



Moment Tensor Components with Their Corresponding Eigen Values and Eigen Vectors for Earthquakes $M_w \geq 6.0$ in Northern and Southwestern of Pakistan

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ABSTRACTS

Self-generated earthquake catalogue of $M_w \geq 6.0$ upto 300 km depth was prepared for the inversion of six components of moment tensor. Seismotectonic map was drawn with seismicity distribution. The stress map of Pakistan was generated through CASMO which described by the Anderson model (1951). The maximum horizontal stress (S_{Hmax}) acting in the compressional zone generates reverse faulting (RF) causes to form the three distressing mountainous range called Karakorum, Hindukush and Himalayan i-e $S_{Hmax} > S_{Hmin}$. In eastern side of Pakistan, minimum horizontal stress (S_{Hmin}) acting in the extensional zone produced normal fault (NF) i-e $S_{Hmax} < S_{Hmin}$. Focal mechanism was also drawn using source parameters of each event obtained by matlab codes. Stress tensor components were obtained using mathematical equations and out comes were written in the form of 3x3 matrix. All matrices have non trivial solutions and their solution sets were also evaluated in the form of eigen values and eigen vectors. Crustal thickness was displayed by thickness map. The maximum thickness of crust in northern Pakistan i-e 60-70 km whereas in southwestern region this varied from 20– 40 km all around. This variation due to plate motion intensity and velocity of Indian and Eurasian plate collision.

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1. INTRODUCTION

Pakistan is located on the dynamic seismic belt where folds and faults have been observed due to the intra-plate collision between the Indian and Eurasian plates (Kazmi & Jan, 1997; Khalid *et al.*, 2016). These plates are continuously moving towards north at the speed of 15 to 20 cm/year (Patriat & Achahe, 1984; Scotese *et al.*, 1988). The Himalayan Fold and Thrust Belt, Kohistan Magmatic Arc, Suleiman Fold and Thrust Belts are the evident features generated under the action of this collision (Powell, 1979). The seismo-genic stresses are acting on the northern side of Pakistan whereas on the western side these two plates slide pass with each other producing Chamman Fault at Quetta. In the southern part the continental and oceanic plates are colliding with each other to form Makran triple junction called Makran Seismic Zone (MSZ). Transform faulting has dynamic position in this compressional region (Sercombe *et al.*, 1998; Mona Lisa *et al.*, 2004). The Himalayas Mountain Range and dynamic convergent plate boundary in this area is tectonically highly under stress (Kazmi & Jan, 1977). Therefore, Kohistan Ladakh, Nanga Parbat contact shows the flexible shear zone (Treloar *et al.*, 1991). The main tectonic components in this area are Main Mantle thrust (MMT), Main Karakorum thrust (MKT), Dasu-Sassi Fault, Upper Hunza Fault, Krakoram Fault, Trichmir Fault, Nanga Parbat and Raikot Sassi Fault as shown in tectonic map of Pakistan (Figure 1). MKT and MMT represent by Shyok Suture and Indus Suture zones respectively (Yeast & Lawrence, 1984).

The seismic energy accumulates into the earth's crust and released from faults during an earthquake. Stress analysis and fault plane solution (Presti *et al.*, 2013; Khalid *et al.*, 2016) are tools in practice to analyze the orientation and geometry of the earthquake related features. The fault plane solution describes by the first P wave motion from the nearest station and remaining far stations measure the solution using waveform analysis (Pondrelli *et al.*, 2006; Scognamiglio *et al.*, 2009). This solution is based on the stress moment tensor either it is single couple or double couple stress tensor. All orientations of earth are being formed after earthquake geometrically shown on the earth surface by beach ball solution. The forces applied vertically produces normal faulting, which is due to tensional stress and behaves like a single couple moment tensor. On the other hand, when the forces acting on both sides of the block, horizontally reverse faulting produced under the compressional regime. Transform faulting as well as strike slip faulting produced the double couple moment. Their stress values produced the nine independent stress orientations. In this paper, we will discuss the tectonic stress orientation as well as fault geometry in Pakistan and adjacent area with respect to the significant earthquakes.

