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Analysis of Transportation Method in Optimization of Distribution Cost Using Stepping Stone Method and Modified Distribution

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Abstract. This study aims to examine each method to show the characteristics, advantages and disadvantages of the transportation problem solving method. Vogel Approximation Method (VAM) is an initial solution method that is better than other methods and generally produces an initial solution that is close to the optimum result. In this study the Stepping Stone and Modified Distribution methods produce the same total cost. The main difference between these two methods is in the step of using the shortest path. The author concludes that the determination of the optimal method must be adapted to the conditions of the transportation problem to be solved. The meaning depends on the number of problem variables which will include the number of sources, the number of destinations and the time provided to solve the transportation problem.

Keyword: Transportation method, initial solution, optimal solution.

Abstrak. *Penelitian ini bertujuan mengkaji setiap metode untuk memperlihatkan karakteristik, kelebihan dan kelemahan yang dari metode penyelesaian masalah transportasi. Vogel Approximation Method (VAM) merupakan metode solusi awal yang lebih baik daripada metode lainnya dan umumnya menghasilkan pemecahan awal yang mendekati hasil optimum. Dalam penelitian ini metode Stepping Stone dan Modified Distribution menghasilkan total biaya yang sama. Perbedaan utama antara dua metode ini yaitu pada langkah pemakaian jalur terpendek. Penulis menyimpulkan bahwa penentuan metode optimal harus disesuaikan dengan kondisi masalah transportasi yang akan diselesaikan. Maksudnya bergantung pada banyaknya variabel permasalahan yang akan mencakup jumlah sumber, jumlah tujuan serta waktu yang disediakan untuk menyelesaikan masalah transportasi tersebut.*

Kata Kunci*: Metode transportasi, solusi awal, solusi optimal.*

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1. Introduction

Transportation problem is famous in operation research for its wide application in real life [1]. The transportation problem is concerned with finding the minimum cost of transporting a single commodity from a given number of sources to a given number of destinations [2]. Because there

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is only one commodity, a destination can receive requests from more than one source in order to determine how much must be sent from each source to each destination so as to minimize the total transportation costs [3].

The procedure for completing transportation problem is carried out in two stages namely determining a feasible initial solution and conducting an optimization test. The initial solution was determined using the Northwest Corner Method, Least Cost Method, Russell Approximation Method Vogel Approximation Method which was then continued with the optimization testing phase using Stepping Stone Method and Modified Distribution.

Research conducted by Herlawati [4] on optimizing the distribution of product using Stepping Stone and Modified Distribution, the combination of optimal solution completion using the North West Corner – Stepping Stone and Least Cost Method – Modified Distribution obtained the same total transportation cost. In this study, the authors aim to analyze the transportation method commonly used by previous researchers related to transportation problems, namely the method for determining the initial solution with VAM and the optimization test method. The analyzed method will be applied to an example of distribution transportation case.

2. Related Work

2.1 Linear Programming

Linear programming is a mathematical method in the form of linear to determine an optimal solution by maximizing or minimizing the objective function against a constraint [5]. In general, linear programming can be formulated as follows:

Optimize,

$$Z = \sum_{j=1}^{n} c_j x_j \tag{1}$$

subject to:

$$\sum_{j=1}^{n} a_{ij} x_j \geq \text{ or } \leq b_i, \text{ for } i = 1, 2, 3, \ldots, m \tag{2}$$

$$x_{ij} \geq 0, \text{ for } j = 1, 2, 3, \ldots, n \tag{3}$$

2.2 Transportation Problem

The transportation method assumes that the cost of shipping a commodity on a particular route is proportional to the number of commodity unit shipped on that route [6]. The transportation model can be formulated as follows:

Optimize,

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{4}$$

subject to:

$$\sum_{j=1}^{n} x_{ij} \leq s_i \,;\, i = 1, 2, 3, \ldots, m \tag{5}$$

$$\sum_{i=1}^{m} x_{ij} \leq d_j \,;\, j = 1, 2, 3, \ldots, n \tag{6}$$

$$x_{ij} \geq 0 \text{ for all } i \text{ and } j \tag{7}$$

2.3 Unbalance Transportation Problem

According to Supranto [7], There are 2 possibilities that occur to unbalanced transportation problem, namely:

