

Hypothetical Learning Trajectory for Blind Students Using *Braille* Media Fraction Blocks

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Abstract. *Fraction is one of the difficult topics for students with visual impairments. Therefore, it is necessary to develop a learning trajectory that can help the students understand fractions. This design research aims to describe the design process of hypothetical learning trajectory for the addition of fractions using braille fraction blocks. This research is divided into three stages: the preparation for the experiment (design), the implementation of the teaching experiment, and the retrospective analysis. Data collection techniques employed in this research were walkthrough, observation, interview, and test. The product of this research is hypothetical learning trajectory (HLT) for the addition of fractions that contains the following activities: comparing unit block and fraction blocks, comparing the size of different fraction blocks to get the same size fraction blocks (equal fraction), comparing the sizes of two fraction blocks, adding fraction blocks, and determining the fraction block with the same size as the added fraction block. Those five activities were carried out in the two-cycle experimental activities. After the implementation of the activities, the students' answers to exercises showed that the research subjects could add fractions, either with the same denominators or different denominators.*

Keywords: *Hypothetical learning trajectory, fraction, blind students, braille media, fraction block.*

Introduction

The opening of wider employment opportunities for children with special needs, especially blind children, is a phenomenon that has to take more seriously by education practitioners to develop education for blind children. Law Number 20 of 2003 Article 5 paragraph 1 explains that every citizen has the same right to obtain quality education. Furthermore, paragraph 2 states that citizens with physical, emotional, mental, intellectual, and or social disabilities have the right to receive special education. In its implementation, every individual gets the same rights in education, both normal individuals and individuals with disabilities.

Mathematics is one of the important fields that need to be mastered by students with visual impairment to develop their ability to welcome challenges and opportunities in the world of work. This is based on Freudenthal's opinion about the view that mathematics is a "human activity" (Hadi, 2017; Sembiring, 2010), meaning that mathematics is the human activity where students are allowed to learn mathematical activities and can find mathematical ideas or create models of students' thinking (Gravemeijer & Terwell, 2000). Otherwise, almost all human

activities including activities in the world of work use mathematics. This view emphasizes the importance of students' understanding of mathematics. However, not all students, especially students with visual impairments, can understand mathematics easily.

Fraction is one of mathematical topics that are classified as difficult and abstract to understand. The concept of fractions is difficult to understand in concrete terms and allows misconceptions and verbalism (Kania, 2018; Nasution & Putri, 2018). According to Bruce, Chang, Flynn, and Yearley (2013), fraction is such a difficult concept to learn and to teach that it is currently a pedagogical challenge for mathematics educators. One alternative solution to overcome the difficulties of learning fractions is to use the context of everyday life both in the learning process and in the evaluation of learning. Based on previous research regarding the development of short stories of fractions and the development of splitting level in doing multiplication, it is known that the context of daily activities can help students construct their understanding (Agustiani, 2016; Meryansumayeka, Darmawijoyo, Putri, & den Hertog, 2011).

Based on the background described above, the researchers conclude that there is a need to develop a hypothetical learning trajectory (HLT) for the fractions topic to help blind students understand the concept of fractions. An HLT consists of the goal for the students' learning, the mathematical tasks that will be used to promote students' learning, and hypotheses about the process of the students' learning (Simon & Tzur, 2004). The benefits of HLT as a tool to support learning are another form of content pedagogical knowledge (Agustiani, 2015; Wright, 2014). At the beginning of the development process, two research had been carried out by the research team in 2019 to apply Braille teaching aids for blind students in several meetings (Anggraini, Afgani, & Agustiani, 2020; Oktaria, Afgani, & Agustiani, 2019). The results of research on the use of Braille circle and Braille multiplication table are used as a basis for making initial products to be developed through design research.

The development of HLT refers to HLT for fractions using the context of *pempek* (Sary, 2015), mathematics short stories for learning fractions (Agustiani, 2016), and research that examines the effectiveness of the "Ball of Fractions" media model on the ability to understand the concept of fractions in blind students (Nurfadillah, 2015) which have been previously developed by the researchers as a starting point for the development of HLT. The teaching aid developed in this study is the modification of the aids that have been used in previous research (*pempek lenjer*, Braille circle, and Braille multiplication table) with learning characteristics for blind students. One of the main characteristics of learning activities for the blind students is the "fingering" activity so that the teaching aid that will be developed is adapted to this activity and is equipped with Braille letters. Hence, the research question examined in this paper is "How far the activities

designed in HLT can be implemented in the teaching experiment and can be used to support students to learn the concept of adding fractions?