Moment tensor analysis has been utilized for decades as a tool for understanding the faulting process during earthquakes (Guilhem *et al.*, 2014). This analysis provides unique information regarding the fault plane orientations (fault plane solution) and the size of the seismic events of all magnitudes by assuming that these events can be represented by a point source for long wavelengths and relatively larger distances between the epicenters and the recording stations.

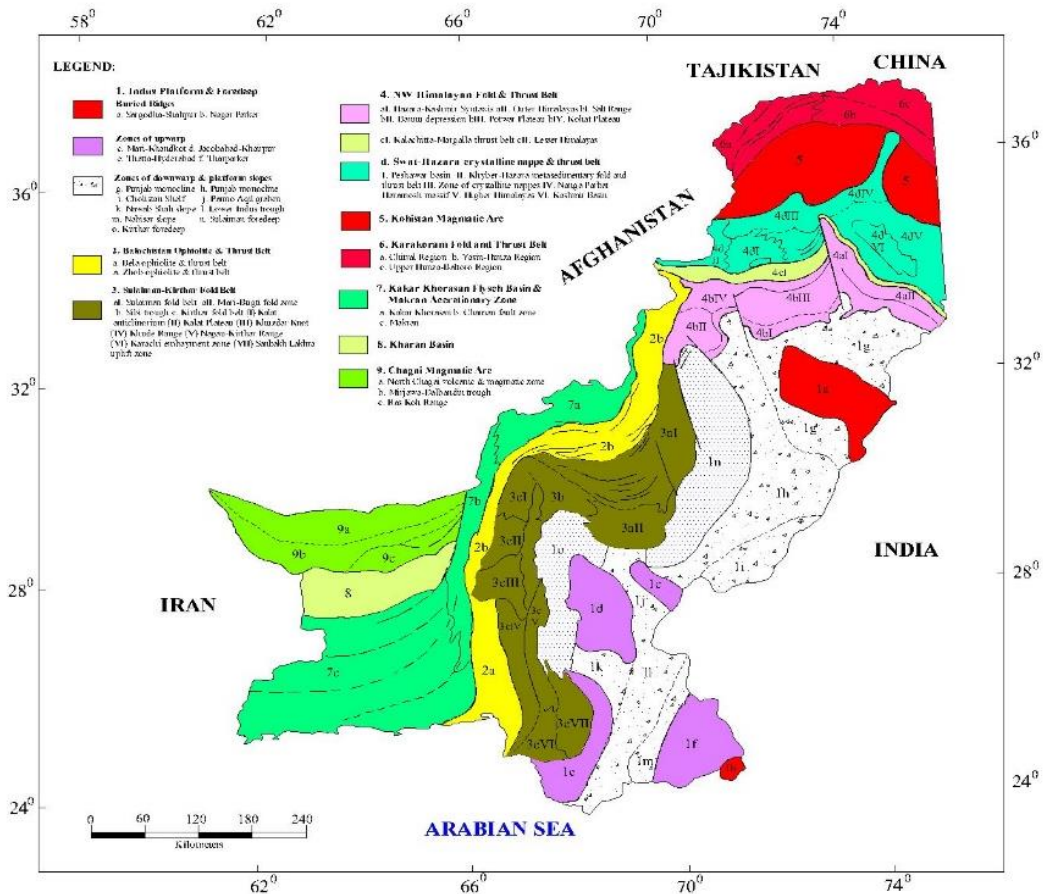


Figure 1. Tectonic features of Pakistan and its surrounding areas revised by Kazmi & Jan 1997.

2. STRESS ANALYSIS AND MOMENT TENSOR

The stress produced under the action of applied forces has nine directional stress components. Three stress components are called principal or normal stresses denoted by σ_x , σ_y , σ_z and remaining six components are called shear stresses denoted by τ_{xy} , τ_{yx} , τ_{yz} , τ_{zy} , τ_{zx} , τ_{xz} . The generalized double couple phenomenon to nine possible couple of forces is demonstrated in Figure 2 described by (Aki & Richard, 2002).

