

Students' Intuition in Solving Mathematics Problems: The Case of High Mathematics Ability and Gender Differences

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Abstract

Students are required to find their appropriate strategies to solve mathematics problems so that intuition is needed. Male and female students have different intuition on mathematical problem-solving. Thus, gender is influencing how to obtain mathematical knowledge. This descriptive qualitative study aimed to analyze the intuition differences of male and female students who have high-level mathematical abilities at secondary school in solving mathematics problems. Data was collected through tests of mathematical problem-solving and interviews then analysed through data reduction, data presentation, and conclusion. This study found that: (1) There are differences in the characteristics of male and female intuition in mathematical problems solving, (2) The intuition of male and female in mathematical problems solving based on Polya's steps is different in re-checking the answers, (3) There are differences in intuition when students solve linear equation system problems. There are differences in intuition between male and female students with high mathematical abilities in each material. Students with problem-solving abilities have affirmative intuition to understand problems, anticipatory intuition for problem-solving plans and solutions, and conclusive intuition to re-examine problems.

Abstrak

Siswa dituntut menemukan sendiri strategi yang sesuai untuk memecahkan masalah matematika, sehingga dibutuhkan intuisi. Siswa laki-laki dan perempuan memiliki intuisi yang berbeda dalam menyelesaikan masalah matematika, sehingga faktor gender mempengaruhi cara memperoleh pengetahuan matematika. Penelitian ini bertujuan untuk mengetahui perbedaan intuisi siswa sekolah menengah atas antara perempuan dan laki-laki yang memiliki kemampuan matematika tingkat tinggi. Penelitian ini menggunakan pendekatan kualitatif dengan jenis penelitian deskriptif. Subjek dalam penelitian ini terdiri dari 1 orang siswa laki dan 1 siswa perempuan dengan tingkatan kemampuan matematika tinggi. Pengumpulan data dilakukan dengan menggunakan instrumen tes yang meliputi tes kemampuan pemecahan masalah matematika dan wawancara. Analisis data meliputi tiga tahap yaitu reduksi data, penyajian data dan penarikan kesimpulan. Hasil penelitian menunjukkan bahwa: (1) Terdapat perbedaan karakteristik intuisi subjek laki-laki dan perempuan dalam menyelesaikan masalah matematika, (2) intuisi subjek laki-laki dan perempuan dalam menyelesaikan masalah matematika berdasarkan pemecahan Polya berbeda pada langkah memeriksa

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kembali, (3) Terdapat perbedaan intuisi pada berbagai materi sistem persamaan linier. Sehingga terdapat intuisi yang berbeda pada setiap siswa laki-laki dan perempuan yang berkemampuan matematika tinggi pada setiap materi. Yaitu siswa dengan kemampuan pemecahan masalah memiliki intuisi afirmatori untuk memahami masalah, intuisi antisipatori untuk rencana penyelesaian dan penyelesaian masalah, dan intuisi konklusif untuk menguji kembali masalah.

INTRODUCTION

The mathematical problems are said to be a problem when the answer is unknown immediately by students, but they need time to think about what steps should be taken to solve the problem. Problem-solving is a goal of learning mathematics and as the main tool for learning at the same time (NCTM, 2000). Therefore, problem-solving skills focus on mathematical learning at all levels, from elementary school to college. By problem-solving in mathematics, students will get to know their way of thinking, diligent habits, curiosity, and self-confidence in unusual situations, such as the situation they will face outside the mathematics classroom. Being a good problem solver can bring great benefits in daily life and work situations.

Intuition is a thought or preference that comes quickly without much reflection. Intuition is a self-evident cognition, can be received directly, holistically, forcefully and exploratively (Fischbein, 1999). Intuition is an immediate cognition in mathematical objects (Giardino, 2010). Liljedahl (2004) argued that mathematical intuition is a spontaneous idea commonly referred to as Aha! Experience. The use of intuition in mathematics leads to errors in concepts understanding, but there are researches (Raman, 2002) which support the importance of intuition in mathematical learning and potential in increasing the understanding of mathematics as well as in mathematical problem-solving. Raman (2002) found that intuitive representation and interpretation can guide students to make the right mathematical claims. In addition, it was found that if one finds a key idea in a proof, then he can associate an understanding of claims in proof with formal evidence of mathematical statements. Burton (1999) studied the involvement of intuition in "mathematical" activities which involve 70 subjects (Mathematicians). Although there were pros and cons about this because of intuition, it was found that many subjects (83%) who acknowledge that the presence of intuition has helped them in their mathematical activities even with varying degrees. The importance of intuition benefits for vocational students which help them produce creative answers, not steps. Students can produce correct answers when intuition with mathematical concepts is formally inline (Westcott, 1968).

Fischbein (1999) states that intuition is categorised into affirmatory intuition, anticipatory intuition, and conclusive intuition. Affirmatory intuition with characteristics: Intuition appears as a statement immediately accepted without justification by formal evidence or empirical support. Certain intuition feeling of intrinsic certainty. Intuition that uses the effect of force on individual reasoning strategies and on the selection from hypothesis and resolution (Coerciveness). Intuition related to the ability to predict behind an empirical supporter (Extrapolativeness). Intuition that is contrary to cognition obtained logically and analytically (Globality). Based on its characteristics, Fischbein (1987) categorizes intuition in three parts, namely; affirmatory intuition, anticipatory intuition, and conclusive intuition. Affirmatory intuitions can be statements, representations, interpretations, solutions that can be individually accepted, self-evident, global and intrinsic certainty. Anticipatory intuition arises when someone works hard to solve a problem, but the solution is not immediately obtained and conclusive intuition is the effort to summarize in general with the main idea of a problem-solving.

