

## Student's Anticipation Profile at Rigor Level in Determining Papaya Tree Root Dimensions

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**Abstract.** *Students with a rigor level of geometric thinking can analytically solve problems, yet the ability may not be readily observable. Thus, an example of how students solve problems merits explorations. Inspired by student's problem solving, this study aimed to examine the student's anticipatory profile in determining Papaya tree roots' dimensions. This qualitative research utilized tests and interview. Two tests were carried out: van Hiele geometric level grouping test for selecting the research participants and the report-based test for the actual project. Seventeen students took the van Hiele test, and one of them, who achieved the rigor level, was selected for the interview. Data obtained from the interview were then analyzed qualitatively. The study showed that students with a rigor level of geometric thinking anticipated analytically. The subject was able to explain a geometric problem systematically, starting from analyzing problems, clarifying details, to presenting arguments clearly and precisely. The findings in this study generate useful information for teachers who train their students to analyze a geometric problem correctly and adequately.*

**Keywords:** *anticipation, analytic anticipation, anticipation rigor level, van Hiele, fractal*

### Introduction

Geometry is a branch of Mathematics that is often used to visualize problems in Mathematics, such as derivatives, area integrals, volume integrals of rotating objects, numbers, limits, and vectors (Ayazgök & Aslan, 2014; Ministry of Education and Culture, 2017). The topic of geometry taught in school was Euclid's geometry. This is in accordance with the syllabus published by the government, in this case the Ministry of Education and Culture (Ministry of Education and Culture, 2017). The objects concerned with Euclid taught are triangles, squares, trapezoid, parallelogram, rectangles, squares, lines, sphere, prisms, pyramids, blocks, and cubes. All these objects are actually rarely found in everyday life, except for blocks and cubes which are similar to boxes or cardboards.

In Euclid's geometry, all objects taught have integer dimensions, comprising of point (dimension 0), line (dimension 1), plane/area (dimension 2), and space (dimension 3). For objects that are often encountered in students' daily life, such as mountains, rocks, trees, clouds, leaves, cracks on the wall, to tree roots, the dimension value cannot be determined. This is because the topics taught in school have not yet led to the development of the discussion of these objects.

The concepts in fractal geometry are needed to solve the problems above, namely to find out the dimensions of these irregular objects, because fractals are able to identify dimensions in real numbers (Chudin et al., 2015; Dewi, Dari, & Elita, 2016). Fractal comes from the Latin *fractus*, which means broken, cracks or irregular. Likewise, the word related to *fractus* is *frangere*, which means breaking up, making irregular parts, but behind the irregular shape there is an order called self-similarity.

Delving into geometry, van Hiele classified geometric thinking into 5 levels, namely (1) visualization level, (2) analysis level, (3) deduction level, (4) informal deduction level, and (5) rigor level. The rigor level almost never appears in many studies because the rigor level is the highest level in geometry (accuracy level). Someone who has reached this level is believed to experience maximized learning in geometry, with substantial potential for further development (Sunardi et al., 2019; Yudianto et al., 2019; Yudianto, 2017). In completing a task, someone needs imagination on how to solve the problem and what results might be obtained. These "possible" results are predictions or forecasts. Prediction related to mental activity such as imagines the results that will be obtained from an event without carrying out a series of activities, while predicting mental and physical activity about a series of steps are carried out from an upcoming event without actually carrying out a series of activities in detail. Both activities cannot be seen directly, so an interpretation is needed. Interpretation is a mental activity explaining something that can be observed from behavior that appears to be someone's opinion or view. These three mental activities are known as anticipation (Jahn & Myers, 2015; Klein et al., 2019). To that end, anticipation is a mental activity about things that have not happened or are presumed to take place to overcome an uncertain situation based on interpretations, predictions, and forecasts.