Method

The type of this research is design research (DR) that aims to describe the design process of hypothetical learning trajectory on fraction topic for blind students using Braille Fractional Block. This research activity started from the odd semester in the academic year of 2019/2020 until the odd semester in the academic year of 2020/2021. The research subjects in this study were two blind students of an extraordinary school in Year 7 and Year 8, namely subject FH (Year 7, low vision or partially blind) for cycle 1 and subject AN (Year 8, totally blind) for cycle 2. The selection of the research subjects employed purposive sampling technique where the research subjects were selected based on the consideration of understanding the prerequisite concept, the type of visual impairments (low vision or partially blind and totally blind), and the ability to communicate well. Experimental activities in this study were carried out offline by using the health protocol of Covid 19 pandemic.

This research is divided into three stages: the preparation for the experiment, the design experiment, and the retrospective analysis (Gravemeijer, 2016; Prahmana, 2017). Based on the implementation theory of design research, these stages take place in several cycles where the research subjects in each cycle come from different classes. The cycles depend on the research objectives. The flow diagram of the research cyclic process is depicted in Figure 1. In this study, the stages of design research were carried out in two cycles.

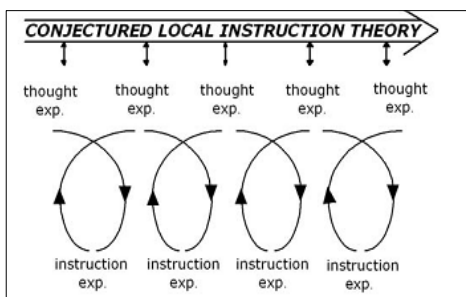


Figure 1. The flow diagram of the research cyclic process (Gravemeijer & Cobb, 2006)

The data collection techniques in this study were the walkthrough technique, observation, interviews, and tests. The walkthrough technique was carried out at the expert review to collect the comments and recommendations from the experts about HLT (Agustina & Zulkardi, 2020). Interviews and observation were carried out to collect data on the implementation of the designed HLT and its impact to support students to conduct the concept of the addition of fractions. The objects observed in this activity are the learning process and student work results in the form of student worksheets and practice questions. Tests were administered to collect data about students'

understanding before and after the implementation of HLT. The instruments were reviewed and validated by experts.

According to Bogdan and Biklen (Moleong, 2019), qualitative data analysis is an activity of working with data which includes activities to organize data, sort data into units by using data code, synthesize them, look for and find the patterns, reduce the important data, and decide what will be described to others. Qualitative data analysis was carried out continuously during research activities and intensively after leaving the research field to identify data that might answer research questions (Moleong, 2019). In general, the steps of analyzing qualitative data include data reduction, data presentation, and conclusion drawing. In this study, the data analyzed are the validation of HLT and the implementation of HLT.

Results and Discussion

Preparing for the Experiment: Delineating an Envisioned Learning Trajectory

The preparation phase for the experimental activities began with reviewing the theory and the body of literature and designing the HLT for learning fractions. The HLT contains mathematics learning objectives, teaching and learning activities, and conjectures of student thinking (Simon & Tzur, 2004). These three components are arranged based on theoretical studies, curriculum analysis on fraction topic for junior high school students, discussions with teachers at school, and several discussions with the research team and students.

The researchers examined the curriculum used for extraordinary schools. In general, their curriculum is the same as the curriculum used for public schools. The difference lies on the achievement target which is set much simpler. Based on the results of the literature review and curriculum analysis, the researchers decided to start the addition of fractions lesson by introducing the concepts of fractions and equivalent fractions as these two preliminary topics are highly important to strengthen. Besides, the researchers emphasize that the lesson that we presented does not contain HOT (High Order Thinking) achievements. Rather, it focuses on the understanding of the basic concepts of fractions, equivalent fractions, and the addition of fractions.