The terms of gender have differences in terms of dimensions. This term refers to the socio-cultural dimension of a man and woman, while gender is related to a man and woman (Santrock, 2011). Gender is a part of self-concept that involves identifying individuals as men and women. So, it can be concluded that the understanding of gender is the difference in roles, functions, and

responsibilities between men and women which are the results of social construction and can change according to the development of the age (Baron, 2012). The differences in male and female learning achievement were caused more by differences in intelligence levels. Men are more active than women. However, the activity of men causes men to be more difficult to manage. This causes men to have lower learning achievement than women (Rushton, 2010). Some researchers believe that the influence of factors gender (the influence of male-female differences) in mathematics is due to biological differences in the brains of boys and girls which are known through observation, that girls, in general, are superior in the field of language and writing, while boys are superior in the field of mathematics because of its better spatial abilities (Geary: 2000). However, according to Mullis (2011) that girls consistently get better achievements than boys in the class; the results of the research described show gender differences in solving mathematical problems. Some results indicate the existence of gender factors in mathematics learning, but on the other hand, several studies reveal that gender does not have a significant effect on mathematics learning.

Because intuition-based learning is a learning activity that provides every student with all kinds of intuition based on mathematical abilities, the way students learn varies depending on their strengths and weaknesses. Because it assesses the progress of student learning in the same way for each student will not cause student intuition appropriately. The questions given by the teacher must be contextual so that students have the opportunity to connect their learning outcomes with their real experiences, their own world, and the wider community (Bouillion & Gomez, 2001). The solution to problem-solving can be done in various ways, not necessarily appropriately with the teacher. So that students can provide creative answers according to their intuition.

A good situation in learning mathematics is where students' intuition with mathematical concepts is formally inline (Jatisunda & Nahdi, 2019). Often students in trivial situations interpret mathematical facts by referring to concrete reality and regard formal evidence as excessive demands. The implication is that students are directed to understand formal deductive-thinking mathematics. Intuitive acceptance of mathematical statements does not exclude the necessity to meet a formal deductive mathematical structure, strictly by axiomatic. The situation that often occurs in teaching mathematics is that student acceptance is intuitively at odds with formal mathematical concepts and results in cognitive conflict and even cognitive bias that can hinder students from learning mathematics. In this case, learning must reconstruct mathematical intuition and students' initial knowledge. It helps students overcome these difficulties by making them aware of the conflict and helping to understand the facts in mathematics that lead to understanding the correct concepts.

When designing learning activities that are based on improving mathematical problem-solving skills, teachers should pay attention to students' intuition. In this study, there is anticipatory intuition which is generally contradictory, and the intuition is based on the senses and imagination and tends to not be able to solve the problem correctly and incompletely. Therefore, the teacher in designing learning activities that can train students with such intuition so that intuition is obtained which can be used to solve problems correctly. This study reveals the differences in the differences in intuition-male and female-female in solving Polya-based mathematics. There are differences in intuition on male and female subjects in solving math-based problems, and there is in various materials.

Researchers researched high school because the survey said that high school students had lower mathematical abilities than vocational students. Therefore, researchers want to see the intuition possessed by male high school students and female high school students when solving a problem based on their level of ability. The purpose of this study was to determine the differences in the intuition of female and male high school students who have high, moderate and low-level mathematical abilities in solving mathematical problems.

METHODS

The approach in this study used a qualitative approach with a descriptive research type. Qualitative research used in this study is due to research to examine differences in the intuition of students of Secondary school in mathematical problem-solving in terms of mathematical abilities and gender differences, while the main instrument is the researcher themselves—data analysis in a qualitative approach. The activities in qualitative data analysis include three stages, namely the stage of data reduction, data presentation, and conclusion drawing (Miles and Huberman, 1992).

The research was conducted at SMAN 3 and SMKN 2 Banda Aceh in class X. This class has the same number of male and female students. The sample is 6 students. The purpose of using the learning achievement test is to classify students' mathematical abilities which are classified as high, medium, and low.

The selected subjects are initialized with the name MZLTA and ANPTA with the following information:

1. MZLTA
 - MZ is the name of the subject.
 - L means Male
 - T is high-level mathematical skills
 - A is the initial of the school
2. ANPTA
 - AN is the name.
 - P means Female.
 - T is high-level mathematical skills
 - A is the initial of the school

The mathematical problem-solving ability test (TKPMM) consists of TKPMM 1 and TKPMM 2 while being interviewed when the student resolving the problems.

In Table 1 below can be seen the characteristics of two subjects, intuition, which are ANPT and MZLTA, with high-level mathematics.

Tabel 1. The Characteristics and Intuition type of the research subjects

Subject	Material	Polya's Steps	Intuition Characteristic	Intuition Type
MZLTA	Linear Equation System of Three Variables	Understanding the problems	<i>Direct, self-evident and Coerciveness</i>	Afirmatori Intuition
		Developing the problem solving plan	<i>Intrinsic Certainty, Globality and Coerciveness</i>	Antisipatori Intuition
		Implementing the problem solving plan	<i>Coerciveness and Intrinsic Certainly</i>	Antisipatori Intuition
		Re-test the problem	-	-
	Linear Equation System of Two Variables	Understanding the problem	-	-
		Developing the problem solving plan	<i>Extrapolative and Coerciveness</i>	Antisipatori Intuition
		Implementing the problem solving plan	<i>Coerciveness and Globality</i>	Antisipatori Intuition
		Re-test the problem	-	-
	Linear Equation System of Three Variables	Understanding the problem	<i>Direct, self-evident and Coerciveness</i>	Afirmatori Intuition
		Developing the problem solving plan	<i>Intrinsic Certainly dan Coerciveness</i>	Antisipatori Intuition