Anticipation is divided into five types, namely (1) impulsive anticipation, (2) rigid anticipation, (3) explorative anticipation, (4) analytic anticipation, and (5) internalized anticipation (Maswar, 2019; Yudianto, 2017; Yudianto & Sunardi, 2015). Impulsive anticipation is a person's way of thinking spontaneously followed by an action that comes to mind without analyzing the problem situation and without considering the relevance of existing actions; this is known as hasty or impulsive forecasting. Rigid anticipation is a person's way of thinking by defending his opinion without evaluating and reconsidering what he understands or not, while at the same time considering possible alternatives. Exploratory anticipation is a person's way of thinking and exploring ideas that might be appropriate to acquire a better understanding of a problem. Analytical anticipation is a person's way of thinking in analyzing problems and setting goals based on logic, mathematical analysis, and logical analysis.

The studies related to anticipation have started to gain their traction in the world, which was initially initiated by Harel's (2008) ideas. His ideas enthuse initiatives related to cognitive conflicts that can help someone improve the way they think and how to understand them. There is a link between mental actions, ways of thinking, and how to understand someone in solving a problem (Houston, 2010). Harel discussed the idea with Lim (2007), as written in his dissertation related to mental action and anticipation in algebraic material.

In Indonesia since 2015, the theory of anticipation has been developed by Sunardi & Yudianto (2015), who reported that students with high ability can be classified with respect to anticipation analytic in solving cube problems. Based on the results of tests and interviews, students were able to point out logical and sensible answers. In the same vein, Yudianto (2015) reported that students with impulsive anticipation have 6 characters, including: (1) students read questions only once, (2) students do not describe questions in detail, (3) students do not combine the criteria in questions, (4) students cannot link between things that are known and asked, (5) students work on the questions in a hurry, and (6) students do not consider alternative problem solving. Yudianto (2015b) also reported that solving the integral problem using internationalized anticipation can be further analyzed on the basis of the results of tests and interviews which indicate that the subject remains firm in his opinion that what he has done is correct without considering alternative answers. Sunardi & Yudianto (2016) reported that students who anticipate analytically will gain more advantage in solving geometric problems. Rohmah (2017) reported that students who have high linguistic intelligence and high logical-mathematical intelligence have the characteristics of analytic anticipation in solving algebraic problems. Students seem to be trying to analyze problems, identify goals, imagine cause and effect, and consider alternative problem solving.

In solving a problem, anticipation has an important role, in addition to thinking and understanding. Anticipation theory was then developed in Indonesia (Maswar, 2019; Rohmah, 2017; Sunardi & Yudianto, 2016; Yudianto, 2015b), but the results of these studies have not revealed in-dept understanding concerned with the subject with the highest level of geometric thinking (rigor) in solving problems non-Euclid geometry. The present study involved four researchers from Indonesia and two international researchers who collaborated on research delving into anticipation study with specific emphasis on Euclid's geometry. To the best of authors' understanding, no research results have reported how a person solves a problem with certain anticipation. Therefore, the present study aimed at investigating students' anticipation profile of rigor level in solving the root dimension problem of Papaya trees.

## Method

This study employed qualitative approach, with specific focus on describing the anticipation of rigor level students in determining the dimensions of the papaya tree roots. The study recruited 17 students who were given the van Hiele test. The study revealed that 1 student met the rigor level condition and had high abilities with a Grade Point Average (GPA) of 3.92 (scale 4) calculated by focusing on specific subjects in Mathematics (other than general subjects of University and Education).

This research was conducted on students in the Mathematics Education Study Program, Faculty of Teacher and Training Education, University of Jember. These subjects had taken the Fractal Geometry course in the academic year 2020/2021 and had sat the van Hiele test. Fractal Subject was a study group subject in Geometry and denoted an optional subject worth 2 credits. The students learned the application of fractal dimension theory in everyday life and then perform dimensional calculations with predetermined data several years ago. The study employed data collected 10 years or 20 years ago, based on trusted sources such as the Central Statistics Agency (BPS). Afterward, the course began to focus on predicting and interpreting likelihood of what happens now and in the future. In this assignment, students were given problems related to the calculation of root dimensions at the roots of papaya trees and were asked to predict the growth of papaya trees around each house. Because there was only one student who met the rigor level, that student was immediately recruited as the research subject. The subject's name was coded in accordance with the first letter of his name, which included given, middle, and family name.

