

NUMERICAL STUDY OF THE EFFECT OF WALL THICKNESS AND INTERNAL PRESSURE ON VON MISES STRESS AND SAFETY FACTOR OF THIN-WALLED CYLINDER FOR ROCKET MOTOR CASE

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Abstrak

Peneitian ini menyelidiki tegangan von Mises yang terjadi pada silinder berdinding tipis dan faktor keamanan untuk tabung motor roket karena pengaruh ketebalan dinding dan tekanan internal. Dimensi panjang silinder adalah 500 mm, diameter luar 200 mm, dan tebal cap 30 mm. Ketebalan dinding divariasikan 6, 7, 8, dan 9 mm, sedangkan tekanan internal divariasikan 8, 9, dan 10 MPa. Analisis tegangan dilakukan menggunakan metode elemen hingga dengan perangkat lunak Ansys Workbench 2019 R3. Hasil simulasi menunjukkan bahwa tegangan maksimum von Mises menurun dengan meningkatnya ketebalan dinding. Tegangan von Mises maksimum meningkat dengan meningkatnya tekanan internal. Material memiliki faktor keamanan lebih tinggi dari 1,25 untuk semua variasi ketebalan dinding dan tekanan internal. Hal ini berarti bahwa material dapat menahan beban statis. Proses verifikasi dilakukan dengan membandingkan hasil analisis elemen hingga dengan perhitungan analitik untuk tegangan hoop maksimum dan tegangan aksial maksimum dengan kondisi batas tetap. Hasil tegangan hoop maksimum dan tegangan aksial maksimum menggunakan analisis elemen hingga dan perhitungan analitik tidak berbeda secara signifikan. Persentase kesalahan antara perhitungan analitik dan analisis elemen hingga < 6%.

Kata kunci: Tekanan Internal, Studi Numerik, Tabung Motor Roket, Silinder Berdinding Tipis, Ketebalan Dinding

Abstract

This paper investigates the von Mises stress that occurs in thin-walled cylinders and safety factors for rocket motor cases due to the influence of the wall thickness and internal pressure. Dimensions of the cylinder length are 500 mm, outer diameter is 200 mm, and cap thickness is 30 mm. The wall thickness is varied 6, 7, 8, and 9 mm, while the internal pressure is varied 8, 9, and 10 MPa. Stress analysis is performed using the finite element method with Ansys Workbench 2019 R3 software. The simulation results show that the maximum von Mises stress decreases with increasing wall thickness. The maximum von Mises stress increases with increasing internal pressure. The material has a safety factor higher than 1.25 for all variations in wall thickness and internal pressure. It means that the material can withstand static loads. The verification process is done by comparing the results of finite element analysis with analytical calculations for maximum hoop stress and maximum axial stress with a fixed boundary condition. The results of maximum hoop stress and maximum axial stress using finite element analysis and analytical calculations are not significantly different. The percentage of errors between analytical calculations and finite element analysis is less than 6%.

Keywords: Internal Pressure, Numerical Study, Rocket Motor Case, Thin-Walled Cylinder, Wall Thickness

INTRODUCTION

The rocket motors are used in launch vehicles, missiles, and spacecraft (Rajesh et al., 2017). It is an important part of rockets that contain fuel to drive rockets (Wibawa, 2018b). It consists of a tube, nozzle, cap, insulator, and igniter. It operates by releasing high-temperature gas through the nozzle to produce thrust (Wibawa, 2020). It works using principles such as pressure vessels because they store propellant fuel. In general, rocket motor works in environments with high pressure and high temperature (Emrich, 2016).

The rocket motor tube design depends on internal pressure and the material used (Wibawa et al., 2020b) (Wibawa et al., 2020a). Rocket motor case that works at high internal pressure generally uses thick-walled cylinders, while at low internal pressure use thin-walled cylinders. Materials that have high strengths typically use thin-walled cylinders, while materials with low strengths use thick-walled cylinders. Rocket motors generally are designed to withstand the pressure exerted inside the casing and provide structural stability (Babu et al., 2015).

Internal pressure and wall thickness are the main factors affecting the maximum von Mises stress in thin-walled cylinders (Wibawa et al., 2020a). This study predicts von Mises stress and safety factors in thin-walled cylinders with variations in wall thickness and internal pressure.

In this study, the rocket motor case design uses a thin-walled cylinder with a length of 500 mm, an outer diameter of 200 mm, and a cap thickness of 30 mm. The wall thickness is varied 6, 7, 8, and 9 mm, while the internal pressure is varied 8, 9, and 10 MPa.