1. If the demand exceeds the supply $(s_i < d_j)$, then dummy sources are made that keep deficiencies as much as:

$$\sum_{j=1}^{n} d_j - \sum_{i=1}^{m} s_i \tag{8}$$

2. Otherwise, if the supply exceeds the demand $(s_i > d_j)$, then a dummy source is created that will absorb the excess as much as:

$$\sum_{i=1}^{m} s_i - \sum_{j=1}^{n} d_j \tag{9}$$

2.4 Transportation Method

2.4.1 Vogel Approximation Method (VAM)

Characteristic of VAM method is the penalty cost which is the difference between the first smallest distribution cost and the second smallest distribution cost. Thus the main priority of this method is transportation costs. VAM algorithm according to Wijaya [8] as follows:

1. Calculate the opportunity cost for each row and column. The opportunity cost for each row i is calculated by subtracting the smallest c_{ij} value in that row from the c_{ij}value one level larger in the same row. The opportunity cost column is obtained in a similar way. These costs are a penalty for not selecting the box with the minimum cost.
2. Select the row or column with the greatest opportunity cost (if there is a tie in the values of penalties then select the cell where maximum allocation can be possible). Allocate as much as possible to the box with the minimum c_{ij} value in the selected row or column. For the smallest c_{ij}, x_{ij} = minimum (a_i, b_j). It means the biggest penalty is avoided.
3. Adjust supply and demand to show allocations already made. Eliminate all rows and columns where supply and demand has been exhausted.

4. If all supply and demand have not satisfied, go back to step 1 and recalculate the new opportunity cost. If all supply and demand constraints have been satisfied, then the solution has been complete.

2.4.2 Stepping Stone

Stepping Stone Method is a way to change the initial solution into an optimal solution. These steps will reduce transport costs by including a non-basic into the solution. According to Bu'ulolo [9] the steps of the Stepping Stone Method are as follows:

1. Select an unused blank cell that you want to evaluate.
2. Find the closest path (horizontal and vertical movement only) of this unused rectangle through the quadrilateral's footing back to the original unused rectangle.
3. Plus (+) and minus (-) signs appear alternately at each corner of the cell from the nearest path, starting with a plus sign in an empty cell.
4. Add up the unit costs in a rectangle with a plus sign as a sign of additional costs. The cost reduction is obtained by adding up the unit costs in each negative cell.
5. Repeat steps 1 to 4 for other blank cells, and compare the evaluation results of these blank cells. Choose the most negative evaluation value**.**
6. Change the path of the selected cells by allocating the smallest number of units from the cells marked with minus and adding them to cells marked with plus.
7. Repeat steps 1 to 6 until all empty cells have positive cost changes that indicate an optimal solution.

2.4.3 Modified Distribution (MODI)

The MODI method (U-V method) is a variation of the Stepping Stone method which is based on the dual formulation. In the MODI method there are the following formula:

$$c_{ij} = F_i + D_j \tag{10}$$

According to Sudrajat [10] the steps of the Modified Distribution Method are as follows:

1. Find out the initial feasible solution of the transportation problem.
2. Compute F_i and D_j values for each row and column by applying the formula $c_{ij} = F_i + D_j$ to each cell that has an allocation. for each base variable and the first row are assigned a value of 0 ($F_i = 0$).
3. Compute the cost change, d_{ij}, for each empty cell using $d_{ij} = c_{ij} - F_i + D_j$
4. Allocate as much as possible to the empty cell that will result in the greatest net decrease cost (most negative d_{ij}). Allocate according to the stepping stone path for selected cell.
5. Repeat steps 2 through 4 until all d_{ij} values are positive or zero.

3. Methodology

To work on the stages in researh need to be arranged systematic research methods. As for the stages, it can be arranged in diagram from like Figure 1 below:

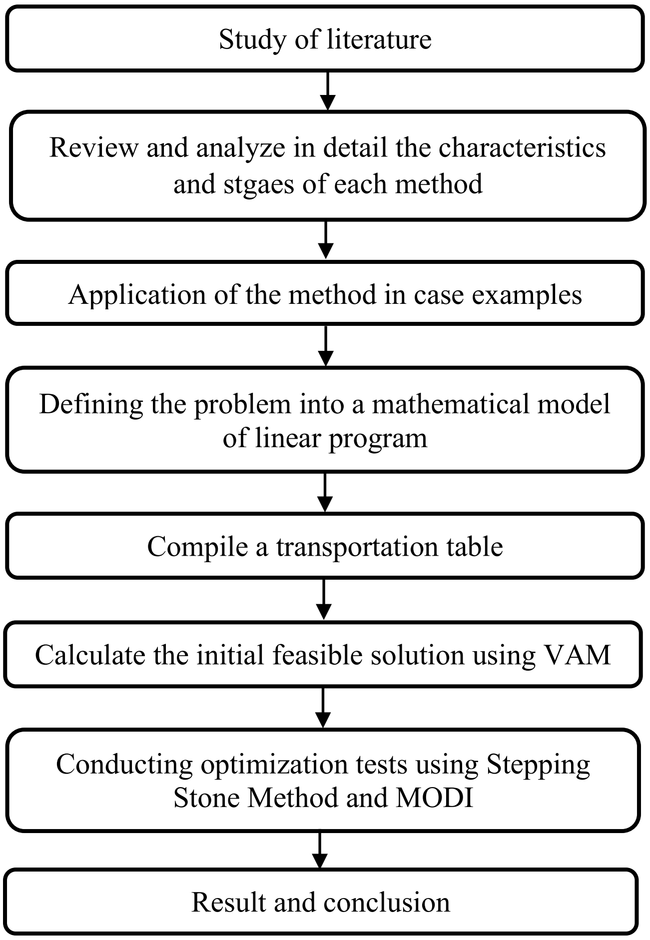


Figure 1. Diagram of Methods and Research Stages

4. Result and Discussion

Table 1 shows that PDAM kabupaten Minahasa Utara aims to optimize water demand by optimizing 5 sources to 5 destinations. The source comes from Tambuk Terang (F_1), Sumur Bor 1 (F_2), Sumur Bor 2 (F_3), Matungkas (F_4), and Mata Air Padang (F_5). While the demand from the destination is Airmadidi (D_1), Kauditan (D_2), Maumbi (D_3), Kolongan (D_4), and Tatelu (D_5).

Table 1. Data on Supply Capacity, Demand, and Transportation Costs for PDAM Distribution from Sources to Destination Areas

	D_1	D_2	D_3	D_4	D_5	s_i
F_1	3.071	3.808	M	M	M	51.300
F_2	2.916	3.991	M	M	M	10.800
F_3	3.420	2.883	M	4.114	M	30.780
F_4	3.183	M	M	M	3.649	15.540
F_5	M	M	3.649	3.183	M	69.120
d_j	53.100	39.600	32.200	45.400	7.240	177.540

Objective function:

Minimize $Z = 3.071x_{11} + 3.808x_{12} + Mx_{13} + Mx_{14} + Mx_{15} + 2.916x_{21} + 3.991x_{22} + Mx_{23} +$
$2.916x_{21} + 3.991x_{22} + Mx_{23} + Mx_{24} + Mx_{25} + 3.420x_{31} + 2.883x_{32} + Mx_{33} +$
$3.420x_{31} + 2.883x_{32} + Mx_{33} + 4.114x_{34} + Mx_{35} + 3.183x_{41} + Mx_{42} + Mx_{43} +$
$3.183x_{41} + Mx_{42} + Mx_{43} + Mx_{44} + 3.649x_{45} + Mx_{51} + Mx_{52} + 3.649x_{53} +$
$3.183x_{54} + Mx_{55}$

subject to:

$$\sum_{i=1}^{5} x_{ij} = s_i \ , i = 1,2,3,4,5$$

$$\sum_{j=1}^{5} x_{ij} = d_j \ , i = 1,2,3,4,5$$

The problem solving for PDAM water distribution cost optimization can be seen on the Table 2 by Vogel Approximation Method to determine initial feasible solution.

Table 2. Allocation of Supply and Demand usingVogel Approximation Method

Source	**Destination**					s_i
	D_1	D_2	D_3	D_4	D_5	
F₁	3.071 34.000	3.808 17.300	M	M	M	51.300
F₂	2.916 10.800	3.991	M	M	M	10.800
F₃	3.420	2.883 22.300	M	4.114 8.480	M	22.300 30.780
F₄	3183 8.300	M	M	M	3.649 7.240	8300 15.540
F₅	M	M	3649 32.200	3183 36.920	M	36.920 69.120
d_j	34.000 44.800 53.100	39.600	32.200	8.480 45.400	7.240	177.540

$Z = (34.000x_{11}) + (17.300x_{12}) + (10.800x_{21}) + (22.300x_{32}) + (8.480x_{34}) + (8.300x_{41}) +$
$(7.240x_{45}) + (32.200x_{53}) + (536.920x_{54})$