The instruments used in this study were tests and interviews. The test consisted of two kinds of van Hiele level clustering tests and a report-based problem solving test related to the determination of the root dimensions of papaya trees. The van Hiele test used in the study was a test developed by Usiskin (1982) and translated into Indonesian by Sunardi (2006). The test consisted of 25 items. The scoring applied 5 questions bound to van Hiele's levels. Student needed to answer every 5 questions in the test. When he can answer at least 3 questions accurately, his ability was classified at a specific level. If he only answers 2 questions correctly, then his ability is between two adjacent levels. If only 1 question is answered accurately, then his ability does not belong to that level, implying that it is included in a lower level.

The problem given to the subject was a non-Euclid problem that asked the subject to calculate the dimensions of the papaya tree roots. The subject was then asked to work on a report for three days. To ensure the credibility of the subject's work, interviews (triangulation) were carried out using unstructured interview guidelines. Interviews between researchers and subjects were coded to determine the order of questions and answers between the subject and the

researcher. For example, P-5 means the question by the researcher is in the 5<sup>th</sup> order, and PTP-9 means the answer from the subject is in the 9<sup>th</sup> order.

Data analysis is the process of finding and compiling a conclusion systematically based on test data and interview results. The data analysis was carried out by organizing data into categories, breaking down the data into units, synthesizing the data, arranging the data into patterns, choosing which data were important and what will be studied, and making conclusions. These steps aim at ensuring that the findings are easily understood by the authors and readers. Because the research data was qualitative in nature, the analysis used qualitative analysis which dealt with interpreting words arranged in an expanded text.

## Results and Discussion

Based on information from the subject, the subject made observations on papaya (*Carica papaya L*) aged approximately 60 days old. The subject pulled the tree from the ground carefully so that the roots of the tree remained intact. The roots with soil were washed first so that they became clearly visible, as shown in Figure 1. For more accurate calculation on the dimensions of papaya roots, the subject focused only on the roots of the papaya, as shown in Figure 2.



Figure 1. Papaya tree



Figure 2. Papaya tree roots

The subject explained in-detail that the object he chose was in accordance with the task given by the researcher, as seen in the following interview excerpt.

- P-5 : *What is this, Miss? Are you sure this is relevant to the task?*  
 PTP-5 : *This is the root sir ... (smile). Yes sir hmm ... (ANTICIPATION) this is suitable to the task.*  
 P-6 : *Try to explain why it is suitable.*  
 PTP-6 : *This is a fractal, Sir ... because the shape is messy ... random ... irregular.*

Based on the explanation and interview excerpts, the subject was confident with his work and understood the task given to her. This finding is in line with Moragues-Faus, Sonnino, & Marsden (2017) who say that the principle of fractal geometry seems suitable to describing root system, because the branching in the root leads to self-similarity which is a characteristic of fractal objects.

In explaining the process passed in completing the task, the subject looks relaxed and competent. This can be seen from the way the subject explained the preparation to the implementation of the papaya tree root. Although she was a little confused by the researcher's question (P-5), the subject had anticipated analytically. The subject demonstrated logical thinking on what was being discussed and immediately clarified what was assigned (Maswar, 2019; Sunardi & Yudianto, 2016).

To calculate the root dimension, the subject used a calculation using the Box-counting fractal dimension. The steps included the following: (1) the subject started to make a *grid* in the shape of a square with an L side consisting only of a square of size  $r_1$  which therefore became  $L = r_1$ . Then the analysis proceeded to calculating the number of boxes ( $N_1$ ) on the grid covering the object, resulting in equation  $N_1 = 1$ ; (2) the subject divided the grid so that it consisted of 4 squares of size  $r_2 = \frac{L}{2}$ . Then the analysis proceeded to calculating the number of boxes ( $N_2$ ) on the grid covering objects; (3) the subject continued the grid division up to n iterations. The smaller the box size ( $r_n$ ) was arranged on the grid, and then the total segments of the squares ( $N_n r_n^2$ ) on the grid covering the object the closer to the actual object area were calculated (Yudianto, 2019).

Based on the subject's reports, as investigated through interviews, the subject carried out the gridding process for stages 1, 2, and 3 on the papaya root object as shown in Figure 3.