The numerical study is done using the finite element method with Ansys Workbench 2019 R3 software, which is widely used for rocket motor case analysis (Ramanjaneyulu et al., 2018) (Niharika & Varma, 2018) (Shaheen & Gupta, 2015) (Kumar et al., 2016) (Özaslan et al., 2018) (Kashyap & Murugesan, 2016) (Teja & Subramanyam, 2019). The finite element method is a numerical mathematical

technique for calculating the structural strength of engineering components by dividing objects into mesh shapes (mesh), a smaller element so that calculations can be arranged and carried out. The finite element method aims to determine the structure or component can safely withstand a predetermined load. The finite element method is widely used to predict stresses in thin-walled and thick-walled cylinders (Mohamed, 2018).

MATERIAL AND METHOD

Thin-walled and thick-walled are the classification of pressure vessels based on their dimensions. Thin-walled pressure vessels are pressure vessels with a wall thickness (t_w) smaller than $1/20$ internal diameter (D_i). Thick-walled pressure vessels are pressure vessels with a wall thickness (t_w) greater than $1/20$ of their inner diameter (D_i).

Thin-walled cylinder stress will occur in three directions: hoop (circumferential), longitudinal (axial), and radial. The maximum stress equation for thin-walled cylinders in the hoop, longitudinal and radial directions is:

$$(\sigma_h)_{max} = \left(\frac{P_i D_i}{2t} \right) \quad (1)$$

$$(\sigma_l)_{max} = \left(\frac{P_i D_i}{4t} \right) \quad (2)$$

$$(\sigma_r)_{max} = -p_i \quad (3)$$

Where:

p_i = internal pressure (MPa)

D_i = inner diameter of cylinder (mm)

t = wall thickness of cylinder (mm)

The analysis of the rocket motor case is performed by numerical methods. The numerical method is done by the finite element method with Ansys Workbench 2019 R3 software. It is one of the most common numerical methods used to calculate every physical phenomenon, such as structural or fluid behavior, heat transfer, and electromagnetic component engineering. This method allows each product design to be analyzed in detail and reduces the number of physical prototypes making it easier to develop better products with a faster time. One of the most popular software used in finite element analysis is Ansys. Ansys is widely

used to measure the stress concentration in thick-walled and thin-walled cylinders (Mohamed, 2018).

Solving physical law problems that depend on space and time is usually expressed in partial differential equations (Partial Differential Equations). In most cases and geometry, partial differential equations cannot be solved by analytical methods. However, an equation approach can be built, usually based on various types of discretization. This discretization method approaches partial differential equations with numerical model equations, which can be solved using numerical methods. The solution to numerical model equations is the approach of real solutions to partial differential equations.

The finite element method procedure allows the continuum to be discretized into a limited number of parts (elements) and emphasizes that the characteristics of the continuous domain can be estimated by assembling the same

properties of discrete elements per node. This process is known as discretization. The values between nodes are determined from polynomial interpolation using the computational matrix method, and the accuracy of the results depends on discretization, the accuracy of the assumed interpolation form, and the accuracy of the computational solution method. The finite element method is very popular because of its ability to model many numerical problems regardless of geometry, boundary conditions, and loading.

Figure 1 shows a cylindrical geometry model. Because the cylinder shows symmetry in the longitudinal (axial) direction, it can be modeled with its upper half (Dadkhah & Zecher, 2008). This study uses a 90-degree segment of the solid cylinder model or eighth cylinder (Figure 2). It is because the pressure applied to the fluid in the cylinder is closed, the pressure will be continued equally and evenly in all directions (Lawrence, 2012).

