

Students' Metacognitive Ability in Mathematical Problem Solving through the Problem-based Learning Model

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Abstract. *Students lack metacognitive ability despite its vital role in mathematical problem solving. The problem-based Learning (PBL) model is one of the learning models to improve metacognitive ability in problem solving. This study aimed to analyze the students' metacognitive ability in mathematical problem solving through PBL and examine its improvement. This present study applied the explanatory sequential mixed-method design. The population was the Year 11 students from one of the senior high schools in South Aceh Regency, Indonesia. Data collection was conducted using three instruments: pre-test, post-test, and interview guidelines. The pre-test and post-test data were analyzed using the t-test, while students' metacognitive ability was analyzed qualitatively. The results showed that students' metacognitive ability in mathematical problem solving through the PBL model was increased. Furthermore, students' metacognitive ability was at the semi-reflective use, the strategic use, the aware use levels for high-ability, medium-ability, and low-ability groups.*

Keywords: *problem solving, metacognitive ability, problem-based learning model*

Introduction

Based on mathematics learning objectives, students must have problem solving skills, including understanding, designing mathematical models, solving, and interpreting the solutions (Depdiknas, 2006). Problem solving activities include cognitive activities of applying previous knowledge and experience; problem solving activities are said to be successful if they can generate a new conclusion as a solution (Lester & Kehle, 2003, Vijayan & Joshith, 2018). Appropriate learning methods, techniques, and strategies can foster and enhance problem solving activities (Yazgan & Bintas, 2005, Tertemiz, Celik & Dogan, 2014). One effort to improve and develop problem solving skills is by involving metacognition.

Metacognition is one's knowledge and awareness of her/his cognitive processes and the ability to monitor, manage and evaluate them (Flavell 1976). Metacognitive ability in problem solving is classified into six levels: Tacit use, Aware use, Semistrategic use, Strategic use, Semi-reflective use, reflective use (Lauren, 2010). The components observed at each level are the planning, monitoring, and evaluation stages. Tacit use is a form of thinking when one decides without thinking, such as a student using a strategy or skill for trial and error and solving a problem randomly. Aware use is a type of thinking when one is aware of what and why of

the thought. For example, a student knows when to apply the problem solving stage and explains why the step was chosen. Semistrategic use is when students realize the mistakes in their thinking processes when solving mathematical problems, and they need help to obtain the right strategy. Strategic use is when students can manage their thinking processes using specific strategies that can increase the accuracy of their thinking. For example, a student knows and selects specific strategies or skills in solving mathematical problems. Semi-reflective use is a type of thinking that applies reflection before, after, or during the thinking process by considering the continuous improvement of the thinking results, but not always applied to each step in solving mathematical problems. Reflective use is applying reflection before, after, or during the thinking process by considering the further improvement of the thinking results.

Metacognitive ability aims to regulate one's cognitive activity in solving problems and completing tasks (Okoza & Aluede, 2014). Involving students' metacognitive ability means making students aware of their learning styles and implementing the most effective problem solving strategies (Huang & Ricci, 2016). Using metacognitive ability in solving mathematical problems helps students control their cognitive activities and find appropriate solutions. Therefore, students must have the metacognitive ability and apply it to solve mathematical problems; however, students' metacognitive ability is lacking as they are not aware of their mistakes and confused in determining information and the problem solving process (Arum, 2017). Furthermore, students are not fully aware of the thinking process and can not correct the errors in the problem solving steps (Siagian, Saragih & Sinaga, 2019). The preliminary research conducted by researchers at school studied, a senior high school in South Aceh, Indonesia, showed that students' metacognitive ability was poor. Based on the analysis results of student answers and the interview results, the researchers found that students' metacognitive ability in problem solving satisfied the planning indicator. Students could mention the important information, things asked, and reasons for choosing the concept, however they could not explain all steps done to solve the problem and only explain what they wrote. They also could neither find the solutions nor problem solving process. Metacognitive ability in mathematical problem solving is classified as poor when it only involves one metacognition activity: planning (Sudia, 2015). Thus, students' metacognitive ability should be developed by applying the Problem-based Learning (PBL) model.