Subject	Material	Polya's Steps	Intuition Characteristic	Intuition Type
ANPTA	Linear Equation System of Two Variables	Implementing the problem solving plan	Coerciveness and Intrinsic Certainty	Antisipatori Intuition
		Re-test the problem	-	-
		Understanding the problem	Direct, self-evident and Globality	Afirmatori Intuition
		Developing the problem solving plan	Intrinsic Certainty, Extrapolative and Coerciveness	Antisipatori Intuition
		Implementing the problem solving plan	Coerciveness Globality and	Antisipatori Intuition
		Re-test the problem	-	-

The subject of MZLTA in the Linear Equation System of Three Variables material has affirmatory intuition and anticipatory in Understanding the problems, Developing the problem-solving plan, and Implementing the problem-solving plan. While the Re-test, the problem does not have conclusive intuition. The Linear Equation System of Two Variables material only has intuition on the Developing the problem-solving plan and Implementing the problem-solving plan. Whereas in Understanding the problem and Re-testing the problem does not have intuition.

The ANPTA subject in the Linear Equation System of Three Variables material has affirmatory intuition and anticipation in Understanding the problems, Developing the problem-solving plan, and Implementing the problem-solving plan. While the Re-test the problem does not have conclusive intuition. In the material of Linear Equation System of Two Variables, it has intuition on Understanding the problem, Developing the problem-solving plan and Implementing the problem-solving plan. Whereas in the Re-test the problem does not have intuition. The characteristics possessed at each step of the solution between the subjects MZLTA and ANPTA vary.

FINDING AND DISCUSSION

The characteristics, description and intuition of the subjects are:
 MZLTA subject

1. Material of Linear Equation System of Three Variables

$$\begin{aligned} x + 2y + 3z &= 52000 \\ x + y + 2z &= 35000 \\ \hline -y + z &= 17000 \end{aligned}$$

$$\begin{aligned} 2x + 3y + 4z &= 39500 \\ x + y + 2z &= 35000 \\ \hline x + 2y + 2z &= 4500 \end{aligned}$$

$$\begin{aligned} x + 2y + 2z &= 4500 \\ x + 2y + 3z &= 52000 \\ \hline -z &= 47500 \\ z &= 9500 \end{aligned}$$

$$\begin{aligned} y + z &= 17000 \\ y &= 17000 - 7500 \\ y &= 7000 \end{aligned}$$

$$\begin{aligned} 2x + 3z &= 39500 \\ 2x &= 39500 - 21500 \\ 2x &= 18000 \\ x &= 8500 \end{aligned}$$

a) 5. Biji : x
 K. Biji : y
 C. Kembar : z
 $x + 2y + 3z = 52000$
 $x + y + 2z = 35000$
 $2x + 3y + 4z = 39500$
 $z = 9500$

b) Marga tepung yang paling mahal = Cakra Kembar (9500)
 c) $30x = 30(8500) = 255000$
 $\frac{9}{10} \times 255000 = 229500$
 d) $30y = 30(7000) = 210000$
 $\frac{9}{10} \times 210000 = 189000$
 e) $30z = 30(9500) = 285000$
 $\frac{9}{10} \times 285000 = 256500$
 f) $30x + 30y + 30z = 30(x + y + z)$
 $= 30(255000)$
 $= 7650000$
 $\frac{1}{10} \cdot 7650000 = 765000$

g) setiap harinya, toko tepung tersebut akan mendapat penghasilan sebesar Rp 765.000 jika setiap tepung dijual sebanyak 30 kg (sangat untung 10% untuk karyawan). Dan karyawan di toko tersebut bisa mendapat hasil dari potongan 10% setiap harinya sebesar Rp 765.000 jika setiap tepung tersebut dijual sebanyak 30 kg

Based on the test results and MZLTA's interview in Understanding the SPLTV problem, the characteristics of intuition are; Direct, Self-evident and Coerciveness. Direct cognition, self-evident

cognition, is accepted as an individual feeling without further checking and verification. MZLTA immediately understood the SPLTV problem by writing, known and asked, the results of the interview were "I made it known and asked from the questions" (Fischbein, 1999). According to Polya, the characteristics of students who understand the problem is they can express questions and answers as follows: (a) what information can be known from the problem, (b) what is the main problem that need to be solved, (c) are there any important conditions to note from the problem.

Whereas the Coerciveness that exists when MZLTA understands the problem, MZLTA felt confident about what is known and asked based on what it has written, the results of the interview quote "When I read the question, I was immediately able to catch that it was known and asked as I wrote, I was very sure like that". Coerciveness is the intuition has the nature of leading towards something that is believed (Fischbein, 1999). This means that individuals tend to reject alternative interpretations that will contradict their intuition. From this, it can be concluded that MZLTA has affirmatory intuition. In contrast, students mention what is known and asked directly and smoothly according to the characteristics of high-level students that will try to achieve success regardless of profit or loss, good fortune or bad luck (Stoltz, 2004).