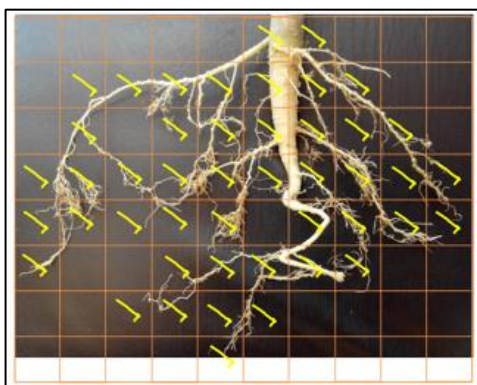


Figure 3. Gridding stage 1

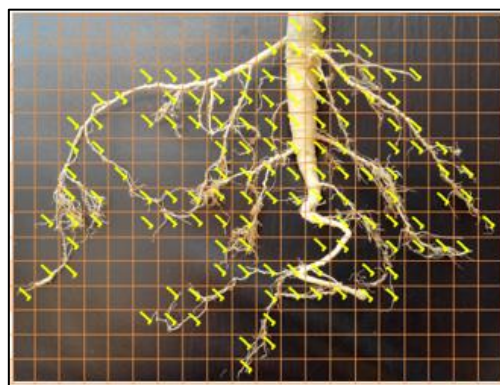


Figure 4. Gridding stage 2

Figure 3 is a stage 1, box-counting method, with a box size of 2 cm. The number of squares on the grid covering the objects was 46, and the total squares on the grid were 80. Figure 4 is a stage 2, box-counting method with a box size of 1 cm. The number of squares on the grid covering the object was 129, and the total grid on the grid was 300.

Figure 5 is a stage 3 box-counting method with a box size of 0.5 cm, the number of squares on the grid covering the object is 415 and the total squares on the grid are 1200.

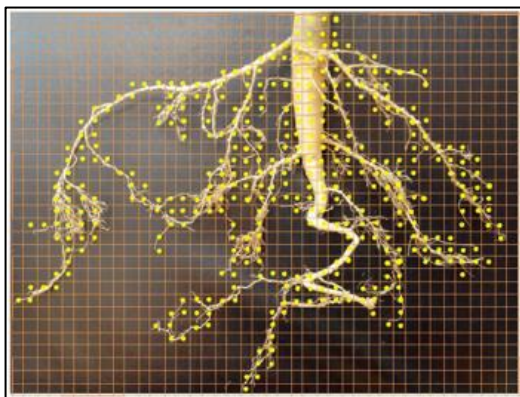


Figure 5. Gridding stage 3

The subject explained that the gridding process simply went through three stages, namely at a size of 2 cm (stage 1), 1 cm (stage 2), and 0.5 cm (stage 3). The subject explained that in order to achieve even more accurate results add steps 4, 5 and so on would be suitable. This can be seen from the following interview excerpt.

- P-11 : Why did you only make three stages for this one? (pointing to the subject report results)*
- PTP-11 : (3 seconds pause) because this is enough for you to see the complete picture. (ANTICIPATION)*
- P-12 : What does that picture mean? (confirm what the subject has said)*
- PTP-12 : This is it Sir ... (pointing to the three pictures above). This is 2 cm. This is 1 cm. This is 0.5 cm. If you want to be more accurate, it can be up to n cm, sir. (what is meant by n is the nth row according to the pattern above)*

In explaining *box-counting* method, making a grid, determining the scale at stages 1, 2, and 3, and up to giving dots on the tree root image, the subject looked observant and critical as well as communicative. This is in accordance with the results of the interview, demonstrating that the subject anticipated the tasks analytically. In the activity of drawing and explaining a problem, someone who anticipates problem analytically tends to be fluent and has a good way of communication, including the ability to use computers in solving a problem (Adams, Pegg, & Case, 2015; Ulmer, 2017).

After determining only three iterations to be used in the report, the subject then considered it  $r_1 = 2\text{ cm}$ ,  $r_2 = 1\text{ cm}$ , and  $r_3 = 0,5\text{ cm}$ . From each iteration, the number of squares (N) on the grid that covers the object is calculated, as shown in Table 1.