PBL is a model providing authentic and meaningful problems for students as a stepping stone for investigation (Arends, 2007). The characteristics of PBL is challenging problems with complex and real-life context encouraging students to seek knowledge from multiple sources and opinion exchange (Jaisook, Chitmongkol & Thongthaw, 2013). PBL can also develop students' high learning metacognition and improve mathematical problem solving skills

(Rahman, Yurniwati & Bintoro, 2018). In addition, the PBL model is effective in improving students' problem solving and metacognitive skills (Siagian, Saragih & Sinaga, 2019). This is also in line with the research of Rofik (2018) and Wulandari (2018).

PBL consists of five steps: orienting students to problems, organizing students to learn, assisting individual and group investigations, developing artifacts or the work and presenting them, and analyzing and evaluating the problem solving process (Arends, 2007). The PBL model can be viewed based on the learning and problem characteristics. The characteristics of PBL learning are proposing problems, relating to other disciplines, authentic investigations, creating and exhibiting works, and collaborating in small groups (Arends, 2004, Johar & Hanum, 2016). On the other hand, the PBL model's characteristics are everyday life problems with complexity level and open-ended problems with many solutions or solving strategies that encourage students to be curious and identify strategies and solutions (Rusman, 2012, Fathurromah, 2016).

Several studies on students' metacognition in mathematical problem solving using the PBL model have been conducted. Rofik (2018) argued that in solving mathematical problems, the metacognitive ability is strongly influential. Students with low metacognitive ability will have difficulty implementing appropriate strategies in solving mathematical problems; high-ability students are more systematic in solving mathematical problems than moderate- and low-ability students. Another study by Siagia, Saragih, and Sinaga (2019) revealed that the second trial of PBL learning materials developed effectively improved students' problem solving skills, as indicated by the increase of the pretest and posttest results. Wulandari (2018) also reported that students with high problem solving skills were at the metacognition level of reflective use and use strategy; medium-ability students were at the aware use level, and low-ability students were at the tacit use level. Among the challenges experienced by students in mathematical problem solving are difficulties in understanding facts, concepts, principles, and procedures.

Based on the studies mentioned above, none has studied the improvement of students' metacognitive ability through the PBL model. Therefore, this research on students' metacognitive ability in mathematical problem solving through PBL was conducted. The research questions are as follows: Is there any increase in students' metacognitive ability in mathematical problem solving through the PBL model? and What is the level of students' metacognitive ability in mathematical problem solving through the PBL model?

Method

This research employed an explanatory sequential mixed-method approach where the researchers first collect quantitative data before analyzing and managing the results to explain in

detail with qualitative research (Creswell, 2016). The research was conducted at one of the public senior high schools in South Aceh, Indonesia. The population was 26 Year 11 science and mathematics stream students, while the samples were 12 Year 11 students. The limited samples in this study were because the research conducted during the COVID-19 pandemic, so it must comply with health protocols. The interview subjects were three students selected who met the high, medium and low categories. One student represented each category.

The instruments were pre-test and posttest problems of metacognitive ability and interview guidelines. Previously, experts validated the tests, and the results showed that the metacognitive ability test items were good and could be used. The tests were also tested for empirical validity, the results met the valid criteria, with the reliability level of 0.94. These results mean that the problems have good reliability. The test items for the difference between items meet sufficient criteria. The difficulty level trial shows that the problems are with moderate difficulty. The pre-test and post-test metacognitive ability in mathematical problem solving was scored on each metacognitive ability aspect met: planning, implementation, monitoring and evaluation. The score is one to four, with the description for each score. One of the metacognitive ability test problems in mathematical problem solving is presented in Figure 1.

A toy company will make a money box of used goods. The company plans to create a money box made of a used can with a surface area of $448\pi \text{ cm}^2$ and a lid that covers the can at 2 cm depth. If the radius of the can base is $x \text{ cm}$, and the height is $h \text{ cm}$, help the company determine the maximum volume of cans for the money box! Explain your answer!

Figure 1. Metacognitive ability test problem in mathematical problem solving

This present study applied technical data triangulation to examine the credibility of the data by re-checking data from the same source with a different technique. In this case, it was comparing the results of the metacognition ability test and the interview results. Data analysis in this study was conducted in two ways: the quantitative data analysis technique: paired sample t-test using SPSS 16 after satisfying the normality criteria. Furthermore, N-Gain with the criteria of Meltzer (2002) was used to calculate the magnitude of the increase in metacognitive ability in mathematical problem solving based on the pretest and posttest scores, as shown in Table 1. Qualitative data analysis involved the interview data in the form of words (the reduction stage), data presentation and concluding.