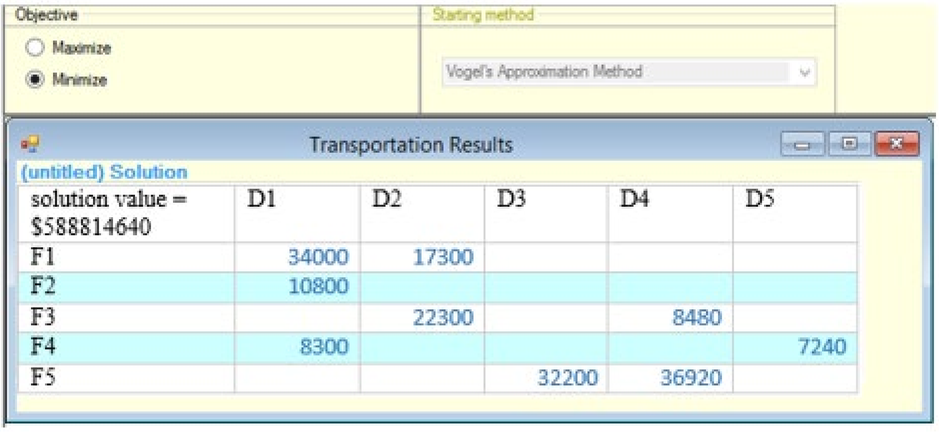
$Z = 34.000\,(3.071) + 17.300\,(3.808) + 10.800\,(2.916) + 22.300\,(2.883) + 8.480\,(4.114) +$
$8.300\,(3.183) + 7.240\,(3.649) + 32.200\,(3.649) + 36.920\,(3.183)$

$Z = 104.414.000 + 65.878.400 + 31.492.800 + 64.290.900 + 34.886.720 + 26.418.900 +$
$26.418.760 + 117.497.800 + 117.516.360$

$Z = 587.814.640$

The total operational cost of distributing PDAM water using the VAM is Rp588.814.640.

To prove the analytical calculation is accurate, it can be examined using QM software for Windows, the output is as follows:



solution value = $588814640	D1	D2	D3	D4	D5
F1	34000	17300			
F2	10800				
F3		22300		8480	
F4	8300				7240
F5			32200	36920	

Figure 2. VAM Allocation using POM-QM

By using the initial feasible table of the VAM, the next step to be checked that the number of base cells is complete $(m + n - 1)$. In Table 2 there are 9 base cells that have been complete. So that the initial feasible for VAM has been complete for optimal testing using Stepping Stone method.

Table 3. Optimum Solution Table using Stepping Stone Method

Source	**Destination**					s_i
	D_1	D_2	D_3	D_4	D_5	
F_1	3.071 34.000	3.808 17.300	M	M	M	51.300
F_2	2.916 10.800	3.991	M	M	M	10.800
F_3	3.420	2.883 22.300	M	4.114 8.480	M	30.780
F_4	3183 8.300	M	M	M	3.649 7.240	15.540
F_5	M	M	3649 32.200	3183 36.920	M	69.120
d_j	53.100	39.600	32.200	45.400	7.240	177.540

From Table 3 select the blank cells to find the repair index. Draw a closed path from an unoccupied cell. Calculate the repair index by adding the unit costs found in each box containing a plus sign (+), followed by subtracting the unit cost in each box containing a minus sign (-). If all the net cost change are greater than or equal to zero, an optimal solution has been reached.

Table 4. Closed loop for unoccupied cells

Unoccupied Cell	Closed Path	Net Cost Change
X_{22}	$X_{22} - X_{12} + X_{11} - X_{21}$	3.991 – 3.808 + 3.071 – 2.916 = 338
X_{31}	$X_{31} - X_{32} + X_{12} - X_{11}$	3.420 – 2.883 + 3.808 – 3.071 = 1.274