The literature review focuses on relevant previous studies as references for the stages of learning mathematics for blind students, teaching aids for blind students, and learning difficulties frequently experienced by blind students. The relevant research on HLT development is HLT products for learning fractions in the context of *pempek lenjer* developed by previous researchers (Sary et al., 2015) and mathematics short stories for fraction learning (Agustiani, 2016). Meanwhile, the literature on the development of teaching aids includes the research that examines the effectiveness of the "Ball of Fractions" media model on the ability to understand the concept

of fractions in blind students (Nurfadillah, 2015). These two studies serve as the basis for the development of activities in HLT and the development of teaching aids.

After studying the theory and curriculum, the research team listed the important parts in introducing the concept of addition of fractions both for adding fractions with the same denominators and adding fractions with different denominators. Next, the research team discussed the activities that blind students might carry out and the aid that was most suitable for them. Based on the review of several relevant research, the research team concluded that the “fingering” activity was the basic activity most likely to be carried out by blind students (Saputri & Wangid, 2013; Pratama et al., 2018). The HLT was developed by adapting learning activities to the context of *pempek* in the daily learning activities of blind students, namely “fingering”. The aid was developed by modifying the context of *pempek* and Broken Ball into Braille Media Fractions Blocks. Braille Fraction Blocks is a nailed block media inscribed with Braille letters. This media is considered quite simple and easy to use for fingering activities, both fingering the size of the blocks and fingering the Braille letters written on the blocks.

The media used in this study is the Braille Fraction Blocks. The material used to make the media is a wooden block of size 5 cm x 5 cm. The blocks are cut 30 cm in length to make a whole unit block. To make smaller block pieces, some of the 30 cm unit blocks are sawn into 2, 3, 4, 5, and 6 pieces. From this process, fraction blocks of the same size are obtained. After the blocks are cut, the blocks are painted green, blue, and orange with oil paints. The blocks are placed under the sun for two days to let the paint dry. The fraction block was named according to the size of the block using Braille on one side of the block. Braille letters are made using plywood nails on one side of the block. This process took a month. One set of braille fraction blocks consists of 1-unit block, $2\frac{1}{2}$ fraction blocks, $3\frac{1}{3}$ fraction blocks, $4\frac{1}{4}$ fraction blocks, $5\frac{1}{5}$ fraction blocks, and $6\frac{1}{6}$ fraction blocks.

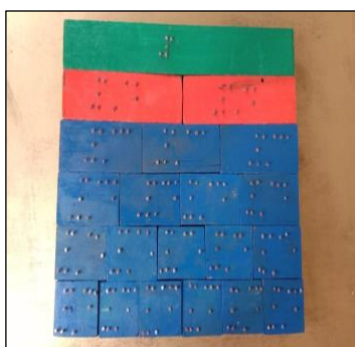


Figure 2. Braille fraction block

Hypothetical learning trajectory design for the addition of fractions with the same denominators and with different denominators using Braille Fraction Blocks contains activities of comparing unit blocks and fraction blocks, comparing the sizes of different fraction blocks to

get the same size fractions, comparing the sizes of combined equal fraction blocks, adding two combined equal fraction blocks, and determining the fraction block that is the same size as the added fraction block. These five activities were carried out for both topics with a description of the activities adjusted to the contents. Activities on HLT were carried out well in the experimental activities. After the implementation of the experimental activities, the results of the exercises showed that the subjects could understand well the concept of the addition of fractions with the same denominators and different denominators. Several important findings will be discussed in the following section.

Preparing the Experiment: Expert Review

During the expert review, the instruments were evaluated by three experts who were educators from various institutions with relevant scientific and research backgrounds. One of them is an expert in HLT design research and the others are experts in the development of mathematics learning aids for blind children. Those three experts assessed whether or not the HLT and HLT supporting instruments that have been prepared are suitable for experimental activities. The instruments were assessed based on their expertise both theoretically and practically. Some common recommendations from the experts regarding the HLT include the order of activities based on the level of thinking, the selection of variations and the level of difficulty of the proof given to students, and the scaffolding used in the activity sheet (Agustiani & Nursalim, 2020). This expert review was carried out by a email review, which took approximately one month to complete. The experts suggested many revisions for the HLT. Suggestions and recommendations from the validators on the HLT were recorded on the validation sheet. They were used as the basis to revise and improve the HLT. The revisions include: 1) language usage in which the experts suggested to use simple language for students; 2) the order of activities based on the level of thinking; 3) the activities related to blind students’ main learning activity; and 4) teacher question (scaffolding) related to process of conducting the concept of addition of fractions. The following table presents the HLT as a result of the expert review.