Table 1. The values of  $r$  and N from each iteration

$N^{\text{th}}$ iteration	$r_n$ (cm)	$N_n$
1	2	46
2	1	129
3	0.5	415

After obtaining the data in Table 1, the calculation of the dimensions of the papaya tree roots used the formula  $D = \lim_{r_n \rightarrow 0} \frac{\log N_n}{\log \left(\frac{1}{r_n}\right)}$ , where the box-counting dimension was estimated as the slope of the linear regression. This happened when plotting  $\log\left(\frac{1}{r_n}\right)$  was expressed as values on the X-axis and  $\log N_n$  as the value on the Y-axis, forming a linear equation  $\ln N(r) + D \ln\left(\frac{1}{r}\right) + k = 0$ . D is the box-counting dimension, and r is the size of the grid on the grid. N(r) the number of squares on the grid covering the object, and k is a constant (Yudianto, 2019).

The subject used a formula  $\ln N(r) + D \ln\left(\frac{1}{r}\right) + k = 0$  and Graphmatica software for linear regression plotting, so that the slope was easily calculated, which indicated the value of D. Graphmatica software is mathematical software that can draw graphs of functions. This can be seen in Figure 6.

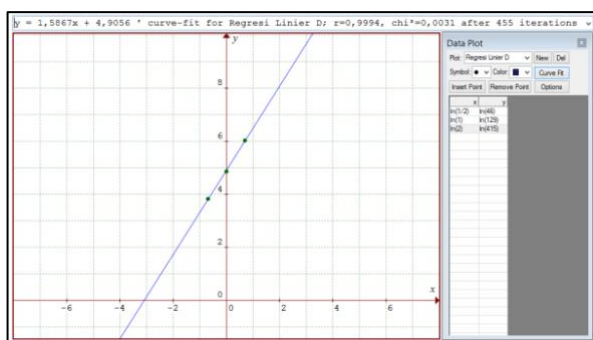


Figure 6. Linear regression to determine the value of D

The subject explained the regression results in Figure 6. She mentioned that when the values (in the form of dots in the image) were the regression lines of the standard values, then the line was the same as the correlation line. The line is also called the match line where the line is closest to the points, so that the equation of the line is obtained as follow  $y = 1,5867x + 4,9056$ . Thus, it can be determined that the dimensions of papaya roots are 1.5867. This can be seen from the following interview excerpt.

- P-17 : Why did you draw this conclusion (pointing to result D)?
- PTP-17 : This is the result of this software, Sir ... The equation is  $y = mx + k$  (refers to the formula)
- P-18 : Continue
- PTP-18 : Hmm ... (ANTICIPATION) m is the slope, (ANTICIPATION) so you can immediately determine the dimensions (showing the explanation from the formula book that came from)
- P-19 : Are you sure about this skill, Miss?
- PTP-19 : Sure Sir
- P-20 : Try to do it in another way, Miss (check the subject's ability)
- PTP-20 : I'll try ir sir



- P-21* : Please do it in 30 minutes only (subject was asked to rework on the task using another method to determine the ability to analyze it)  
*PTP-21* : OK Sir (the subject starts working)

Based on the interview above, it can be concluded that the subject understood what was assigned to her by applying the formula available and then explained the dimensional relationship with linear regression. In this case, the linear regression aimed to determine the dimensions of the papaya tree roots. The anticipation used by the subject was analytic anticipation, where the subject explained in-detail the meaning of the slope and then explained the relationship between slope and linear regression. In this case, the gradient manifested the dimension. The dimension obtained is 1.5867, with the root dimensions found in the interval of  $1 < \text{dim} < 2$ . In Euclid's geometry, the 1<sup>st</sup> dimension is a line and the 2<sup>nd</sup> dimension is a plane or area.

The subject started to work on the problem given to do it in other ways but still employed the box-counting method. The results of the subject's work can be seen in Figure 7.



Figure 7. Left hemisphere (red) and right hemisphere (green) roots

The subject divided the root image into two asymmetrical (different) parts. The subject explained that she was using local box-counting dimension calculations ( $D_{local}$ ) on each left and right hemisphere. This was later be compared to and linked to the distribution of root patterns. The left and right division was based on the location of the main root. Using the same pattern as pictures 3 to 6, involving the same size and the same calculations, the analysis generated the findings as shown in Figure 8.