Because there are no negative values in Table 4, then the solution is optimal. So the total transportation costs are:

$$Z = (34.000x_{11}) + (17.300x_{12}) + (10.800x_{21}) + (22.300x_{32}) + (8.480x_{34}) + (8.300x_{41}) + (7.240x_{45}) + (32.200x_{53}) + (536.920x_{54})$$

$$Z = 34.000\,(3.071) + 17.300\,(3.808) + 10.800\,(2.916) + 22.300\,(2.883) + 8.480\,(4.114) + 8.300\,(3.183) + 7.240\,(3.649) + 32.200\,(3.649) + 36.920\,(3.183)$$

$$Z = 104.414.000 + 65.878.400 + 31.492.800 + 64.290.900 + 34.886.720 + 26.418.900 + 26.418.760 + 117.497.800 + 117.516.360$$

$$Z = 588.485.520$$

From Table 5, F_i show as row value dan D_i as column value. If the value of one row has been obtained, then the value of the column associated with rectangle. Evaluation of non basic variables by assuming one of the values of F_i or D_j with any given integer, for example: $F_1 = 0$ so that it can be found by the formula $F_i + D_j = c_{ij}$. So that the value of D_i and F_i shown in Table 5.

Table 5. Optimum Solution Table using Modified Distribution

Source	Destination					s_i
	D_1=3.071	D_2=3.808	D_3=5.505	D_4=5.039	D_5=3.537	
$F_1 = 0$	3.071 34.000	3.808 17.300	M	M	M	51.300
$F_2 = -155$	2.916 10.800	3.991	M	M	M	10.800
$F_3 = -925$	3.420	2.883 22.300	M	4.114 8.480	M	30.780
$F_4 = 112$	3183 8.300	M	M	M	3.649 7.240	15.540
$F_5 = -1.856$	M	M	3649 32.200	3183 36.920	M	69.120
d_j	53.100	39.600	32.200	45.400	7.240	177.540

Calculate repair index which is the value of empty rectangle use the formula $d_{ij} = c_{ij} - F_i + D_j$.

Table 6. Closed loop for unoccupied cells

Closed Path	$c_{ij} - F_i + D_j$	Net Cost Change
G_2K_2	3.991 + 155 – 3.808	338
G_3K_1	3.420 + 925 – 3.071	1.274

Because there are no negative values in Table 6, then the solution is optimal. So the total transportation costs are:

$$Z = (34.000x_{11}) + (17.300x_{12}) + (10.800x_{21}) + (22.300x_{32}) + (8.480x_{34}) + (8.300x_{41}) + (7.240x_{45}) + (32.200x_{53}) + (536.920x_{54})$$

$Z\ =\ 34.000\ (3.071) + 17.300\ (3.808) +\ 10.800\ (2.916) +\ 22.300\ (2.883) + 8.480\ (4.114) + 8.300\ (3.183)\ +\ 7.240\ (3.649)\ +\ 32.200\ (3.649)\ +\ 36.920\ (3.183)$

$Z\ =\ 104.414.000\ +\ 65.878.400\ + 31.492.800\ + 64.290.900\ +\ 34.886.720 + 26.418.900 + 26.418.760\ +\ 117.497.800\ +\ 117.516.360$

$Z\ =\ 588.485.520$

From the evaluation of unoccupied cells using Stepping Stone and Modified Distrubution, the optimal total operational cost is Rp588.485.520 to allocate supply to all demand. The distribution route changes are as follows:

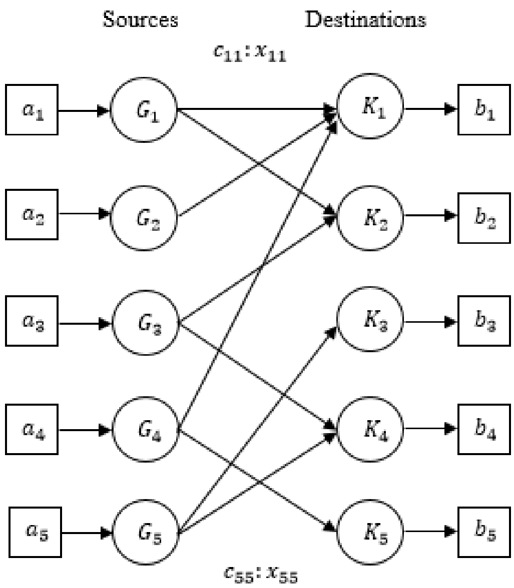


Figure 3. Distribution Route of Stepping Stone Method dan MODI

5. Conclusion

The results obtained after using VAM – Stepping Stone Method and VAM – Modified Distribution resulted in the same transportation costs of Rp588.814.640. From the results of the method analysis, the authors conclude that the selection of the optimal method must be adjusted to the conditions of the transportation problem to be solved, which depends on the number of problem variables that include the number of sources and the amount and time provided to complete.

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