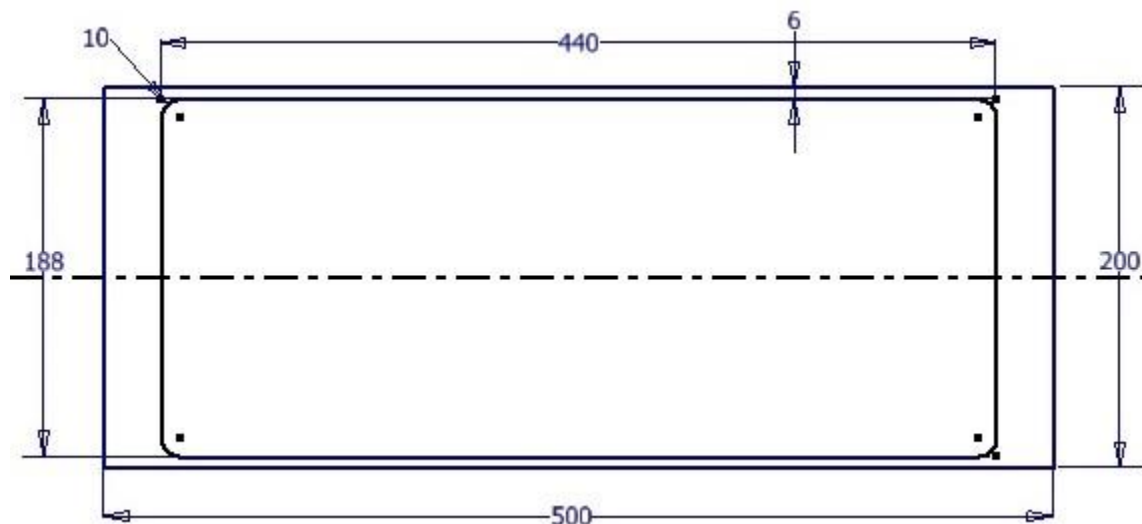


Fig. 1. Thin-Walled Cylinder Design with A Wall Thickness Of 6 Mm and Length of The Cylinder Of 500 mm.

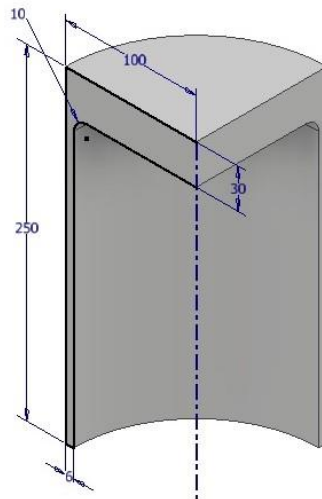


Fig. 2. One-eighth part of a cylinder with a wall thickness of 6 mm and length of the cylinder of 500 mm.

The following assumptions are made theoretically on thin-walled cylinders: Aluminum material 6061 homogeneous and isotropic; The cylinder remains flat even after being subjected to internal pressure; All cylindrical surfaces will expand or contract independently.

The boundary conditions using Ansys Workbench 2019 R3 software are

shown in Figure 3. The finite element analysis parameters are shown in Table 1. The size of the elements used is set uniformly, which is 3 mm. The results in the number of nodes and elements for each variable being slightly different, but not too significantly.

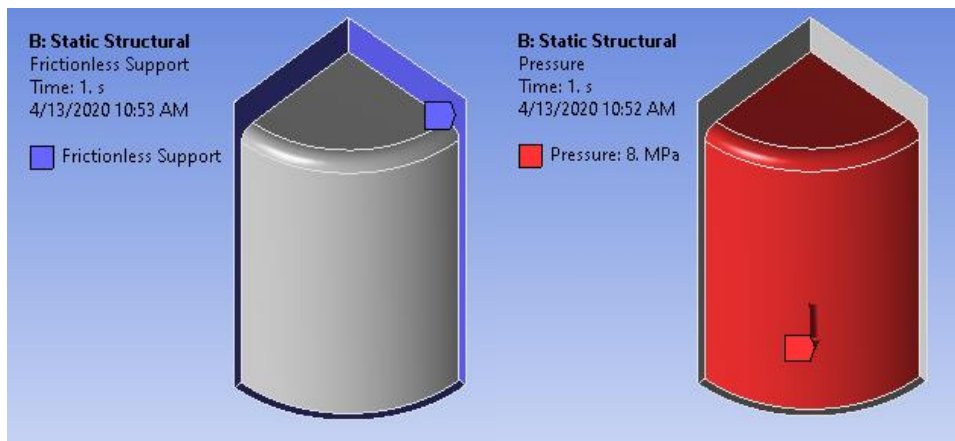


Fig. 3. Boundary conditions: type of constraint (left) and loading condition (right).

Table 1. Finite element analysis parameters.

Parameter	Notation	Value
Length of cylinder	L	500 mm
Outer diameter	D_o	200 mm
Wall thickness	t	6, 7, 8, and 9 mm
Internal pressure	P_i	8, 9, and 10 MPa
Cap thickness	t_c	30 mm
Element size	-	4 mm

The criteria for a rocket motor tube material are lightweight and have high strength. Aluminum 6061 was chosen for the material (Wibawa, 2018a). The mechanical properties of Aluminum 6061 are shown in Table 2.