Table 1. HLT prototype 2

No	Activity	Goal	Description of the Activity
<i>Topic 1: The addition of fractions with the same denominators</i>			
1	Comparing a unit block and fraction blocks	<ul style="list-style-type: none"> - Students understand the fraction concept - Students understand the kinds of fraction 	Students are given unit blocks to finger. Then, the students are asked to finger a $\frac{1}{2}$ fraction block to compare its size with the unit blocks. Students are asked to find how many $\frac{1}{2}$ fraction blocks are needed to make a unit fraction block. This activity is also carried out for the $\frac{1}{3}$ and $\frac{1}{4}$ fractions.

Table 1. HLT prototype 2 (continue)

No	Activity	Goal	Description of the Activity
2	Comparing the size of different fraction blocks to get equivalent fractions	Students understand the equivalent fraction concept	Students are given a $\frac{1}{2}$ fraction block. Then the students are asked to finger a $\frac{1}{4}$ fractions block and the students are asked if the two blocks have the same size. Then the students are asked to combine two blocks of fractions and finger their size. Students are asked, "Do the two blocks have the same size now?" Another block given for comparing is the $\frac{1}{3}$ fraction block.
3	Comparing the sizes of combined equal fraction blocks	Students understand the comparison of the size of combined equal fraction blocks (fraction with the same denominators)	Students are asked to finger a $\frac{1}{4}$ fraction block and a combination of two $\frac{1}{4}$ fractional blocks. Then, the students were asked, "Which block is bigger? $\frac{1}{4}$ or the combination of two $\frac{1}{4}$ blocks?". Another block is given for the fraction ratio of $\frac{1}{3}$.
4	Adding two combined equal fraction blocks (fraction with the same denominators)		Students are asked to finger a $\frac{1}{4}$ fraction block. Then the students are asked to take additional blocks of the $\frac{1}{4}$ fraction to combine with the existing blocks. Then, the students were asked, "Does the size get bigger? What is the combined result of the two $\frac{1}{4}$ fraction blocks?". In the same way, students were asked to do activities for $\frac{1}{3}$ and $\frac{1}{6}$ fraction blocks.
5	Determining the fraction block that is the same size as the added fraction blocks	Students understand the simple form of the result of addition of fractions. Students understand about adding fractions with the same denominators	Students are given a $\frac{2}{4}$ block. Then, students compare the size of the $\frac{2}{4}$ block fraction with several other single fraction blocks. This activity aims to introduce students to simplifying fractions.
<i>Topic 2: The addition of fraction with different denominators</i>			
1	Comparing unit block and fraction blocks	<ul style="list-style-type: none"> - Students understand the fraction concept - Students understand the kinds of fraction 	<i>The description is the same as activity 1 in topic 1</i>
2	Comparing the size of different fraction blocks to get equivalent fractions	Students understand the equivalent fractions concept	<i>The description is the same as activity 2 in topic 1</i>
3	Comparing the sizes of combined fraction blocks with different denominators	Students understand the comparison of the size of combined fraction blocks with different denominators	Students are asked to take two blocks of different size fractions (fraction blocks with different denominators). Then, the students are asked which fraction block is bigger. The blocks that are compared include a $\frac{1}{3}$ fraction block and a $\frac{1}{6}$ fraction block, a $\frac{1}{3}$ fraction block and two $\frac{1}{6}$ fraction blocks, a $\frac{1}{2}$ fraction block, and a combination of two $\frac{1}{4}$ fraction blocks.

Table 1. HLT prototype 2 (continue)

No	Activity	Goal	Description of the Activity
4	Adding fraction blocks with different denominators	Students understand about adding fractions with different denominators	Student A is asked to finger a $\frac{1}{2}$ block fraction and a $\frac{1}{3}$ block fraction. Students are directed to find the least common multiple (LCM) of 2 and 3. Then, students are asked to find the combined $\frac{1}{6}$ fraction that is equivalent to $\frac{1}{2}$ and is equivalent to $\frac{1}{3}$ and add the two fractions (fractions with a denominator of 6). The result is $\frac{5}{6}$.
5	Determining the fraction block that is the same size as the added fraction blocks	Students understand the simple form of the result of addition of fractions. Students understand about adding fractions with different denominators	<i>The description is the same as activity 5 in topic 1</i>