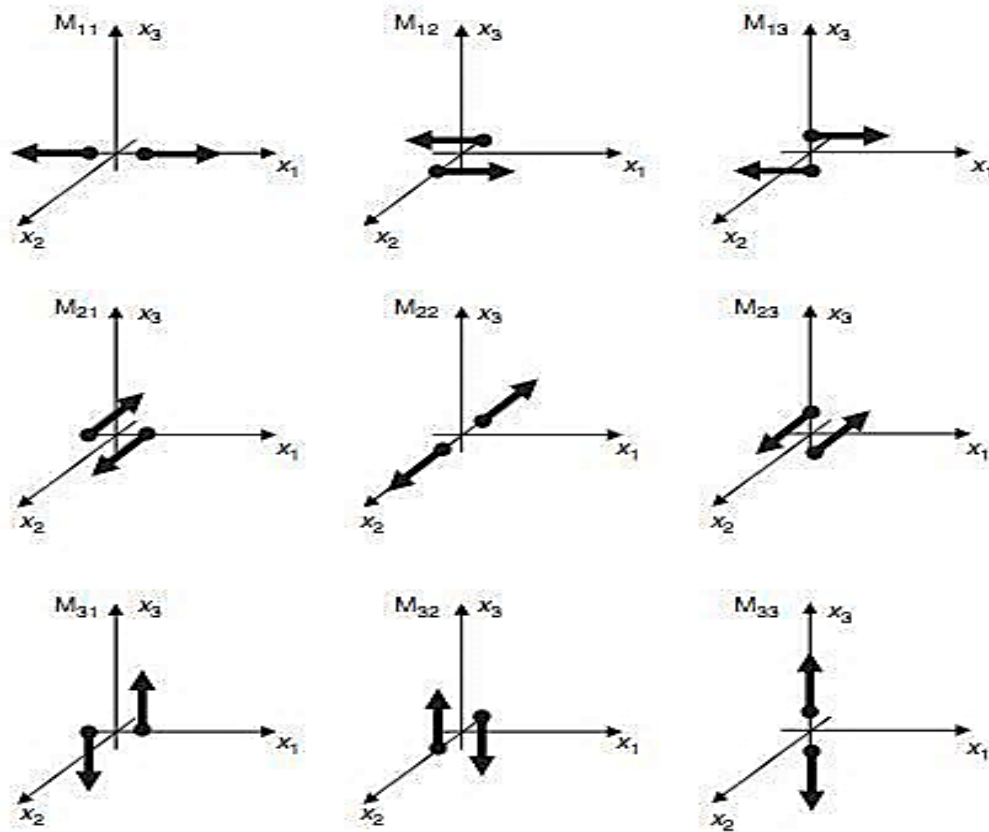


Figure 2. Nine components of stress tensor presented by Aki & Richard (2002).

2.1 Anderson model of stress distribution

To study the directional stresses in lithosphere the model proposed by Anderson (1951) is used (Figure 3). This model shows these directional stresses act on the body in the form of S_v , S_{Hmax} and S_{Hmin} at depth of S_1 , S_2 and S_3 respectively. Where S_v is vertical stress due to overburden, S_{Hmax} the maximum principal horizontal stress and S_{Hmin} is the minimum principal horizontal stress.