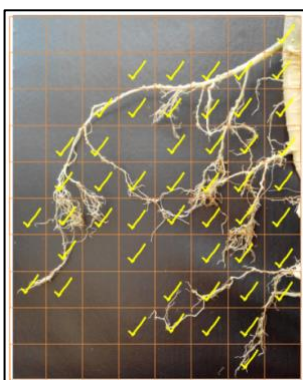


Figure 8. Left: Gridding stage 1

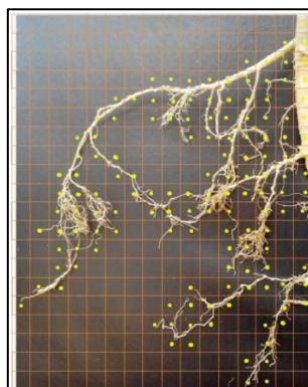


Figure 9. Left: Gridding stage 2

Figure 8 uses the box-counting method at the left root with a box size of 2 cm. The number of squares on the grid covering the object is 49, and the total grid on the grid is 80. Figure 9 shows the use of the box-counting method at the left root with a box size of 1 cm. The number of squares on the grid covering the object is 139, and the total grid on the grid is 320.

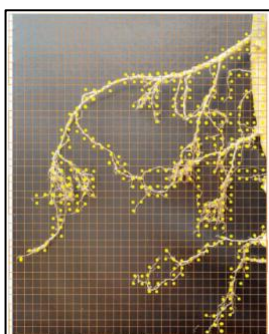


Figure 10. Left: Gridding stage 3

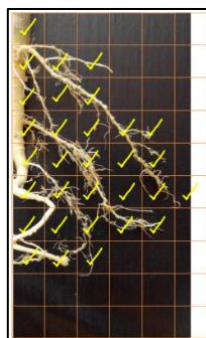


Figure 11. Right: Gridding stage 1

Figure 10 uses the box-counting method at the left root with a box size of 0.5 cm. The number of squares on the grid covering the object is 379, and the total grid on the grid is 1280. Figure 11 uses the box-counting method at the left root with a box size of 2 cm. The number of squares on the grid covering the object is 31, and the total grid on the grid is 60.

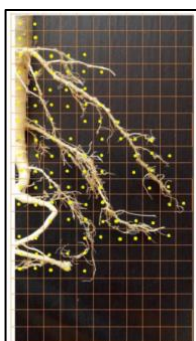


Figure 12. Right: Gridding stage 2

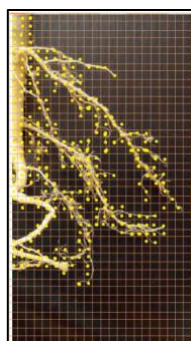


Figure 13. Right: Gridding stage 3

Figure 12 uses the box-counting method at the left root with a box size of 1 cm. the number of squares on the grid covering the object is 100 and the total grid on the grid is 240. Figure 13 uses the box-counting method on the left root with a box size of 0.5 cm. The number of squares on the grid covering the object is 309, and the total grid on the grid is 960.

The subject explained again that the gridding process for the left and right continued through 3 stages involving sizes of 2 cm, 1 cm, and 0.5 cm. Making the grid was done one by one this was because the left and right were not symmetrical, so the calculation had to be done one by one. This can be seen from the following interview excerpt.

- P-25 : *Why do you keep 3 iterations, Miss?*  
 PTP-25 : *To be more sure, sir ... maybe more than 3 packs, it's just that this is the same as the previous result (making sure it is the same as the previous work)*  
 P-26 : *This is a lot of pictures, right?*

- PTP-26 : Because this is sir... hmm (ANTICIPATION)... I did not draw symmetrical because it was taken from the main root so that the left and right sides were not the same, sir.
- P-27 : I see... if it's made the same, can it be counted on one side right away?
- PTP-27 : Obviously, you can't sir ... you can make the symmetry, but later the ticks you have to see if there is a root in this box ... the rest is empty sir (explain in detail)
- P-28 : Oh .. ok

Based on Figure 13 and interview excerpts, it can be seen that the subject really understands what is being done and is sure of the answer. Furthermore, the subject begins to calculate with the same steps as the first task, namely by making a table and then performing calculations as shown in Table 2.