Tabel 1. *Criteria for N-gain score*

Gain Score	Interpretation
$g > 0,7$	Excellent
$0,3 < g \leq 0,7$	Medium
$g \leq 0,3$	Low

Results and Discussion

Based on the research questions, the following sections will explain the increase in students' metacognitive ability in mathematical problem solving using the PBL model.

The improvement of Students' Metacognitive Ability in Mathematical Problem Solving Using the PBL Model

Data on the improvement of students' metacognitive ability were generated by comparing the pretest and posttest scores and analyzing them using statistical tests. The magnitude of the difference of the pretest and posttest scores resulted in an N-Gain value. Table 2 presents the descriptive statistics of the pretest, posttest, and N-Gain score data.

Table 2. Data on students' metacognitive ability scores in mathematical problem solving

Ability	Skor	N	\bar{x}	SD
Metacognition	Pretest	12	28.17	7.94
	Posttest	12	58.08	7.85
	N-Gain	12	0.73	0,5
Maximum score	= 70			

Table 2 shows that the mean of metacognition ability pretest is 28.17 for the ideal maximum score of 70, while it is 58.08 for the posttest score. These results indicate an increase in students' metacognitive abilities in mathematical problem solving after learning through the PBL model. The N-Gain score of 0.73 indicates that the increase of students' metacognitive ability is very good ($\alpha = 0.05$).

The normality tests of the pretest and posttest scores were conducted by using the Shapiro Wilk test ($\alpha = 0.05$); the results are displayed in Table 3.

Table 3. Normality test results of the mean pretest of metacognition ability

Results	Shapiro –Wilk			Conclusion
	Statistics	Df	Sig-	
Pretest	0.955	12	0.708	H ₀ accepted
Posttest	0.933	12	0.411	H ₀ accepted

Table 3 reveals that the significance of pretest and posttest scores of students' metacognition abilities are = 0.708 and 0.411, ($\alpha = 0.05$), and H₀ is accepted. This result indicates that the sample comes from a normally distributed population. Based on the test results, it is known that students' initial abilities and final abilities are normally distributed.

The hypothesis of this study was ‘there is an increase in students' metacognitive ability in solving mathematical problem solving through the PBL model’. The detailed hypotheses are as follows.

H₀ : $\mu_1 = \mu_2$, There is no increase in students' metacognitive ability in mathematical problem solving through the PBL model

$H_a : \mu_1 > \mu_2$, There is an increase in students' metacognitive abilities in mathematical problem solving through the PBL model.

Hypothesis testing was under taken by paired-sample t-test. The H_0 is rejected if $t_{count} > t_{table}$ and the sig. (2-tailed) or p-value $< \alpha = 0.05$. The results of the paired-sample t-test of students' metacognitive ability are presented in Table 4.

Table 4. The paired-sample t-test results of students' metacognitive ability

		Paired Differences					T	Df	Sig(2-tailed)
		Mean	Std. Deviation	Std. Error Mean	5% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	pretest–posttest	29.917	3.825	1.104	29.987	29.846	27.096	11	.000

Table 4 shows p-value is 0.000 ($\alpha = 0.05$), t_{count} is 27.096 and t_{table} is 1.79. Thus, H_0 is rejected and H_a is accepted. It can be concluded that the hypothesis of "there is an increase in students' metacognitive ability in mathematical problem solving through the PBL model", is accepted. In other words, there is an increase in students' metacognitive ability in mathematical problem solving through the PBL model.

The N-Gain test was conducted to examine how well the students' metacognitive ability in mathematical problem solving through the PBL model was improved. The summary of the N-Gain results is displayed in Table 5.

Table 5. N-Gain results of metacognitive ability

Descriptive Statistics					
	N	Minimum	Maximum	Mean	Std. Deviation
NGain_Score	12	.50	1.00	.7387	.15480
Valid N (listwise)	12				

Table 5 reveals that mean of N-Gain of students' metacognitive ability in mathematical problem solving is 0.738, indicating that the increase in students' metacognitive ability was very good.