In the Anderson model, horizontal principal stresses are always unequal. It is either be less or greater than the vertical stress. When the vertical stress is more in divergent movement area ($S_1=S_v$), the gravity promotes the normal faulting. Whereas when S_{Hmax} and S_{Hmin} go over the S_v ($S_3=S_v$) the compression regime (thinning) is entertained the reverse faulting. (Barba *et al.*, 2010) Finally, the transform faulting shows the intermediate stress behavior ($S_2=S_v$). This is the stage where horizontal stress is greater than the overburden vertical stress. So mathematically, expression for the reverse faulting described as $S_{Hmax} \geq S_v \geq S_{Hmin}$. The overburden stresses along the fault line inside the earth at any location also a major cause of earthquake. After the earthquake, the fault can be express geometrically on the earth surface using beach ball. To find out the earthquake focal mechanism, study of stress analysis and seismogenic forces operating in the region is the essential part (Montone *et al.*, 2004; Zoback & Zoback, 1980). The fault divides into two planes. These planes are formed by three major components i.e., strike lies between 0° to 360° , dip ranges from 0° to 90° and rake/slip lies in -180° to 180° . Nodal Plane 1 and Nodal Plane 2 also called fault plane and auxiliary plane respectively. These planes contain two major axes: one is “P” pressure axis and other “T” tensional axis. The third axes M shows the movement plane. A stress map was drawn using world stress map (WSM) database that indicates the directional forces and their fault

mechanism. Only A, B and C quality (add refer) data was used in stress map of Pakistan bounded by coordinates from ($24^{\circ} - 38^{\circ}$ latitude) and ($60^{\circ} - 78^{\circ}$ longitude). The CASMO stress map is showing the stress indication of faults. In the stress map, different colors were used for different regimes identification. Lambert projection was used for preparing the stress map with their focal mechanism inversion.

3. METHODOLOGY AND DATA SET

To study the moment tensor inversion and focal mechanism earthquake stress orientation 14 number of events were selected for self-generated earthquake catalogue (SGEC) from the local and international earthquake databases like Pakistan meteorological department (PMD), United State Geological Survey (USGS) and centroid moment tensor (CMT). The epicenter, focal depth and magnitude of each event with their unique ID are presented in **Table 1**.

Each event contains the information about the seismic moment, rupture length, fault length, magnitude and displacement (Wells & Coppersmith, 1994). The moment tensor solution describes the nine couple forces in the Cartesian coordinate system as given in the equations (1) – (7). In the double couple source, rectangular or Cartesian component can be written in the form of direction cosine using strike, dip and rake of the fault plane and the scalar seismic moment M_o (Aki & Richards, 2002). We are familiar about fault parameters i.e., strike (ϕ), dips (δ) and rake/slip (λ). The numerical values source parameters can be found out by using moment tensor directional components (Aki & Richards, 2002).

$$M_{xx} = -M_o (\sin\delta \cos\lambda \sin 2\phi + \sin 2\delta \sin\lambda \sin^2\phi) \quad (1)$$

$$M_{xy} = M_{yx} = M_o (\sin\delta \cos\lambda \cos 2\phi + 0.5 \sin 2\delta \sin\lambda \sin 2\phi) \quad (2)$$

$$M_{yy} = M_o (\sin\delta * \cos\lambda \sin 2\phi - \sin 2\delta \sin\lambda \cos^2\phi) \quad (3)$$

$$M_{yz} = M_{zy} = -M_o (\cos\delta * \cos\lambda * \sin\phi - \cos 2\delta * \sin\lambda * \cos 2\phi) \quad (4)$$

$$M_{zz} = M_o (\sin 2\delta * \sin\lambda) \quad (5)$$

$$M_{zx} = M_{xz} = -M_o (\cos\delta * \cos\lambda * \cos\phi + \cos 2\delta * \sin\lambda * \sin\phi) \quad (6)$$

where M_o is the seismic moment defined as:

$$M_o = \mu DA \quad (8)$$

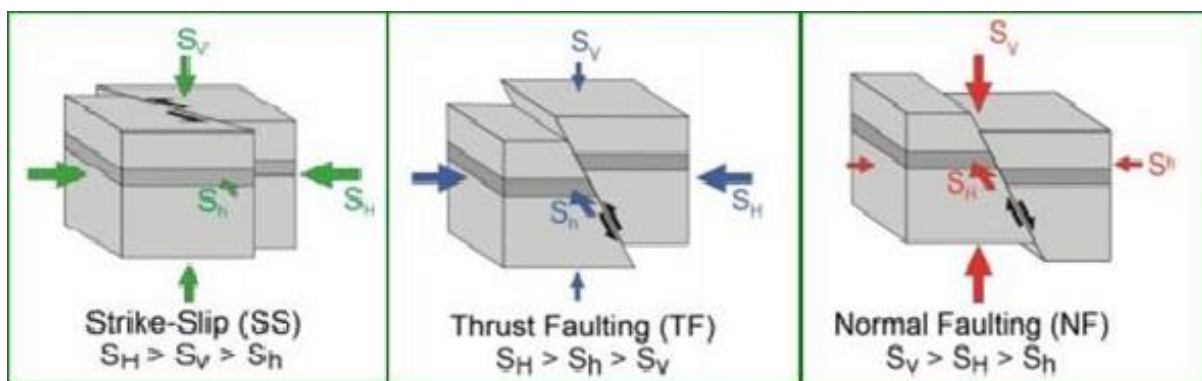


Figure 3. Modified Anderson model (1951) about fault orientation due to applied stresses.

here μ is modulus of rigidity typically taken as $3 \times 10^{10} \text{ Nm}^{-2}$ for crust and $7 \times 10^{10} \text{ Nm}^{-2}$ mantle, D is displacement of fault and A is rupture area of the fault zone measures in (m^2) . M_o can also be calculated from moment magnitude M_w by (Hiroo & Kanamori, 1979).

$$M_w = \frac{2}{3} \text{Log}(M_o) - 10.73 \tag{9}$$

The source parameters of the significant earthquake are describing using inversion of moment tensor (Bock, 2002). Focal mechanism or fault plane solution, explain by beach ball using polarity of P wave first motion at the observatory station which produce the strong relation between earthquake and seismic faults geometry (Anderson et al., 1993). The fault parameters of nodal plane 1 and Nodal plane 2, azimuth and plunge values determine by MATLAB code for seismic inversion (Efthimios & Sokosa, 2008).

3.1 Eigen vectors and eigen values of moment tensor solution

Stress tensor is a linear operator which provide the traction of vector t from normal vector n^{\wedge} . In the seismology, seismologist always used stress tensor in the form of Cartesian geometry as 3×3 matrix. This is fully description of stress tensor in Cartesian coordinate system. To describe the fully state of stresses, stress tensor contained six independent components at any points in medium. In the field of science and technology eigenvectors and eigenvalues are frequently used. Specifically, in the field of matric transformation, system of simultaneously equations, mathematical differential expression, tensor analysis, stress orientation and axis transformation, eigenvectors and eigenvalues are imperative. In physics, earth rotation model (geodesy), geophysical simulation of earth motion belongs to cubic polynomial called characteristic polynomial which have root. These roots are called eigen values and their vectors are called eigen vectors. in coordinate system rigid bodies and their orientation with respect to their axis transform one axis to another called tensor transformation. The complicated shapes of rigid bodies definitely have some rotational direction. This phenomenon is called axis of inertia and these are determined by obtaining eigen values and eigen vectors. This inertia tensor also called moment of inertia.

Table 1. Self-Generated Earthquake catalogue (SGEC) of events $M_w \geq 6.0$.

Event ID	Region	Date	Latitude	Longitude	Focal Depth(km)	M_w
1A	PAKISTAN	16-Mar-1978	29.83	66.43	39.2	6.1
2A	PAKISTAN	10-Aug-1987	29.65	63.72	157.2	6
3A	PAKISTAN	4-Mar-1990	28.66	66.16	28	6
4A	PAKISTAN	17-Jun-1990	26.75	65.25	15	6.1
5A	PAKISTAN	20-May-1992	32.95	71.27	15	6
6A	PAKISTAN	27-Feb-1997	29.74	68.13	15.3	7.1
7A	PAKISTAN	5-Oct-2005	34.38	73.47	12	7.6
8A	PAKISTAN	8-Oct-2005	34.7	73.12	12	6.4
9A	PAKISTAN	28-Oct-2008	30.4	67.48	17.2	6.4
10A	PAKISTAN	29-Oct-08	30.29	67.57	12	6.4
11A	PAKISTAN	18-Jan-2011	28.61	63.9	52.3	7.2
12A	PAKISTAN	24-Sep-2013	26.7	65.04	12	7.8
13A	PAKISTAN	28-Sep-2013	27.11	65.5	15	6.8
14A	PAKISTAN	7-Feb-2017	25.01	63.25	14	6.4