Table 2. Mechanical properties of Aluminum 6061 material (Wibawa, 2018a).

Material	Density (gr/cm ³)	Young's Modulus (GPa)	Poisson's Ratio	Yield Strength (MPa)	Tensile Strength (MPa)
Aluminum 6061	2.7	68.9	0.33	275	310

RESULTS AND DISCUSSION

One failure theory is based on maximum distortion energy, known as the von Mises criterion. Von Mises stress becomes the determining factor whether the material will fail or not. Von Mises stress calculates the stress combination at a certain point that will cause failure. The material will fail when von Mises stress exceeds the strength of the material. The von Mises theory is the theory of plasticity that applies best to ductile materials, especially for metal materials such as Aluminum 6061. The von Mises stress is also known as the equivalent stress.

Figure 4 shows the von Mises stress for a 6 mm (left) and 7 mm (right) wall thickness with constant internal pressure,

which is 8 MPa. The simulation results show that the maximum von Mises stress decreases with an increased wall thickness (Table 3). The relationship between wall thickness and maximum von Mises stress is inversely proportional.

Figure 5 shows the von Mises stress for an 8 MPa (left) and 9 MPa (right) internal pressure with constant wall thickness, 9 mm. The simulation results show that the maximum von Mises stress increases with increasing internal pressure (Table 3). The relationship between the fillet radius and the maximum von Mises stress is linear. The maximum von Mises stress occurs in the cap area. It is a critical area.

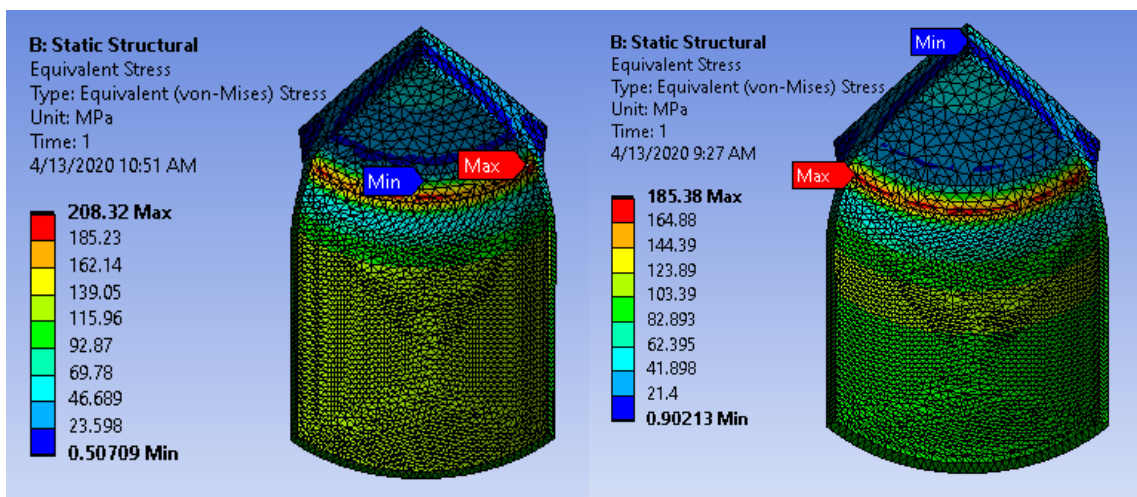


Fig. 4. The von Mises stress for a 6 mm (left) and 7 mm (right) wall thickness with constant internal pressure, which is 8 MPa.