MZLTA subject used the SPLTV concept based on the command on the problem when compiling the problem-solving plan. The question has been presented in the Linear Equation System of Three Variables and is trying hard to think about the problem-solving step. Because of the emergence of intuition after trying to work hard on the problem by looking at the information from the text of the question, it is said that what was in his mind at the earliest times was a global idea. So, the preparation of the problem-solving plan is very neat (Fischbein, 1999). MZLTA has the characteristics of Intrinsic Certainty intuition when forming the Linear Equation System of Three Variables at point a, which was linked with something that has been studied but cannot explain in detail based on the multiplication requirements of a matrix. Interview quote that "Indeed, it can't be multiplied directly with Kg every type of flour and grapes". According to Intrinsic Certainty is Certainty of intuition cognition is usually associated with certain feelings of intrinsic certainty (Fischbein, 1999). Intrinsic means that there is no external support needed to obtain a direct certainty (either formally or empirically). MZLTA also has the characteristics of Globality intuition which is difficult to explain what method is used. Globality is a global cognition that is contrary with cognition obtained logically, sequentially and analytically (Fischbein, 1999). MZLTA also believes that his plan was based on mathematical thinking in real terms after MZLTA examined the questions, so it has the characteristics of Coerciveness intuition. The interview results quote that "More confident as the step I wrote". Coerciveness is the nature of leading towards something that is believed (Fischbein, 1999). Thus, it can be concluded that MZLTA has anticipatory global intuition and intuition in the form of mathematical thinking in real. Students mention what will be planned directly and smoothly according to the characteristics of high-level students that will try to achieve success regardless of profit or loss, good fortune or bad luck (Stoltz, 2004).

In resolving the problem, MZLTA believes that the plan is written based on mathematical thinking in real terms after he looks at the problem, so he has the characteristics of Coerciveness intuition. The results of the interview quote that "More sure of the solution I wrote". Coerciveness is the character that leads to the direction that is believed (Fischbein, 1999). MZLTA has the characteristics of Intrinsic Certainty intuition when forming a Linear Equation System of Three Variables at point a, linking with something that has been studied but cannot explain in detail based on the multiplication requirements of a matrix. Interview quote that "Indeed, it can't be multiplied directly with Kg every type of flour and grapes". Intrinsic certainty is a certainty of intuition cognition is usually associated with certain feelings of intrinsic certainty (Fischbein, 1999). Intrinsic means there is no external support is needed to obtain a direct certainty (either formally or empirically). It can be concluded that MZLTA has global anticipatory intuition and intuition in the form of real mathematical thinking. While in solving problems students do not easily give up and

successfully solve the problems. This is by the characteristics of high-level students who are always ready to face the obstacles that exist and not easily give up (Stoltz, 2004).

Re-testing the matrix problem can be concluded that MZLTA does not have conclusive intuition because he only believes in the results he found without checking in other ways whether the answer is correct or not. The results of the interview quote, "I'm sure that I don't have to check again". Conclusive intuition is an effort to summarize in general with the main of a problem solving (Fischbein, 1987). Students also do not re-check the results to ensure correct answers. Actually, in convincing students' answers emphasize the results of achievement in accordance with the characteristics of the type of climbers (Stoltz, 2004).

2. Linear Equation System of Two Variables Material

2. Polya x
Jill y

$$\begin{array}{r} 5x + 3y = 18000 \\ 3x + 7y = 27000 \end{array} \quad \begin{array}{r} 15x + 9y = 54000 \\ 15x + 21y = 147000 \\ \hline -12y = -93000 \\ y = 7750 \end{array} \quad \begin{array}{r} 5x + 3y = 18000 \\ 5x = 18000 - 10500 \\ = 7500 \\ x = 1500 \\ 72x = 18000 \end{array}$$

$$4(12x) + 7(12y) = 4(42000) + 7(93000) \quad 4(16000) + 7(47000)$$

$$= 168000 + 651000 \quad = 64000 + 329000$$

$$= 819000$$

In understanding Linear Equation System of Two Variables's problem, can be concluded that MZLTA cannot immediately understand the problem, but he need to repeat on the text of the question to understand it, so he does not have the characteristics of intuition. In accordance with Polya's statement (1985), students are said to understand if they can express the questions and answers as follows: (a) what information can be known from the questions, (b) what is the main problem of the questions that require solving, (c) is there any important conditions that need attention from the questions. It can be concluded that MZLTA does not have an *affirmatory intuition* in understanding, because according to *Fischbein* (1987), *affirmatory intuition* can be in the form of statements, representations, interpretations, solutions that can be individually accepted, self-evident, global and intrinsically sufficient. In accordance with the opinion from Stoltz (2004), students have no enthusiasm when faced with a problem.

During the problem-solving plan development, it concluded that MZLTA has intuition characteristics including Extrapolative and Coerciveness. Extrapolative characteristics happen when MZLTA guess the steps used are based on what has been learned. During the thinking plan step, the students have to be able to think about what steps are important and support each other to solve the problems they face (Polya, 1987). The right-thinking ability can only be done if students have been provided with sufficient knowledge before, which mean that the problem is not new at all but similar or approaching. Whereas the MZLTA believes that the results obtained through the steps that have been implemented, so he has Coerciveness intuition characteristic. MZLTA also does not work step by step, so he has a jump in thinking. This is in accordance with the characteristics of anticipatory intuition based on *Fischbein* (1987), which presents the steps in carrying out a problem-solving plan globally. Thus it can be concluded that MZLTA has anticipatory intuition. Besides that because in making the plan obtained by students after observing the previous pattern,

the type of intuition is anticipatory intuition. Students still have the ability to face difficulties even though their abilities are small. This is in accordance with the characteristics of high students (Stoltz, 2004).

MZLTA has intuition characteristics, including Coerciveness and Globality during problem-solving of Linear Equation System of Two Variables. Coerciveness characteristics occur when MZLTA believes in concepts and formulas or equations that have been prepared in the second stage with the steps used are correct. According to Polya (1985), the considerations taken related to the statement, were at this stage, students carry out the calculation process in accordance with the plans that have been prepared, also provided with all data's and information's needed, so that students can solve the problems they face properly and correctly. When MZLTA is difficult to explain why it must be just the left variable, there are Globality intuition characteristics. The considerations taken related to the statement, where at this stage students carry out the calculation process in accordance with the plans that have been prepared, also provided with all data's and information's needed, so that students can solve the problems they face properly and correctly (Polya, 1985). From these explanations, it can be concluded that MZLTA has anticipatory intuition based on its characteristics after solving problems and producing real mathematical thinking. This is in accordance with the characteristics of high-level students namely stopping in the midst of challenges, easily to be discouraged, avoiding obligations, retreating and rejecting the coming opportunities (Stoltz, 2004: 18-37).