Table 2. The value of r and N from each iteration in each section

N <sup>th</sup> iteration	r <sub>n</sub> (cm)	N <sub>n</sub>	
		N <sub>left</sub>	N <sub>right</sub>
1	2	49	31
2	1	139	100
3	0.5	379	309

The subject used a formula  $\ln N(r) + D \ln(\frac{1}{r}) + k = 0$  and Graphmatica software for linear regression plotting. These allowed the analysis to determine the slope, which is the value of D from both D left and D right. This can be seen in Figure 14.

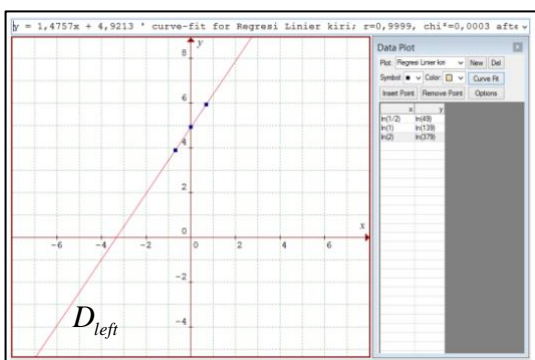


Figure 14. Linear regression for determining value  $D_{left}$

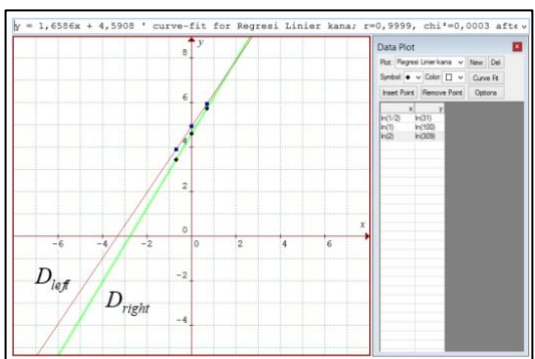


Figure 15. Linear regression for determining value  $D_{right}$

Figure 14 shows the calculation of the dimensions of the tree roots on the left, where the lines are in the closest position to the points, while Figure 15 describes the calculation of the root dimensions on the right. For Figure 15, the plotting results of the calculating the dimensions of the left tree roots still appear, so that they appear side by side and can be used as a comparison. Based on the regression results in Figure 14, the line equation for the left root object is  $y = 1.4757x + 4.9213$  while the regression results in Figure 15 generates a line equation of  $y = 1.6586x + 4.9056$ . From the two images above (Figures 14 and 15), different dimensions are obtained, namely 1.4757 and 1.6586. The subject explained that these two roots had different dimensions, meaning that the left dispersion pattern is not developed better than the right one. To show that the calculation using the second method is also correct, the subject showed the result of the local calculation, as shown below.

$$D_{local} = \frac{D_{left} + D_{right}}{2} = \frac{1.4757 + 1.6586}{2} = \frac{3.1343}{2} = 1.56715 \approx D$$

This can be seen from the following interview excerpt.

- P-30 : *Hmm ... not exactly the same (trying to provoke subject analysis)*  
 PTP-30 : *Yes, it's impossible Sir ... (smile)... because of this approach, but only the second digit is different, Sir*  
 P-31 : *Hmm (smile)... yes but why is it different? What can be concluded from this?*  
 PTP-31 : *Now it can be seen between the left and right roots sir ... which one is thicker? (ask me)*  
 P-32 : *The left one is thicker (picks an answer that doesn't match the subject's expectations)*  
 PTP-32 : *As you can see, there are more of them on the left side, sir. But if... (ANTICIPATION) we see from its dimensions, the right side had a better result, Sir.*  
 P-33 : *How come, Miss?*  
 PTP-33 : *Because the left dimension is lower than the right one, Sir. This means that the density of the roots is not good for growth.*  
 P-34 : *I see... okay*

At the end of the section, the subject concluded the density of the roots between the left and right roots. She mentioned whether the left root was better than the right side root, or vice versa. The right root was denser, possibly because the water absorption was better than the left. The wider dimensions of the roots (not up to the 2<sup>nd</sup> dimension), the better the papaya tree root density and this has implications for the absorption of water and elements in the soil. From the above explanation, the subject can be classified in analytic anticipation inasmuch as the subject appropriately presented an argument or opinion in accordance with mathematical reasoning (Ulmer, 2017).