In any medium for any stress tensor it is always possible to determine the direction of n vector such as there is no shear stress across the plane to normal n . In rock mechanics, +ive sign represents the compressional forces and tensional forces denoted by -ive. Traction of any vectors defined by \hat{t} (Figure 4).

$$t(\hat{n}) = (tx, ty, tz) \quad (10)$$

$$t(-\hat{n}) = -t(\hat{n}),$$

t is normal to the plane called normal stress and its parallel stress called shear stress. Thus, according to the Shearer (2009),

$$\begin{aligned} t(\hat{n}) &= \lambda \hat{n} = \tau(\hat{n}) \\ \tau \hat{n} - \lambda \hat{n} &= 0 \\ (\tau - \lambda I) \hat{n} &= 0 \end{aligned} \quad (11)$$

where I is identity matrix and λ is scalar quantity. This has non-trivial solution and give the eigen values and their corresponding eigen vectors when it is a nonsingular matrix i.e., $\det[A]=0$. Such that

$$\det[\tau - \lambda I] = 0 \quad (12)$$

This give the cubic characteristics polynomial and have three solutions in which $\lambda_1, \lambda_2, \lambda_3$ are the eigen values. $\lambda_1, \lambda_2, \lambda_3$ are simply the principal stresses. Eigen matrices/Eigen vectors used to determine the numerical values of individual component. These vectors problems are usually solved by MATLAB or Mathematica software or simply solve the mathematical expression of Eq. 12. These source parameters are utilizing to find the moment tensor values, which can be written in 3x3 matrixes as shown in matrix A . Seismic moment tensor is always symmetric. Therefore, the M_{xy} and M_{yx} are always equal due to the same magnitude but opposite direction (Aki & Richard, 2002).

$$M = \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \quad (A)$$

where σ and τ are the shear and normal stresses. Moment magnitude (M_w) was consider in whole analysis yet the other magnitude can be found using conversion by different author.

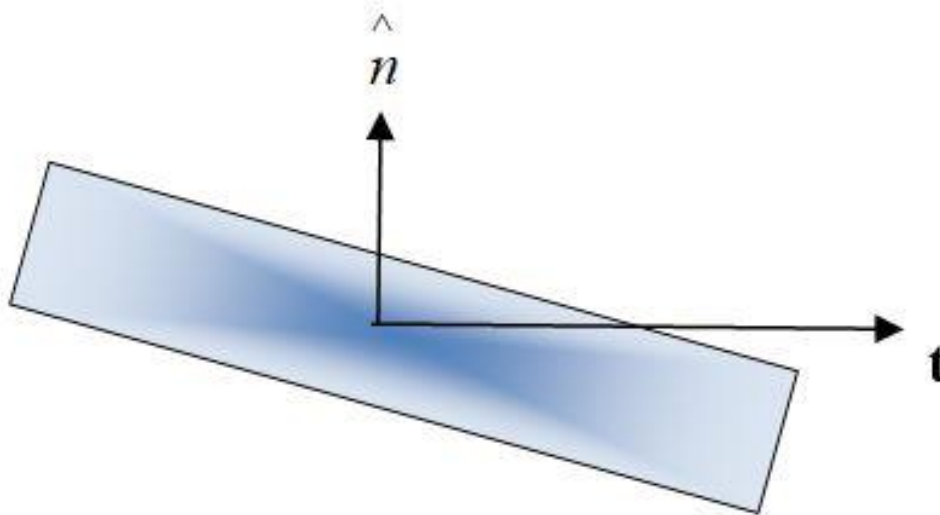


Figure 4. Graphically representation of traction of vectors.