Students' Metacognitive Ability in Mathematical Problem Solving through the PBL Model

a. Results of High-Ability Student's Metacognitive Ability Analysis and Interview

Figure 2 describes the responses of the high-ability students to the test items. Later, based on the student's solution in Figure 2, an interview was conducted between the researcher (P) and student (S) and the following is the interview excerpt.

- P01 : Can you explain what to be solved in this problem!
 S01 : What is the value of x for the maximum volume of the can and the maximum volume of the can?
 P02 : Explain the concept you chose and why choosing it!
 S02 : Because a can is the same as a cylinder, I use the cylinder concept, namely the surface area and volume of a cylinder.
 P03 : Why you are sure that your answer is correct
 S03 : I am sure because I tested the point test too, where $x = 8$ cm, I substituted the area concept, and I got the area of the money box is $448\pi\text{cm}^2$, the same as what is known in the problem.

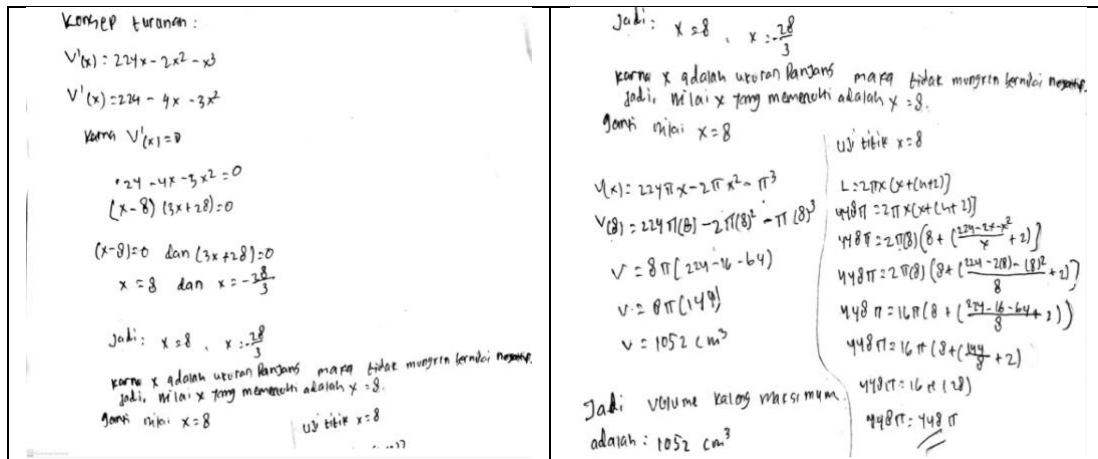


Figure 2. The solution of the high-ability student

The test and interview results indicate that the metacognition ability of Student 1 in mathematical problemsolving after learning with the PBL model was at the Semi-reflective use level. This finding was identified from the student’s solution. At the planning stage, the student understood the problem well. S/he could explain the problem and the strategies used. At the monitoring stage, the student believed that the steps done to solve the problem were correct, and realized that there was an error in the final result, as known from her/his answer: "I think, generally, it was correct and in line with the instructions of the problem, but I made mistakes in determining the maximum volume of the can, so the final result is incorrect". The was also confident that s/he could apply the same strategy to other similar problems. At the evaluation stage, the student could evaluate the solution to correct the mistakes.

b. Results of Medium-Ability Student’s Metacognitive Ability Analysis and Interview

Medium-ability student's responses to the test problem are presented in Figure 3. Based on the student’s solution in Figure 3, an interview was conducted between the researcher (P) and student (S), and the following is the interview excerpt.

- P01 : Can you explain the problem to be solved!
 S01 : The problem to be solved is the length of the cube edge for the maximum box volume.
 P02 : Explain the concept you chose and why choosing it!

- S02 : To solve this problem, I used the concepts of square and rectangle surface area and the volume of a cuboid.
- P03 : Explain why you are sure that your solution is correct!
- S03 : I am not sure about my solution because I made an error when determining the x. I made a mistake on the factoring part. I found out when I substituted the x to the formula for the cylinder surface area. I did not get the result that the surface area of the can was 448 cm³, as known from the problem.