The Design Experiment: 2 Cycles of Teaching Experiment

The teaching experiment is an important phase in evaluating both the HLT design and the teaching process. Simon (Bakker & Van Eerde, 2015) states that "mathematical teaching cycle" gives opportunity for teachers to predict before the occurrence of students' mental activities (through teaching experiment), then try to find the students' thinking processes related to those hypotheses in the teaching experiment. Each teaching experiment activity had an average duration 120 minutes. The teaching experiment was implemented by two preservice teachers from mathematics education program. They taught and interacted with blind students before the implementation of the teaching experiment. The preservice teachers were part of the research team selected by the researchers based on GPA, proposal selection, and interview. The researchers chose two preservice teachers based on the number of subtopics of adding fractions, the addition of fractions with the same denominators, and different denominators. The preservice teachers were also trained to read and write braille fonts used in making media and in the implementation of the teaching experiment for three months. Before the preservice teachers implemented the teaching experiment, they had translated activities to braille.

The teaching experiment activities were carried out offline during the Covid-19 pandemic. To prevent the spread of the Covid-19 virus, the teaching experiment was carried out in compliance with health protocols. Those are restricting people who involved in the teaching experiment, limiting experiment time, washing hands and checking body temperature before starting the teaching experiment, using a mask and face shield during the teaching experiment, preparing hand sanitizers, and maintaining a safe distance whenever possible. The teaching experiment phase consists of four experimental activities, that is, two activity cycles with two different research subjects for two topics. Each topic comprises five activities carried out for two face-to-face meetings.



Figure 3. The implementation of health protocols in the experimental activities

The experimental activities in cycle 1 involved subject F. Subject F is a male student with partial blindness. As mentioned earlier, topic 1 in the experimental cycle 1 contains five activities. The implementation of this first experiment performed quite well; four out of five activities ran without problems. However, activity 2 that is comparing the size of different fraction blocks to get the same size of fractions encountered a few obstacles. It is most likely because the student was slow to understand the activity. From this activity, it can be concluded that the student understood the concept of the equivalent fraction to the size of a block, where the size of a $\frac{1}{2}$ fraction block is the same as the size of two $\frac{1}{4}$ fractional blocks. However, the student still found it difficult to understand the concept of equivalent fraction using the fraction symbol. He did not immediately understand that the fraction $\frac{2}{4}$ is the same as the fraction $\frac{1}{2}$. The following is the conversation between the teacher and the student in the teaching experiment of topic 2 in the experimental cycle 1. In this excerpt, the second activity is presented to show the student's understanding of the equivalent fraction (i.e., subject F).

- Preservice teacher 2* : Please take two fraction blocks $\frac{1}{6}$ and $\frac{1}{3}$. Finger the blocks and tell me which one is bigger?
- Subject F* : $\frac{1}{3}$
- Preservice teacher 2* : Please take some $\frac{1}{6}$ fraction blocks. Arrange the blocks and compare their size with the $\frac{1}{3}$ fraction block.
(Subject F is arranging some $\frac{1}{6}$ fraction blocks until the size is the same as the $\frac{1}{3}$ fraction block)
- Preservice teacher 2* : Finger the blocks and tell me how many $\frac{1}{6}$ fraction blocks I need to make a $\frac{1}{3}$ fraction block.
- Student F* : I need two $\frac{1}{6}$ fraction blocks
- Preservice teacher 2* : Okay, we continue with $\frac{1}{2}$ and $\frac{1}{6}$. Please take two fraction blocks $\frac{1}{6}$ and $\frac{1}{2}$. Finger the blocks and tell me which is one bigger?
- Subject F* : $\frac{1}{2}$
- Preservice teacher 2* : Please take some $\frac{1}{6}$ fraction blocks. Arrange the blocks and compare their size with the $\frac{1}{2}$ fraction block.
(Subject F is arranging some $\frac{1}{6}$ fraction blocks until the size is the same as the $\frac{1}{2}$ fraction block)
- Preservice teacher 2* : Finger the blocks and tell me how many $\frac{1}{6}$ fraction blocks I need to make a $\frac{1}{2}$ fraction block.
- Student F* : three $\frac{1}{6}$ fraction blocks is the same as a $\frac{1}{2}$ fraction block