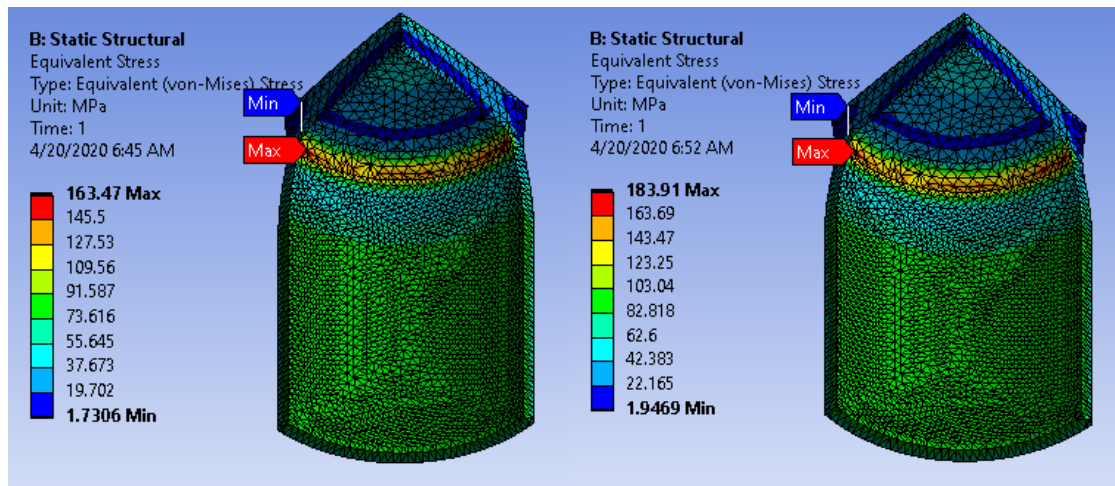


Fig. 5. The von Mises stress for an 8 MPa (left) and 9 MPa (right) internal pressure with constant wall thickness, which is 9 mm.

Table 3. Effect of Variations In Wall Thickness And Internal Pressure On Maximum Von Mises Stress

Wall thickness (mm)	Internal pressure (mm)	Number of Nodes	Number of Elements	Maximum von Mises Stress (MPa)
6	8	37774	21102	208.32
7	8	38675	21625	185.38
8	8	37784	20811	175.52
9	8	35895	19613	163.47
6	9	37774	21102	234.36
7	9	38675	21625	208.55
8	9	37784	20811	197.46
9	9	35895	19613	183.91
6	10	37774	21102	260.41
7	10	38675	21625	231.73
8	10	37784	20811	219.40
9	10	35895	19613	204.34

Safety factors are used to evaluate the safety of components or structures, even though the dimensions used are minimum (Wibawa, 2018c). The safety factor can be based on either the maximum tensile stress limit or the yield stress of the material. Yield strength is the minimum stress when a material begins to lose its elastic properties, that is, the nature of the material to return to its original shape when the load or force is removed. Maximum tensile strength is the maximum stress a material can reach before breaking. The safety factor is aimed at preventing yield deformation, while the safety factor is at maximum tensile strength aimed at preventing collapse. A safety factor of less than 1 indicates a permanent failure of a design.

In this study, the safety factor is based on maximum tensile strength because the rocket tube is not a component that is used repeatedly. It is because of the nature of the rocket motor case, which is a disposable component.

Table 4 shows the effect of wall thickness and internal pressure on the safety factor. Simulation results show that when the wall thickness increase, the safety factor also increases. The relationship between wall thickness and the safety factor is linear. In contrast, when the internal pressure increases, the safety factor decreases. The relationship between internal pressure and the safety factor is inversely proportional.

The finite element analysis results show that the material has a safety factor

higher than 1.25 for all variations in wall thickness and internal pressure. It means that the material can withstand static loads. It means that the material can

withstand static loads because the safety factor value for a material can withstand static loads is 1.25-2.00 (Dobrovolsky & Zablonky, 1978).

Table 4. The Effect Of Wall Thickness And Internal Pressure On The Safety Factor.

Wall thickness (mm)	Internal pressure (MPa)	Minimum safety factor
6	8	1.49
7	8	1.67
8	8	1.77
9	8	1.90
6	9	1.32
7	9	1.49
8	9	1.57
9	9	1.69
6	10	1.19
7	10	1.34
8	10	1.41
9	10	1.52

The verification process is done by comparing the value of the maximum hoop stress and maximum axial stresses between analytical calculations and finite element analysis. The smaller the percentage of errors shows the results of finite element analysis are increasingly valid.

Figure 6 shows the maximum hoop stress (left) and maximum axial stress (right) with a wall thickness of 6 mm and an internal pressure of 8 MPa, i.e., 129.46 MPa and 60.72 MPa.

Simulation results show that when the wall thickness increases, maximum hoop stress and maximum axial stress decrease (Table 5). In contrast, maximum hoop stress and maximum axial stress increase when the internal pressure increases (Table 5).