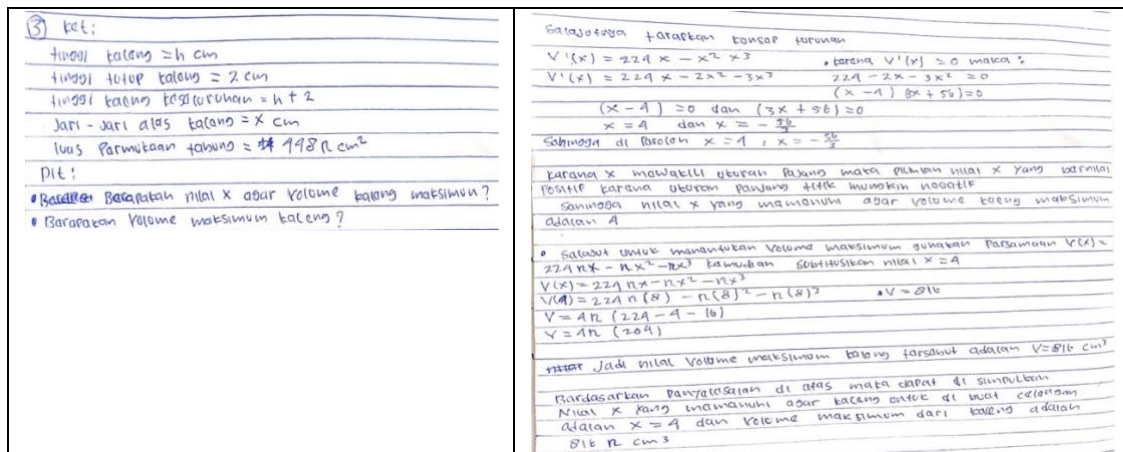


Figure 3. The solution of the medium-ability student

The test and interview results reveal that the metacognition ability of Student 2 in mathematical problem solving after the PBL model learning is at the Strategic use level. This was identified based on the student’s responses. At the planning stage, Student 2 understood the problem as s/he could express it clearly. The student had no difficulty and confusion in determining the concept and the calculation. He could also explain most of what he wrote. At the monitoring stage, the student realized the errors of concept and calculation method, and s/he provided supporting reasons of her/his thinking. The student did an evaluation at the evaluation stage but did not write it down on the answer sheet. However, the solution was incorrect because of the error in factoring the quadratic equation.

c. Results of Low-Ability Student’s Metacognitive Ability Analysis and Interview

The responses of low-ability students to the given test item can be seen in Figure 4. Based on the student’s solution in Figure 4, an interview was conducted and the following is the interview excerpt.

- P01 : Can you explain the information you know? Is your answer correct?
- S01 : The height of the cylinder I made was incorrect, it should be $h + 2$ cm
- P02 : Why did you not finish your answer!
- S02 : I forgot the formula for finding the cylindervolume, so after I found the overall height of the can, I stopped.
- P03 : In your opinion, do you think your answer is in line with the problem instruction? Explain!
- S03 : My answer is incorrect; I could only solve the problem up to determining the height of the can and did not continue.

$\text{Luas Permukaan Kalkes} = 448 \pi \text{ cm}^2$
 $\text{tinggi kalkes} = 2 \text{ cm}$
 $\text{tinggi kalkes keseluruhan} = h + 1$
 $\text{jari-jari kalkes} = x \text{ cm}$
 $\text{Luas Permukaan tabung} = 2\pi r(r+h)$

Dit :
 - berapakah nilai x agar volume kalkes maksimum?
 - berapakah volume maksimum kalkes

$\text{Penyelesaian Luas Permukaan Kalkes} = 2\pi r(r+h)$
 $448\pi = 2\pi r(r+h)$
 $448\pi = 2\pi r(x + (h+1))$
 $\frac{448\pi}{2\pi} = x(x + (h+1))$
 $224 = x^2 + hx + x$
 $x^2 + hx + x = 224$

Selanjutnya tentukan terlebih dahulu nilai h dengan tinggi kalkes
 $x^2 + hx + x = 224$
 $hx = 224 - x - x^2$
 $h = \frac{224 - x - x^2}{x}$

Selanjutnya tentukan volume kalkes dalam bentuk variabel x maka :

Figure 4. The solution of the low-ability student

The tests and interview results indicate that the metacognitive ability of Student 3 in mathematical problem solving after learning the PBL model was at the level of aware use. At the planning stage, students experienced difficulties and confusion in thinking about the concepts and methods of calculation; s/he only explained what s/he wrote. At the monitoring stage, the student was confused and could not continue. At the evaluation stage, the student did not evaluate.