In re-testing, the Linear Equation System of Two Variables problem can be concluded that MZLTA does not have the characteristics because he does not check it in other ways whether the answer is correct or not, so it does not have conclusive intuition. Conclusive intuition is an effort to summarize in general with the main problem solving (Fischbein, 1987). At the stage of checking the solution, students do not carry out any activities. So that it can be said that in examining solutions students do not use their intuition. This is in accordance with the nature of high-level students, which is they do not want to try and reject the opportunity (Stoltz, 2004).

ANPTA subject

1. Linear Equation System of Three Variables Material

1. Ditk. $x = \Delta$ biru
 $y =$ kunci biru
 $z =$ Cakra Kembar

$$\begin{cases} x + 2y + 3z = 52.000 \dots (1) \\ 2x + 3y = 39.500 \dots (2) \\ y + z = 17.000 \dots (3) \end{cases} \text{ a.}$$

$$\begin{array}{r|l} x + 2y + 3z = 52.000 & \times 2 \\ 2x + 3y = 39.500 & \times 1 \end{array} \quad \begin{array}{r} 2x + 4y + 6z = 104.000 \\ \underline{2x + 3y = 39.500} \\ y + 6z = 64.500 \dots (4) \end{array}$$

$$\begin{array}{r} x + 6z = 64.500 \\ \underline{x + z = 17.000} \\ 5z = 47.500 \\ z = 9.500 \end{array} \quad \left. \begin{array}{l} y + z = 17.000 \\ y = 17.000 - 9.500 \\ y = 7.500 \\ \underline{y = 9.500} \end{array} \right\} \text{ b.}$$

$$\begin{array}{l} 2x + 3y = 39.500 \\ 2x = 39.500 - 3(9.500) \\ 2x = 39.500 - 28.500 \\ 2x = 10.500 \\ x = 5.250 \end{array}$$

$$\begin{array}{l}
 c. \quad x = 5.250/\text{kg} \\
 30 \text{ kg/hari} \\
 -10\% \text{ utk karyawan} \\
 x \cdot 30 \text{ kg/hari} = 5250 \times 30 \\
 = 157.500 \\
 -10\% = 157.500 \times \frac{10}{100} \\
 = 15.750 \\
 157.500 - 15.750 = 141.750
 \end{array}
 \quad
 \begin{array}{l}
 d. \quad y = 9.500 \\
 30 \text{ kg/hari} \\
 -10\% \text{ utk karyawan} \\
 y \cdot 30 \text{ kg/hari} = 9.500 \times 30 \\
 = 285.000 \\
 -10\% = 285.000 - 28500 \\
 = 256.500
 \end{array}$$

$$\begin{array}{l}
 e. \quad z = 7.500/\text{kg} \\
 30 \text{ kg/hari} \\
 -10\% \text{ utk karyawan} \\
 z \cdot 30 \text{ kg/hari} = 7500 \times 30 \\
 = 225000 \\
 -10\% = 225.000 - 22500 \\
 = 202.500
 \end{array}$$

$$\begin{array}{l}
 f. \quad x = 30 \text{ kg/hari} = 141.750 \\
 y = 30 \text{ kg/hari} = 256.500 \\
 z = 30 \text{ kg/hari} : \\
 g. \quad \begin{array}{l}
 x \cdot 30 \text{ kg/hari} = 157.500 \\
 y \cdot 30 \text{ kg/hari} = 285.000 \\
 z \cdot 30 \text{ kg/hari} = 225.000 \\
 \hline
 = 667.500
 \end{array} \\
 -10\% = 667.500 - 66750 \\
 = 600.750
 \end{array}$$

g. pada poin f, bukan nilai sisa pendapatan setelah dipotong 10% utk untuk karyawan yg diambil untuk dijumlahkan, melainkan sebelum dipotong 10%, karena hasil pemotongan nilai akan berbeda.

Based on the test results and the interview obtained, ANPTA subject in understanding the matrix problem has characteristics of intuition: Direct, Self-evident and Coercive. *Direct* cognition, *self-evident* cognition, is accepted as an individual feeling without further checking and verification (Fischbein, 1999). ANPTA immediately understood the Linear Equation System of Three Variables problem by writing it down and being asked, the results of the interview were "I made it known and asked about it". The characteristics of students in understanding the problem is that students can express the questions and answers as follows: (a) what information can be known from the problem, (b) what is the core of the problem that requires solving, (c) is there any important requirements that need to be noted from the problem (Polya, 1987). Whereas *Coerciveness intuition* shown when ANPTA feels confident about what is known and being asked based on what it has written, the results of the interview quote that "When I read the question, I was immediately able to catch what was known and asked as I wrote, I was very sure like that". *Coerciveness* is the intuition that has the nature of leading towards something believed (Fischbein, 1999). This means that individuals tend to reject alternative interpretations that will contradict with their intuition. From the explanations above, it can be concluded that ANPTA has *affirmative intuition*. Whereas students mention what is known and asked directly and smoothly according to the characteristics of high-level students that will try to achieve success regardless of profit or loss, good fortune or bad luck (Stoltz, 2004).