4. RESULT AND DISCUSSION

Tectonically Pakistan seized by hidden and visible forces which based on the velocity of plate motion of Indian and Eurasian. Although the situation is concerned with the seismogenic behavior however this may lead to stress dispersion inside crust.

In this work, time domain moment tensor inversion is applied to significant earthquakes ($M_w \geq 6.0$) occurred in the northern and southwestern parts of Pakistan. A self-generated earthquake catalogue (SGEC) was prepared, which constitutes 14 significant seismic events. The stress tensor transformation in the form of components was obtained to determine the Eigen values and Eigen vectors of direction of each event. The seismo-tectonic map of Pakistan and its surrounding region was drawn in **Figure 5** which shows that seismicity distribution of selected events and local tectonic setting. Shallow earthquake can be seen in northern and northwestern side whereas the depth particularly in Hindukush region and Gilgit Balitistan. Earthquakes with depth more than 270 km were also observed in the area extreme towards China, Uzbekistan, and Afghanistan.

4.1 Stress orientation and focal plane solution

The stress orientation under the action of various tectonic activities in and around the Pakistan presented in stress map (**Figure 6**). The faults produced under the action of these seismic sources which classified into strike slip, thrust and normal faults described by different color schemes. Strike slip (SS) and thrust faults (TF) are dominant tectonic features in the southwestern part of Pakistan. Normal faults (NF) and TF are dominant fault systems in the northern part of the country, indicate the continuous collision between the Indian and Eurasian Plates, thus compressional regime is present. In southern part, the Arabian plate slides pass to the Indian plate, thus producing the strike slip faulting on axial belt. The most prominent fault in this region is the Chamman Transform Fault. Stress map shows the maximum horizontal stress (S_{Hmax}), acting in the compressional zone, generated the reverse faulting, the condition $S_{Hmax} > S_{Hmin}$ will satisfy whereas no fault system is observed in the eastern side of Pakistan due to far distance from the collision zone thus, $S_{Hmax} < S_{Hmin}$ holds correctly.