Similar to the experiment of the first topic, the experiment of the second topic comprises five activities. This second experiment had been well-implemented where activities 1, 2, 3, and 5 were conducted without any problems. Yet, a few obstacle was found during activity 4, to wit, the activity of determining the addition of fractions without using the fraction blocks, but rather finding the least common multiple (LCM) at first. Finding the LCM of the denominators is one way to add fractions with different denominators (Gembong, 2020; Putri et al., 2017). However, many students do not realize: 1) why the LCM of two denominators has to be used and 2) how to find the LCM by using the fractions. To help the student, the researchers provided fraction blocks and guided the student to find the LCM of the two denominators of the fractions. In the fourth activity, the student was asked to compare the sizes of the $\frac{1}{2}$ fraction blocks with the $\frac{1}{3}$ fraction blocks arranged vertically (see Figure 4).

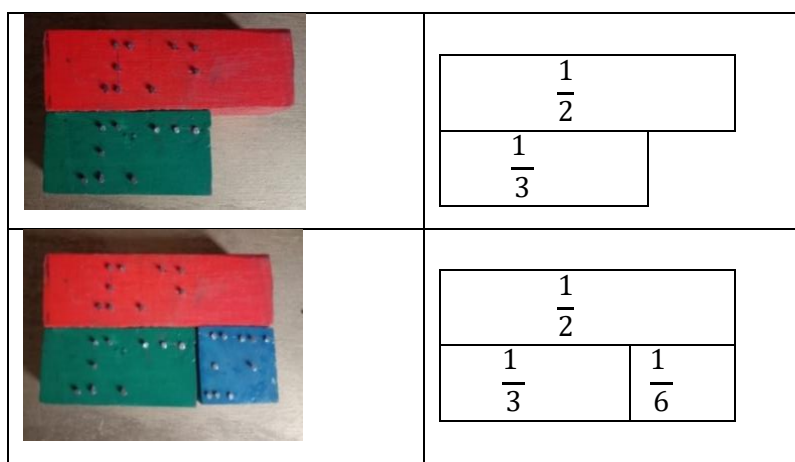


Figure 4. Equalizing the denominators by using braille fraction blocks

After fingering the two fraction blocks, the student noticed that the two fraction blocks had different sizes. The researchers asked the student to finger the two fraction blocks again to find the blank part on the right side of the $\frac{1}{3}$ fraction block. The blank part is the area under the $\frac{1}{2}$ fraction block that is not filled with the $\frac{1}{3}$ fraction block. The researchers subsequently requested the student to take another fraction block to place it in the blank. The first fraction block picked was a $\frac{1}{5}$ fraction block. After placing it in the right side of the $\frac{1}{3}$ fraction block, it turns out that the $\frac{1}{5}$ fraction block did not fit the size of the blank part. The size of the $\frac{1}{5}$ fraction block was bigger than the area in the blank. Then, the student took the $\frac{1}{6}$ fraction block and placed it in the blank area. The $\frac{1}{6}$ fraction block fit the area exactly. The size of the $\frac{1}{6}$ fraction block is similar to the area of the blank. The researchers invited the student to conclude that the fractions $\frac{1}{3}$ and $\frac{1}{2}$ have the common denominator of 6.

Furthermore, the experimental activities in cycle 2 were carried out for subject A. Subject A was a female student with total blindness. The implementation of experiment for topic 1 was

conducted perfectly; all five activities ran without any obstacles. However, different results were obtained during the experiment of cycle 2 in topic 2.

Similar to the previous experimental cycle, the implementation of the experiment of cycle 2 for topic 2 contain five activities. This second experiment ran quite well; no obstacles were found for activities 1, 2, and 5. Nevertheless, we faced challenges in activities 3 and 4, i.e., comparing fractions with different denominators and determining the addition of fractions using fraction blocks. In activity 3, the student made a mistake when determining the fraction that has the bigger value without using the fraction blocks. The student was always correct if the student used braille fraction blocks as comparing fractions. In activity 4, the student experienced the same difficulties as in activity 3. The student considered that the fraction with a larger denominator had a bigger size. When the researchers asked the student to finger the braille numbers on the fractional block twice, the student could answer the questions correctly. Figure 5 is an example of a student’s answers (Subject F) from teaching experiment of cycle 1.