In preparing the problem-solving plan, ANPTA has the characteristics of Intrinsic Certainty intuition when forming Linear Equation System of Three Variables at point a, which is related to

something that has been studied but cannot explain in detail based on the terms of multiplication of a matrix. Interviews quote "its juts like that miss". Intrinsic certainty is the certainty of intuition cognition usually associated with certain feelings of intrinsic certainty (Fischbein, 1999). Intrinsic means that there is no external support needed to obtain a direct certainty (either formally or empirically). ANPTA also believes that the plan was written based on mathematical thinking in real terms after she examined the question, so she has the characteristics of Coerciveness intuition. The results of the interview quote that "I am more confident like the steps I've written". Coerciveness is the nature leading towards something that is believed. Thus, it can be concluded that ANPTA has anticipatory, global intuition and intuition in the form of mathematical thinking in real (Fischbein, 1999). Students mention what will be planned directly and smoothly according to the characteristics of high-level students that will try to achieve success regardless of profit or loss, good fortune or bad luck (Stoltz, 2004).

During the problem solving, ANPTA believes that the preparation of the problem is written based on mathematical thinking in real terms after she looks at the questions, so she has the characteristics of Coerciveness intuition. The results of the interview quote that "More confident like the steps I've written". Coerciveness is the nature leading towards something that is believed (Fischbein, 1999). ANPTA also has Intrinsic Certainty intuition characteristics when completing Linear Equation System of Three Variables. Interview quote "It's just like that, Miss". Intrinsic certainty is the certainty of intuition cognition that usually associated with a certain feeling of intrinsic (Fischbein, 1999). Thus it can be concluded that ANPTA has anticipatory intuition that is global and has an intuition in the form of mathematical thinking in real. Stoltz (2004) mentioned that the students still have the ability to face the problems, eventhough with low ability.

Based on Re-testing the matrix problem activity can be concluded that ANPTA does not have conclusive intuition because she only believes in the results found without checking in other ways whether the answer is correct or not. The results of the interview quote, "No Miss, I'm sure I don't need to check it again". Conclusive intuition is an effort to summarize in general with the core of a problem solving (Fischbein, 1999). At the stage of checking the solution, students do not carry out any activities, in this time means that students do not use intuition. This is in accordance with the nature of high-level students who are unwilling to try and reject the existing opportunities (Stoltz, 2004).

2. Linear Equation System of Two Variables Material

$$\begin{aligned}
 & \textcircled{2} \text{ Dik. } 5p + 3j = 18.000 \\
 & \quad 3p + 7j = 29.000 \\
 & \text{Dit. } 9(12)p + 3(12)j = ? \\
 & \begin{array}{r}
 5p + 3j = 18.000 \quad | \times 3 \rightarrow 15p + 9j = 54.000 \\
 3p + 7j = 29.000 \quad | \times 5 \rightarrow 15p + 35j = 145.000 \quad - \\
 \hline
 -26j = -91.000 \\
 26j = 91.000 \\
 j = 3500
 \end{array} \\
 & \begin{array}{l}
 5p + 3j = 18.000 \\
 5p + 3(3500) = 18.000 \\
 5p = 18.000 - 10.500 \\
 5p = 7.500 \\
 p = \frac{7500}{5} = 1500.
 \end{array} \\
 & 9(12)p + 3(12)j = 9(12)(1500) + 3(12)(3500) \\
 & \quad = 98(1500) + 36(3500) \\
 & \quad = 72000 + 126000 \\
 & \quad = 198.000.
 \end{aligned}$$

In understanding the Linear Equation System of Three Variables problem, it can be concluded that ANPTA can immediately understand the problem by writing down what it 'known and 'asked based on the text of the question so that she has *direct, self-evident* characteristics. In accordance

with the theory of *Fischbein* (1999), *Direct, self-evident* is cognition that is accepted as an individual feeling without requiring further checking and verification. ANPTA found it difficult to explain what she has written so that ANPTA has the characteristics of intuition, including *globality*. According to *Globality*, global cognition is contrary to cognition obtained logically, sequentially, and analytically (*Fischbein*, 1999). Students can be understood if they are able to analyze the questions and write down what is 'known and 'asked in the question. So it can be concluded that ANPTA has *affirmatory intuition* (*Polya*, 1985). In contrast, students mention what is known and ask directly and smoothly according to the characteristics of high-level students that will try to achieve success regardless of profit or loss, good fortune or bad luck (*Stoltz*, 2004).

During a plan completion for resolving linear equation of two variable problems, it can be concluded that ANPTA has the characteristics of Intrinsic Certainty, Extrapolative, and Coerciveness. In accordance with the theory of *Fischbein* (1999), Intrinsic Certainty is the certainty of intuition cognition that is usually associated with certain intrinsic feelings. Intrinsic means that no external support is needed to obtain a direct certainty (either formally or empirically). Extrapolative characteristics occur when ANPTA predicts that the concepts or formulas used have been studied. The stage of the thinking plan, students must be able to think about what steps are important and support each other to solve the problems (*Polya*, 1987). The right-thinking ability can only be done if students have been provided before with sufficient knowledge, which means that the problem faced is not new but similar or approaching. In comparison, Coerciveness occurs when ANPTA believe that the formula or concept arranged is correct. From this it can be concluded that ANPTA has anticipatory intuition based on its characteristics after trying to examine the text and generate mathematical thinking in real. The students can plan some solutions to solve the problems; this is in accordance with the characteristics of campers students, namely responding to the challenges that exist (*Stoltz*, 2004).