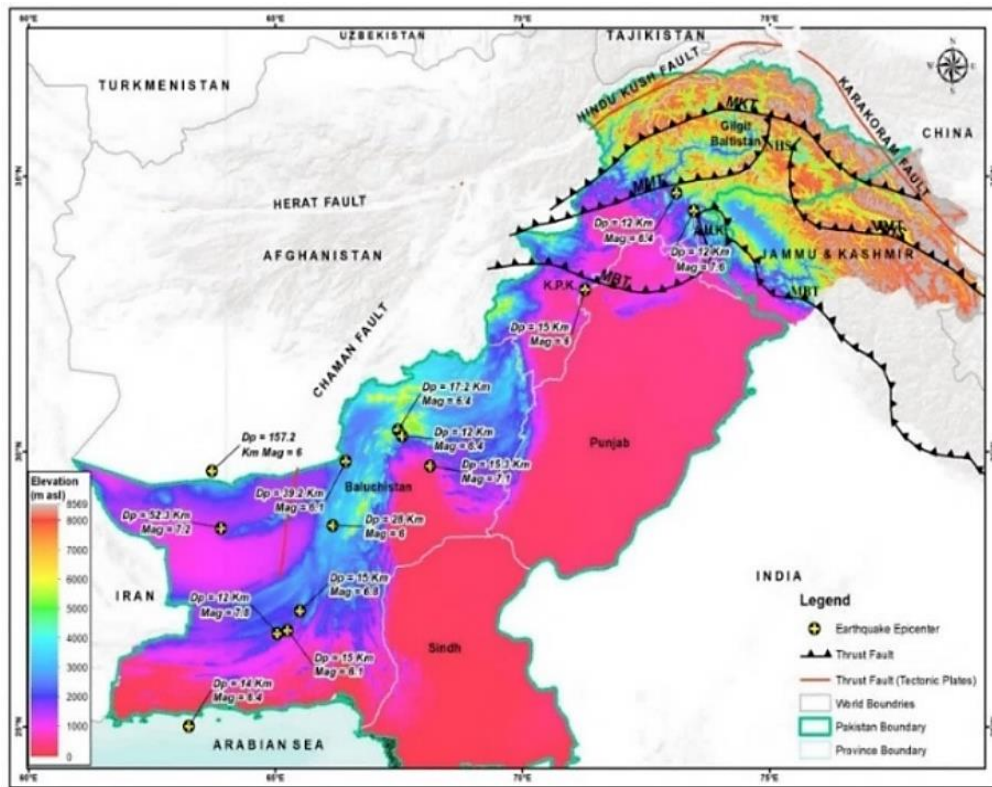


Figure 5. Seismo-tectonic map of selected events of SGEC with depth and M_w .

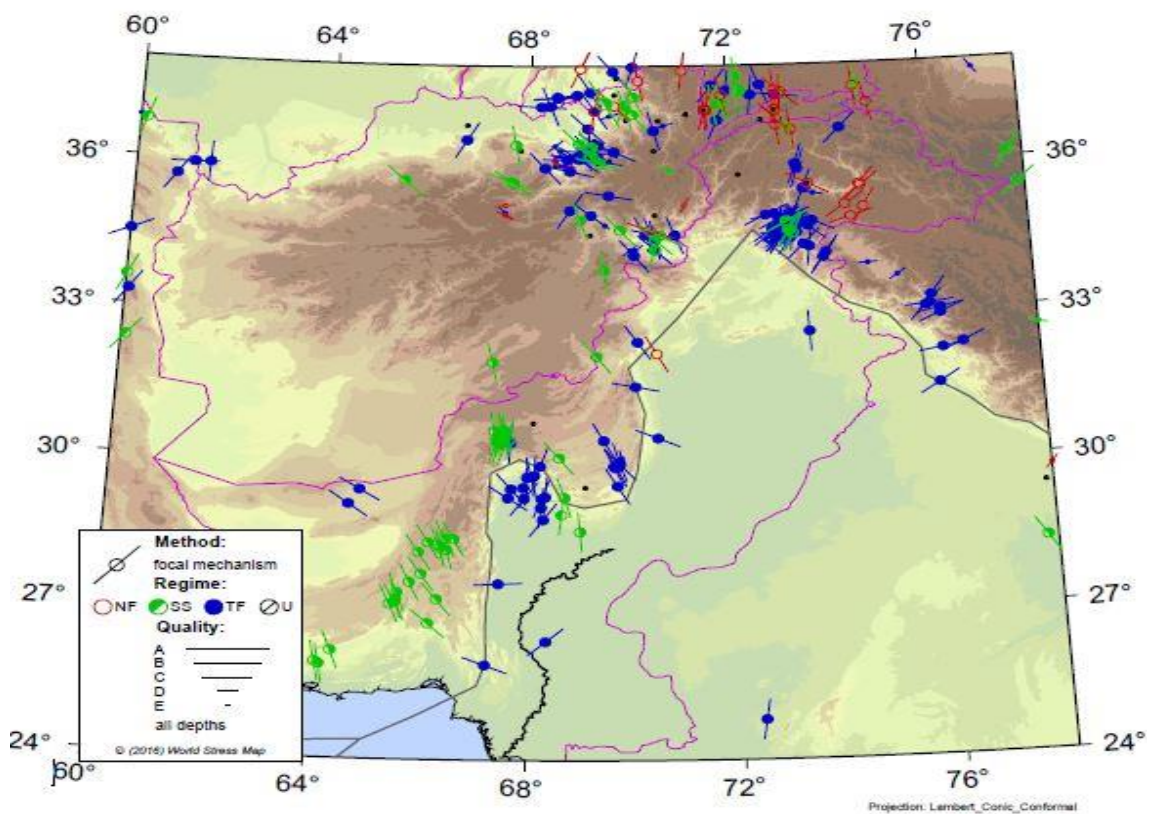


Figure 6. Lambert projection of stress map of Pakistan describes the stress inversion. Normal fault shown by red color, reverse fault represents by green and transform fault marked by blue color.