In completing the given task, the subject worked and explained everything asked, and then she elaborated a very clear argument, even though there were several breaks in the thought

process, which was called anticipation. By implication, there are no significant obstacles. This means that the subject's anticipation is in accordance with their abilities and in accordance with their geometry learning experience (Adams et al., 2015; Lim, 2007; Stepp & Turvey, 2010). According to Rosen (2010) someone with analytical anticipation will go through specific thinking and understanding stage as shown in Figure 16.

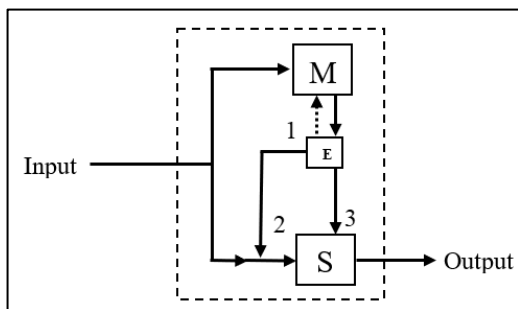


Figure 16. The rosen anticipation system

Input is a problem given to the subject. Addressing the problem, the subject will anticipate what will happen in the future. M is a forecast/prediction model where the subject will obtain details pertinent to the future situation or information. In the other words, the M model that is carried out/owned in the present, yet it also can be done to predict what S model is like. S is the overall anticipation system, while E (*error*) is an error associated with the M model. The relationship between the anticipation system according to Rosen (2010) and anticipation classification according to Yudianto (2021) is manifested in impulsive anticipation, rigid anticipation, and internalized anticipation, which can go directly to S (the anticipation to be used) without going through M. This is because it is spontaneous in its anticipatory activities, while analytic anticipation and exploratory anticipation still have to go through M because it is possible for the subject to add certain steps to help in anticipation. If the resulting M has a slight error (E), then it can go straight to S (3). If the evaluation result is unsatisfactory, then it will return to M (1). By contrast, if the subject evaluates and feels that the results are right, then she will go straight to S (2). In this research activity, the subject goes through M because the subject consistently provides the right reasons in answering what has been done. This means that the subject has a better way of thinking and understanding of the problem given. Thus, the subject way of thinking is directly focused on S (2).

Overall, the subject tends to use analytic anticipation, starting from preparing assignments, selecting objects to calculating dimensions, performing procedures required, to making reports. This is in line with the results of previous studies which state that the rigor level manifests the highest level of accuracy. Someone who is able to think at this level will have a good level of accuracy (Kivkovich, 2015; Yudianto, 2017). The essence of this finding is that if a person is at the rigor level, he anticipates analytically. By the same token, if someone anticipates a problem

analytically, then he may be at the rigor level (specifically on geometry-related topics). The practical implication for education is that if a person anticipates problem analytically, he will be able to solve the problems correctly and appropriately.

## Conclusion

When solving the problem related to the dimensions of roots of papaya tree, the subject who possesses the highest geometrical ability (rigor level of geometric thinking) is able to explain the given task in detail. With good communication and accuracy in delivering reports and arguments, the subject always employs analytical anticipation. The subject is able to explain the procedure systematically, which ranges from analyzing, clarifying, and presenting argument clearly and precisely. The way he delivers an argument was calm, which showed his satisfactory understanding and thinking process before conveying his interpretation to the researchers.

The present study highlights the need to carry out further research related to the types of anticipation carried out by others students with a rigor level of geometric thinking in different levels of education. This future study, however, needs to remain focused on the topic of geometry. Since rigor level of geometric thinking is the highest level in van Hiele's stage, the way to understand problems and how to solve them are so essential that they have to be addressed in order to help students cope with the difficulties when learning geometry.

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