\
 wt_{f_3}(A_i) &= 26n - 2i + 1 && \text{for } i = 2, 3, \dots, n \\
 wt_{f_3}(B_i) &= 18n + 4i - 2 && \text{for } i = 1, 2, \dots, n \\
 wt_{f_3}(C_i) &= 14n - 4i + 2 && \text{for } i = 1, 2, \dots, n.
 \end{aligned}$$

Clearly,

$$\begin{aligned}
 \{wt_{f_3}(a_1)\} &= \{20n - 1\} \\
 \{wt_{f_3}(a_i) | i = 2, 3, \dots, n\} &= \{18n + 1, 18n + 3, \dots, 20n - 3\} \\
 \{wt_{f_3}(b_i) | i = 1, 2, \dots, n\} &= \{2n + 2, 2n + 6, \dots, 6n - 2\} \\
 \{wt_{f_3}(c_i) | i = 1, 2, \dots, n\} &= \{22n + 2, 22n + 6, \dots, 26n - 2\}. \\
 \{wt_{f_3}(A_1)\} &= \{26n - 1\} \\
 \{wt_{f_3}(A_i) | i = 2, 3, \dots, n\} &= \{24n + 1, 24n + 3, \dots, 26n - 3\} \\
 \{wt_{f_3}(B_i) | i = 1, 2, \dots, n\} &= \{18n + 2, 18n + 6, \dots, 22n - 2\} \\
 \{wt_{f_3}(C_i) | i = 1, 2, \dots, n\} &= \{10n + 2, 10n + 6, \dots, 14n - 2\}.
 \end{aligned}$$

Therefore f_3 is an edge odd labeling for every even number $n \geq 3$ and hence $P_2 \times DSF_n$ is edge odd graceful for each even positive integer $n \geq 3$. Thus $P_2 \times DSF_n$ is edge odd graceful for arbitrary positive integer $n \geq 3$. \square

Figure 10. shows an edge odd labeling on $P_2 \times DSF_3$. To make the figure look clear, we

draw the graph into several separate parts. Respect to the labeling f_3 for $P_2 \times DSF_3$ we have

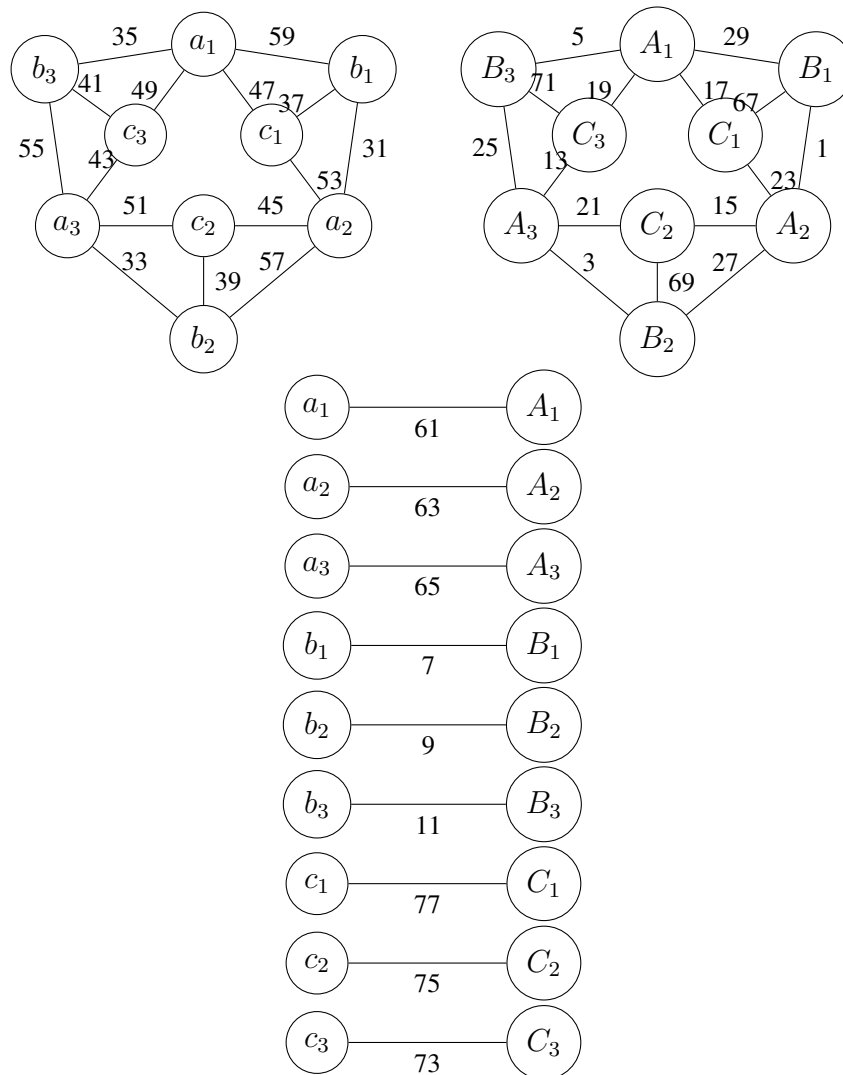


Figure 10. Edge Odd Graceful Labeling f_3 for $P_2 \times DSF_3$

the following vertex weights:

$$\begin{array}{ll}
 wt_{f_2}(a_1) = 17 & wt_{f_2}(A_1) = 53 \\
 wt_{f_2}(a_2) = 15 & wt_{f_2}(A_2) = 51 \\
 wt_{f_2}(a_3) = 13 & wt_{f_2}(A_3) = 49 \\
 wt_{f_2}(b_1) = 56 & wt_{f_2}(B_1) = 26 \\
 wt_{f_2}(b_2) = 60 & wt_{f_2}(B_2) = 30 \\
 wt_{f_2}(b_3) = 64 & wt_{f_2}(B_3) = 34 \\
 wt_{f_2}(c_1) = 58 & wt_{f_2}(C_1) = 28 \\
 wt_{f_2}(c_2) = 54 & wt_{f_2}(C_2) = 24 \\
 wt_{f_2}(c_3) = 50 & wt_{f_2}(C_3) = 20.
 \end{array}$$

ACKNOWLEDGEMENT

This work was done by the support from Universitas Gadjah Mada under Research Grant Year 2019 (Hibah Penelitian Dosen Dana Masyarakat Alokasi Fakultas Tahun 2019).

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