In resolving the Linear Equation System of Three Variables problem, can be concluded that ANPTA believes that the results obtained through the steps that have been implemented, so she has the characteristics of Coerciveness and Globality intuition. ANPTA also does not work step by step, so it has a jump in thinking. This is in accordance with the characteristics of anticipatory intuition based on *Fischbein* (1987) that is presenting globally the steps in implementing a problem-solving plan. Meanwhile, when ANPTA is difficult to explain why it must be just the left variable, so that is characteristics of Globality intuition. Considerations taken related to the statement, where at this stage students carry out the calculation process in accordance with the plans that have been prepared, also provided with all data's and information's needed, so that students can solve the problems properly and correctly (*Polya*, 1985). Thus it can be concluded that ANPTA has anticipatory intuitions based on mathematical thinking in real terms. During problem solving, students do not easily give up and successfully solve problems. This is in accordance with the characteristics of high-level students who are always ready to face the obstacles that exist and not easily give up (*Stoltz*, 2004).

In re-testing the Linear Equation System of Three Variables problem, ANPTA does not have conclusive intuition because she does not re-test the results in other ways. While at the *Polya* stage, the steps to re-examine the problem must be done not to believe the results obtained without checking in other ways. At the stage of checking the solution, students do not carry out any activities, means that in examining solutions, students do not use intuition. This is in accordance with the nature of high-level students. For example, they do not want to try and reject the coming opportunity (*Stoltz*, 2004).

During the understanding of the problem, male and female climber students used the sensory affirmatory intuition on Linear Equation System of Three Variables material while male climber students did not use intuition on Linear Equation System of Two Variables material. During the problem planning stage, male and female climber students use affirmatory intuition based on their

senses and mathematical intuition in real terms on Linear Equation System of Three Variables and Linear Equation System of Two Variables material. While carrying out problem-solving, male and female climber students use sensual and mathematical intuition anticipatory in real terms on Linear Equation System of Three Variables and Linear Equation System of Two Variables material. During the stage of examining the solutions, the climber students did not use conclusive intuition on Linear Equation System of Three Variables and Linear Equation System of Two Variables material.

MZLTA

Linear Equation System of Three Variables material has affirmatory intuition and anticipatory in Understanding the problems, Developing the problem solving plan, and Implementing the problem solving plan. While the Re-test the problem does not have conclusive intuition. In the material of the Linear Equation System of Two Variables only has intuition on the Developing the problem solving plan and Implementing the problem solving plan. Whereas in Understanding the problem and Re-testing the problem does not have intuition.

ANPTA

Linear Equation System of Three Variables material has affirmatory intuition and anticipation in Understanding the problems, Developing the problem-solving plan, and Implementing the problem-solving plan. While the Re-test, the problem does not have conclusive intuition. The material of Linear Equation System of Two Variables has intuition on Understanding the problem, Developing the problem-solving plan, and Implementing the problem-solving plan. Whereas in the Re-test the problem does not have intuition. The characteristics possessed at each step of the solution between the subjects MZLTA and ANPTA vary.

Intuition and characteristic difference of each students based on their Mathematical abilities. Students way of learning are variety depends on their own strengths and weaknesses. It means assessing the learning progress of students in the same way for each student will not lead to appropriate student intuition. The situation that often occurs in teaching mathematics is that student acceptance is intuitively at odds with formal mathematical concept and result in cognitive conflict and even cognitive bias that can hinder students from learning mathematics.

In this case, learning must reconstruct mathematical intuition and basic knowledge of the students. It helps students overcome these difficulties by making them aware of the conflict and help them to understand the facts in mathematics that lead to understanding the correct concepts. The most favourable situation in learning mathematics is where students' intuition with mathematical concepts is formally in line. Usually, students in trivial situations interpret mathematical facts by referring to concrete reality and regard formal evidence as an excessive demand. The implication is that students are directed to understand formal deductive thinking in mathematics. Intuitive acceptance of mathematical statements does not exclude the necessity to fulfil a formal deductive mathematical structure, strictly in accordance with axiomatic.

The next stage of the study will guide the teacher when designing learning activities based on the ability improvement to solve mathematical problems. The teacher should pay attention to students' intuition. In this study, there is an anticipatory intuition that is contradicted generally. Intuition is based on the senses and imagination that tends to be unable to solve the problem correctly and not complete. Therefore, the teacher should design learning activities that can train students with such intuition so that intuition is obtained, which can be used to solve problems correctly. Learning must often be given to problem-solving using the stages of the pattern. It aims to further explore students' intuition in solving mathematical problems.

CONCLUSION

In the subject of SMKN 2 Banda Aceh female students in understanding mathematical problems use affirmative intuition in formulating problem-solving plans and problem-solving using anticipatory intuition, while in re-examining the problem there is only one subject who has conclusive intuition, namely drawing conclusions directly, summarizing in general with the basic

idea of the problem that has previously been occupied. Furthermore, male students in understanding mathematical problems use affirmative intuition, in preparing plans for solving and solving problems using anticipatory intuition, namely the emergence of thought when trying hard to solve the problem then in re-examining the problem there is not a single subject who has good conclusive intuition on the material SPLTV and SPLDV.

On the subject of SMAN 3, Banda Aceh female students understand mathematical problems using affirmative intuition, in formulating problem-solving plans and solutions using anticipatory intuition and then in re-examining the problem, there is only one subject who has conclusive intuition, namely drawing conclusions directly, summarizing in general with the basic idea previously dealt with problems. In male students in understanding mathematical problems using affirmative intuition, in preparing problem-solving plans and solving them using intuition then in re-examining the problem, there is not a single subject who has conclusive intuition in both SPLTV and SPLDV material.

Affirmative intuition on understanding issues, anticipatory intuition for planning problems, and anticipatory intuition for solving problems are all examples of the four different types of intuition that can be found in solving problems using Polya's steps.