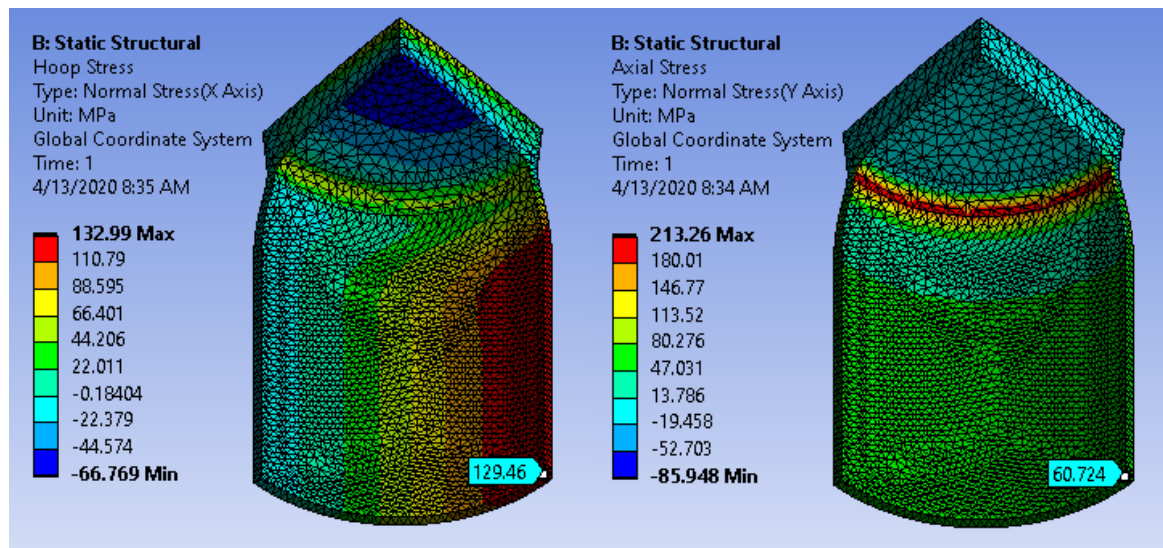


Fig. 6. The maximum hoop stress (left) and axial stress (right) with a wall thickness of 6 mm and an internal pressure of 8 MPa.

Table 5. Comparison of The Maximum Hoop And Axial Stress Between Analytical Calculations And Finite Element Analysis.

Wall thickness (mm)	Internal pressure (MPa)	Analytic		FEA		Error	
		Hoop stress (MPa)	Axial stress (MPa)	Hoop stress (MPa)	Axial stress (MPa)	Hoop stress (%)	Axial stress (%)
6	8	125.33	62.67	129.46	60.72	3.29	3.11
7	8	106.29	53.14	110.46	51.21	3.93	3.64
8	8	92.00	46.00	96.20	44.07	4.57	4.20
9	8	80.89	40.44	85.12	38.52	5.23	4.76
6	9	141.00	70.50	145.64	68.31	3.29	3.11
7	9	119.57	59.79	124.26	57.62	3.92	3.62
8	9	103.50	51.75	108.21	49.58	4.55	4.19
9	9	91.00	45.50	95.76	43.33	5.23	4.77
6	10	156.67	78.33	161.82	75.90	3.29	3.11
7	10	132.86	66.43	138.03	64.01	3.89	3.64
8	10	115.00	57.50	120.26	55.09	4.57	4.19
9	10	101.11	50.56	106.41	48.15	5.24	4.76

The maximum hoop stress and maximum axial stress results using finite element analysis and analytical calculations are not significantly different. The percentage of errors between analytical calculations and finite element analysis is less than 6 percent.

Hoop stress is always tensile, and maximum hoop stress always occurs in the inner radius or the outer radius depending on the direction of the pressure gradient (Skinner, 2018). If the thin-walled cylinder only subjected to external pressure, then the maximum hoop stress occurs at the outer radius ($r = r_o$). In this study, thin-walled cylinders only subjected to internal pressure, so the maximum hoop stress occurs in the inner radius ($r = r_i$).

In closed cylinders, internal pressure works to develop stress along the cylinder's axis. The stress is called axial stress and is usually smaller than hoop stress. In this study, hoop stress is higher than axial stress.

CONCLUSIONS

The simulation results show that the maximum von Mises stress decreases with increasing wall thickness. The maximum von Mises stress increases with increasing internal pressure. The material has a safety factor higher than 1.25 for all variations in wall thickness and internal

pressure. It means that the material can withstand static loads.

The verification process is done by comparing the results of finite element analysis with analytical calculations for maximum hoop stress and maximum axial stress with a fixed boundary condition. The results of maximum hoop stress and maximum axial stress using finite element analysis and analytical calculations are not significantly different. The percentage of errors between analytical calculations and finite element analysis is less than 6 percent.

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