These study results concluded that there is an increase in students' metacognitive ability in mathematical problem solving through the PBL model. These findings are supported by several previous studies (Merantasanai & Dwijanto, 2016; Hamimah & Kartika, 2019; Amir, 2018, Tosun & Senocak, 2013).

In PBL learning, students are trained to develop their metacognitive ability in mathematical problem solving involving everyday lifeproblems. PBL can encourage students to research by integrating theory and practice and apply knowledge and skills to develop appropriate solutions (Savery, 2015). This is also supported by the five syntaxes of the PBL model, starting from orienting students on real-world problems to the final stage of analyzing and evaluating the students' learning outcomes; students are guided at the investigation stage to compile the results, which improving their metacognitive ability through authentic problem solving (Fitriyani, Corembima & Ibrohim, 2015). Besides, PBL syntaxes develop thinking, problem solving, intellectual skills, allow students to experience adult roles through various real situations, and becoming independent learners (Arends, 2008).

The implementation of PBL model stages assists students to build their knowledge through learning activities; students will get used to using and seeking multiple sources of knowledge (libraries, the internet, interviews, and observations), supporting students in assessing their learning progress (Lindinillah, 2007). Another contributing factor is the purpose

of learning with the PBL model, which is to develop intelligence and competence for problem solving, and train and foster students' metacognitive ability in problem solving (Tan, 2013; Tan, Molen & Schmidt, 2016). In addition, applying the PBL model in learning can improve students' metacognition skills because it enables students and interact with their peers in solving problems (Kusumaningtias, Zubaidah & Indriwati, 2013). These advantages support the improvement of students' metacognitive ability in mathematical problem solving through the PBL model.

Based on the results of tests and interviews, it was found that the high-, medium- and low-ability students were at the semi-reflective use, the strategic use, and the aware use level, respectively. In addition, no student was at the lowest level (tacit use). This is because each student could explain their thinking results, the information guiding the problem solving and the problem correctly. Furthermore, each student was also aware of their mistakes. The results also showed no students satisfying the reflective use level because the test instruments used in this study cannot measure such ability. Thus, it is hoped that other studies can develop special instruments to measure and cater to students who are at the reflective use level.

Overall, students' metacognitive ability in mathematical problem solving was within the good category based on student's solutions and interview results. High-ability students could evaluate their solution and re-examine it using logical reasons. High and medium ability students can also provide their reasoning when choosing the concept applied to solve the problem. Furthermore, high- and medium-ability students also realized their errors during the process of mathematical problem solving. Students with high metacognitive awareness after learning with the PBL model could master all stages of problem solving (Achsini, Kartono & Wibawanto, 2019). Other research also concluded that students with the high metacognitive ability through the PBL model could understand problems quickly, analyze problems, use strategies accurately and quickly in the problem solving process, and always re-check the problem solving process; while students with low metacognitive ability are lacking in implementing problem solving strategies and do not re-examine every problem solving process (Rahman, Yurniwati & Bintoro, 2018, Anggo, 2011, Young 2010).

Conclusion

Students' metacognitive ability in mathematical problemsolving was improved through the PBL model. Based on the grouping of the metacognition level, the metacognitive ability of high-, medium, and low-ability students in mathematical problemsolving was at the semi-reflective use, the semi strategic use level, and the aware use level, respectively. No student was at the lowest (tacit use) or the highest level (reflective use). This finding was because the test

instruments used in this study cannot determine the ability of students at the reflective use level. Hence, other studies should develop special instruments to measure and cater to students at the reflective use level.

The PBL model is proven to improve students' metacognitive ability in mathematical problem solving. However, it is suggested for future researchers to develop a more effective PBL-based instrument to measure each level of metacognitive ability in mathematical problem solving.

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