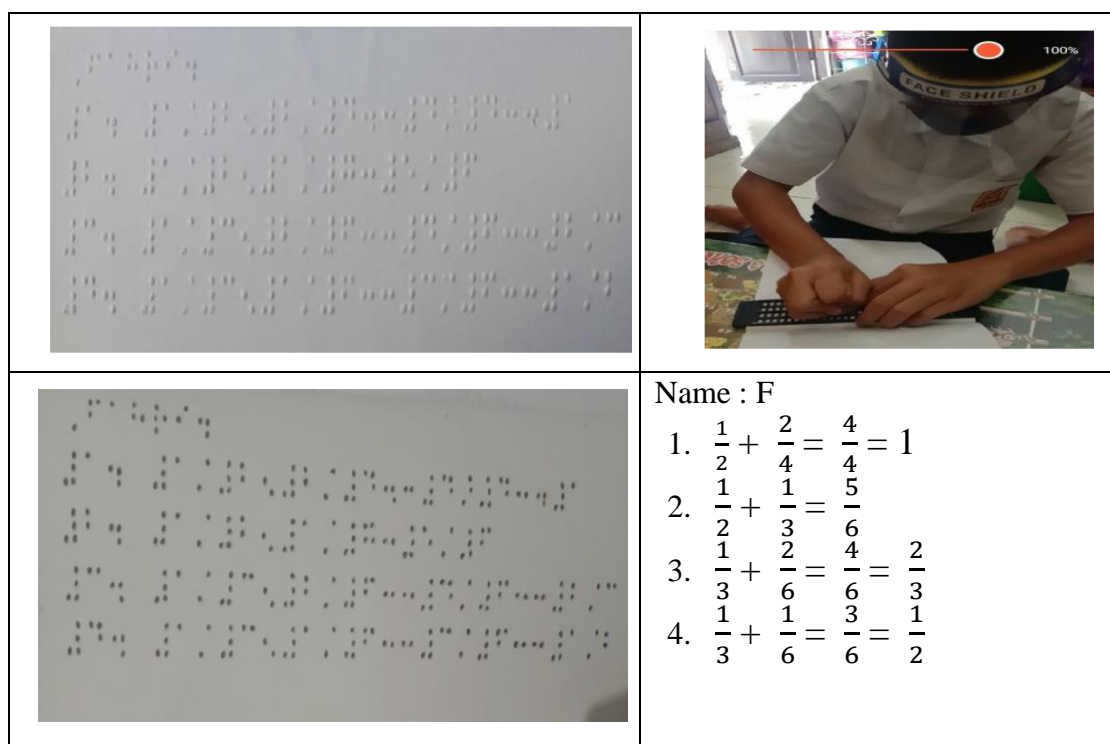


Figure 5. The student’s answers using braille fonts and his translation

The Retrospective Analysis

The research team carried out a retrospective analysis after one experiment activity was completed. Activities in this retrospective analysis aim to evaluate the implementation of HLT in the experimental activities and discuss what needs to be revised for further experiment implementation (Gravemeijer, 2016). The revisions focus not only on the contents of HLT but also on teaching scenarios to support the implementation of activities in the HLT. In this study,

three retrospective analysis (RA) activities were carried out. RA 1 was conducted after the experimental activities of cycle 1 with subject F for topic 1. RA 2 was carried out after the experimental activities of cycle 1 with subject F for topic 2 and the cycle 2 with subject A for topic 1. Last, RA 3 was managed after the experimental activities of the cycle 2 with subject A for topic 2. These RA activities were organized through online meeting applications. In this activity, the research team discussed the implementation of HLT in the experimental activities and decided the revisions for the HLT.

Table 3 shows the decision made for revisions after each retrospective analysis activity. Table 3 shows which parts of the HLT and braille fraction block media have been revised and what revisions have been made. In general, the transformation of the HLT and braille fraction block media to support students' learning activities is evident. It can be seen from the revised versions in RA 1 and RA 2, where the number of fraction types and blocks for each type of fraction in the Braille fraction block package was increased in response to the increase in variety of questions in activity 2 and the use of Braille fraction blocks to introduce the LCM concept as finding the common denominator. This is also consistent with the revisions made in RA 3. As the number of students with different characteristics who use braille fraction block media to support the process of learning fractions, particularly the addition of fractions, was increased, revisions regarding the variation of questions and the modification of the braille letters in the braille fraction block were addressed.