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REFERENCES

- Abidin, Z. (2012). *intuisi siswa dalam memecahkan masalah matematika divergen berdasarkan gaya kognitif field independent dan field dependent*. Disertasi, Unesa Surabaya.
- Agus Sukmana. 2011. A Study of the Role of Intuition in Students' Understanding of Probability Concepts. *Proceeding of the International Conference on Numerical Analysis and ZAQ Optimization (ICeMATH2011)*. UAD. (Artikel web).
- Baron. (2012). *Social Psychology 13th Edition*. New Jersey: Pearson Edition.
- Burton, L. (1999). Why is intuition so important to mathematicians but missing from mathematics education? *For the Learning of Mathematics*, 19(3), 27-32.
- Bouillion, L. M., & Gomez, L. M. (2001). Connecting school and community with science learning: Real world problems and school–community partnerships as contextual scaffolds. *Journal of Research in Science Teaching: The Official Journal of the National Association for Research in Science Teaching*, 38(8), 878-898.
- Caprioara, D. 2015. *Problem Solving- Purpose a Means of Learning Mathematics in School*. *Romania Journal of Social and Behavioral Science University of Ovidius Constanta*, 191, 1859-1864.
- Djamarah, S.B, dan A. Zain. 2002. *Strategi Belajar Mengajar*. Rineka Cipta. Jakarta.
- Enstein, S. (1995). Integration of the cognitive and the psychodynamic unconscious. *American Psychologist*, 49, 709-724.
- Fischbein, E. (1983). Intuition and Analytical Thinking in Mathematics Education. *International Reviews on Mathematical Education*. 15, 2, 68-74.
- Fischbein, E. (1987). *Intuition in Science and Mathematics*. Dordrecht: D. Reidel.
- Fischbein, E. (1994). The Interaction between the Formal, the Algorithmic, and the Intuitive Components in a Mathematical Activity. In R. Biehler, R. W. Scholz, R. Sträßer, & B. Winkelmann (Eds.), *Didactics of Mathematics as a Scientific Discipline* (pp.231-245). Dordrecht: Kluwer Academic Publishers.
- Fischbein, E., Grossman, A. (1997). Schemata and Intuitions in Combinatorial Reasoning, *Educational Studies in Mathematics* 34, 27–47.
- Fischbein, E. (1999). Intuitions and Schemata in Mathematical Reasoning. *Educational Studies in Mathematics*. 38,11–50.

- Geary, D.C., 2000. Sex Differences in Spatial Cognition, Computational Fluency, and Arithmetical Reasoning. *Journal of Experimental Child Psychology*. 77, 337-353.
- Giardino, V. 2010. Intuition & Visualization in Mathematical Problem Solving. *Topoi Research Journal*. Vol.29, hlm. 29-39.
- Jatisunda, M. G., & Nahdi, D. S. (2019). Peran Mathematical Intuition dalam Pembelajaran Matematika. *JUMLAHKU: Jurnal Matematika Ilmiah STKIP Muhammadiyah Kuningan*, 5(2), 12-24.
- Liljedahl, P. G. 2004. *The Aha! Experience: Mathematical Contexts, Pedagogical Implications* Disertasi, Simon Fraser University, Burnaby, BC Canada.
- Markaban, A. 2015. "Profil Kemampuan Spasial dalam Menyelesaikan Masalah Geometri Siswa yang Memiliki Kecerdasan Logis Matematis Tinggi Ditinjau Dari Perbedaan Gender" *Jurnal Daya Matematis* (Vol. 3/No.1/Maret 2015). Jakarta.
- Mullis, et. al. 2011. *TIMSS 2007: International Mathematic Report*. Boston: TIMSS & PIRLS International Study Center.
- Mudrika, Mega Teguh Budiarto. 2013. -Profil Intuisi Siswa Smp Dalam Memecahkan Masalah Geometri Ditinjau Dari Kemampuan Matematika Siswal, *Jurnal Pendidikan Matematika Fmipa, UNESA*. Vol.1 No.1
- Nazariah. (2015). Intuisi Siswa SMK Dalam Memecahkan Masalah Matematika Ditinjau Dari Kemampuan Matematika Dan Perbedaan Gender. Disertasi, UNSYIAH Banda Aceh.
- Polya, G. (1981). *Mathematical Discovery, On Understanding, Learning, and Teaching Problem Solving*. United States of America.
- Polya, George. 1985. *How To Solve It* 2nd ed. New Jersey : Princeton University Press.
- Raman. (2017). Primary School Teachers Self Efficacy in Handing School Bullying: A Case Study. *International Journal of English Literature and Social Science*, 4 (2), 187-202.
- Rustini, T. 2008. *Penerapan Model Problem Solving untuk Meningkatkan Pengembangan Potensi Berpikir Siswa Dalam Pembelajaran IPS di Sekolah Dasar*. *Jurnal*. Universitas Pendidikan Indonesia. Bandung.
- Rushton. (2010). Neuroscience, Play, and Early Childhood Education: Connections, Implications, and Assessment. *Early Childhood Educ J*. 37:351-361.
- Santrock, J.W (2011). *Life Span Development* (13 ed). New York: McGraw-Hill Companies, Inc.
- Triyadi, Rudini. 2013. *Kemampuan Matematis Ditinjau dari Perbedaan Gender*. Jakarta: Skripsi tidak diterbitkan.
- Budi Usodo. 2012. Karakteristik Intuisi Siswa SMA dalam Memecahkan Masalah Matematika Ditinjau dari Kemampuan Matematika dan Perbedaan Gender. *AKSIOMA*, Vol. 01, No. 01, 1 - 14.
- Westcott, M. (1968). *Toward a contemporary psychology of intuition*. New York: Holt, Rinehart, and Winston.