Table 2. Revisions of the HLT

No	Experiment	Discussion Result/Revision Decision
1	The first cycle, Subject F, Topic 1	The variation was made for the questions presented in activity 2 so that students construct the concept based on problem-solving patterns using Braille fraction blocks
2	The first cycle, Subject F, Topic 2, and the second cycle, Subject A, Topic 1	The Braille characters on the fraction block need to be reviewed for revisions Activity 4 (Topic 2): The description of the activity is made in more detail. The activity of equalizing the denominators is not carried out by directly looking for the LCM, but rather by using Braille fraction blocks.
3	The second cycle, Subject A, Topic 2	Activity 2: the teacher should emphasize the concept of a fraction by comparing the denominators, "if the denominator/divisor is larger, the resulting fraction is smaller". Revised Braille Letters on fraction blocks: Braille letters are written in full symbols where the core symbol is represented by a nail that is not completely knocked out and the column symbol is represented by a nail that is struck until it is almost flush with the surface of the block

In the first experimental cycle of topic 1, we discovered that more diverse problems are needed to spark students' interest in learning concepts through the use of media. The finding is

also related to the result of the first experimental cycle of topic 2. The problems regarding the use of fraction blocks to guide the students to find the LCM as the common denominator (in topic 2: addition of fractions with different denominators) also need more variations before the students find the answers of why the LCM of two denominators has to be used and how to find the LCM using the fractions. Besides that, the preservice teachers also provided a good scaffolding as an alternative way to guide students' understanding (see Figure 4 for the preservice teacher's ways to explain the LCM using fraction blocks). Providing variations of problems and appropriate scaffoldings is part of pedagogical content knowledge (PCK). As a result, teachers need to have a good pedagogical content knowledge, especially in teaching blind students. The success of a teacher in providing a scaffolding is inseparable from a good pedagogical content knowledge (Agustiani, 2015; Anwar & Rofiki, 2018).

The use of instructional media in some research, such as Braille fraction blocks, can help students understand fractions from informal to formal level. (Jannah et al., 2019; Sari et al., 2015; Zabeta et al., 2015). However, bridging students' understanding from informal to formal level is not always easy, specifically for blind students who are unable to see the media directly. Along with the revisions of HLT, the Braille fraction blocks used to support the HLT were also revised to facilitate the completion of HLT activities. In the teaching experiment of cycle 1, subject F (a partially blind student) could easily read braille fonts on the fraction blocks. The braille fonts on the fraction blocks were written by using plywood nails on one side of the blocks. The nails represent the black dot of the braille fraction symbols (see Figure 6). Different findings appear in the teaching experiment of cycle 2. Subject A (a totally blind student) was not able to read braille fonts on fraction blocks easily. She needs to repeat fingering the braille fraction symbols on the fraction blocks. After the researchers investigated the phenomena to the parents and teachers of the two subjects, we found that the totally blind student and the partially blind student have different abilities in reading braille fonts. The totally blind student (subject A) needs complete braille fraction symbols on the fraction blocks, which contain nails that represent the black and white dot of the braille fraction symbols. According to the result of previous research, blind students have different ways of reading braille fonts (Susanti & Rudiwati, 2019).




Braille fraction symbol	Initial Braille fraction symbol	Revised Braille fraction symbol
$\frac{1}{2}$ 		

Figure 6. Braille fraction symbol on the fraction blocks

Two-cycle experimental learning using Braille Media Fraction Block was carried out offline. However, in response to pandemic conditions and technological advances that require online learning, the Braille Media Fraction Block can be used with several modifications so that student activities can be conducted online with activity sheet guidelines and teachers as well as with the help of others at home. The modifications emphasize the details of the questions that need to be given to students. It aims to get the appropriate understanding that follows the objectives of each activity.

Conclusion

This study produced a hypothetical learning trajectory (HLT) design for the addition of fractions with the same denominators and different denominators by using Braille media fraction block. The HLT consists of five activities: comparing unit block and fraction blocks, comparing the size of different fraction blocks to get the same size fractions, comparing the sizes of combined equal fraction blocks, adding two combined equal fraction blocks, and determining the fraction block that has the same size as the added fraction blocks. Based on the two-cycle teaching experiment that has been implemented, it can be concluded that the five activities arranged in HLT can be implemented in the teaching experiment and can be used to support students to learn the concept of the addition of fractions, either with the same denominator or the different denominators.

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