Mathematical Reasoning Abilities of Students through a Model of Discovery Learning in Senior High School

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Abstract

Reasoning in mastering the foundations of mathematics and mathematical competence is one that should be owned by students. While a model of discovery learning is one of the models of learning that must be applied at school in accordance with the curriculum learning goals of 2013, the purpose of this research was to describe the mathematical reasoning abilities of students through a model of discovery learning. The research method used was a qualitative exploratory type approach. The data has been obtained through mathematical reasoning tests and interviews. The result of this study is: 1) High ability students only mastered the indicator to propose allegations. The ability to file allegations limited to the ability to estimate answers with the reasons but the evidence provided was not appropriate; 2) Medium-capacity students only mastered the indicator to propose allegations. Ability to file allegations limited to the ability to estimate answers with the reasons but the evidence provided was not appropriate; 3) Low ability students could not master the six indicators of mathematical reasoning ability. In the indicator of filing suspicion, students estimated answers together with the reasons through trial and error, and students did not know the supporting concepts used in showing evidence. Student's inability in mathematical reasoning was caused by the inability to master the concept correctly as well as not able to understand the problem and to provide evidence in solving the problem. Therefore, the application of discovery learning should be carried out continuously and consistently so that students can develop reasoning abilities well.

Keywords: mathematical reasoning, discovery learning.

Introduction

Mathematics is a structured and systematic science that requires a logical way of thinking to conclude correctly. Thinking activity in mathematics is called reasoning. Mathematics and reasoning are two things intact, with the reason that someone is more natural in understanding mathematics and through mathematics one can train his reasoning ability. The reasoning is one of the necessary competencies of mathematics which is essential and supporting feature in mathematics learning (Herbert et al., 2015; Singley & Bunge, 2014). The National Council of Mathematics Teachers (2000) affirms that reasoning and proofing are fundamental aspects of mathematics. It means that reasoning is the foundation for mastering mathematics.

Mathematical reasoning is a logical thinking activity based on facts in solving non-routine problems. As Gardner, et al. (Lestari & Yudhanegara, 2015) argues that mathematical reasoning is the ability to analyze, generalize, synthesize, solve non-routine problems and provide reasons for a problem-solving. Based on Dirjen Dikdasmen No.506/C/Kep/PP/2004, there are six indicators of reasoning ability that must be achieved by the students: (a) ask the alleged; (b) perform mathematical manipulation; (c) draw conclusions, draw up evidence, give reasons or evidence against the truth of the solution; (d) draw conclusions from statements; (e) examine the validity of an argument; and (f) discover the pattern or nature of the mathematical phenomena to make generalizations (Wardhani, 2008). Therefore, based on the opinions of these experts, it can be concluded that the ability of mathematical reasoning is a person's ability in logical and systematic thinking based on empirical facts measured by indicators of mathematical reasoning ability.

Based on the Trends International Mathematics and Science Study (TIMSS) score in 2015, the reasoning of Indonesian students was below the average with a value of 397 from the International average of 500. In line with that, based on survey results of the Program for International Student Assessment (PISA) in 2015, Indonesia scored 386 of the International Average 494 (Nizam, 2016). These surveys showed that the reasoning ability of Indonesian children was still low.

The Government also requires that the organization of the educational curriculum in the schools of 2013 focusing on learning designs that allow student’s ability to build concepts, laws or principles which are commonly known as a scientific approach. One of these scientific approaches that must be applied is discovery learning model. Kemendikbud (2014) emphasized that discovery learning is one of the scientific learning models applied to the learning curriculum in 2013. Furthermore, Ahmad (2015) stated that discovery learning is one of the learning models that gives the students the opportunity to develop and find their understanding so that learning becomes more meaningful. The information received can be quickly absorbed and stored well and can involve the active activities of students in the classroom. In line with that, according to Hosnan (2014), Discovery Learning is a learning model that develops students ' active learning, where students have to find and investigate on its own so that the results of the knowledge gained in the long memory. Based on some senses above, it can be concluded that the model of discovery learning is a learning model that invites the active role of the students, which the material is not presented in the form of the finale. The student should be able to organize themselves through the process of observation, experimentation, and conclusion of withdrawal so that meaningful learning is obtained.

According to Syah (Hosnan, 2014) there are six phases of learning through discovery learning models, namely: 1) phase where students are exposed to something that causes confusion in order to arise the desire to investigate of its own called stimulation; 2) Teacher gives the opportunity to students to identify as many issues that are relevant to the learning materials for the next chosen one and formulated in the form of a hypothesis that is called with a Problem Statement; phase 3) Teacher gives the opportunity to collect data or information relevant to prove as many true or whether the hypothesis is called with Data Collection; 4) Data that has been collected by the students is then processed and interpreted at the level of a certain confidence in the Data Processing phase; 5) students in doing the checks carefully to prove true or whether the hypothesis that has been set for the later associated with the data that has been retrieved (data processing) is called verification; and 6) withdrawal process conclusion that can serve as a general principle and applies to all of the events and the same problem having regard to the results of the verification phase is a generalization.

Discovery learning is a learning model that is considered to facilitate students in finding the concept independently so that learning becomes more meaningful and the material can last longer in memory. However, research conducted by Purnomo (2011) showed different results. Purnomo disclosed that by using a model of the invention of the students become more active, better concept and the cultivation of the students is not dependent on rote formula, but students with the low ability not used in conducting the discovery and need a lot of guidance. On learning with a model of the invention that is done on a group ability in solving problems in students with low ability did not develop because students are just resting on high ability students. This indicates that discovery learning good only applied to students with math skills.

Based on the explanation above, the effort made is to describe students' mathematical reasoning abilities that have been taught through discovery learning models. This research is important to know the thinking process of students' mathematical reasoning in conducting through discovery learning models. Therefore, the objective of this research was to describe the mathematical reasoning abilities of students who are taught through the model of discovery learning.

Research Methods

The method in this research uses the qualitative exploratory type of approach. The subject is 5 of 24 students of class X in a senior high school of Southwest Aceh who represent students with mathematical ability categories of high, medium, and low. From each category, 2 of 5 students were selected from high math skills, 1 of 6 students from medium math skills, and 2 of 13 students of low mathematical ability. Consideration in the selection of the sample was based on criteria: 1) have learned through a discovery learning model, 2) have followed a mathematical reasoning test, 3) answers for students who represent mathematical reasoning, indicators, 4) originality of answers, 5) answering test results that raise question marks and are interesting to express, and 6) cooperative attitude of students who must be interviewed.

Data were collected through mathematical reasoning tests and interviews. The tested test was carried out at the end of the lesson (posttest) on trigonometric material for the competence of the sinus and cosine rules. The test instrument has been validated and has been tested. The test was given in the form of a description of four questions containing six indicators of mathematical reasoning. Conducted interviews were unstructured so that researchers more easily in digging the data needed to support the data that obtained earlier. Data analysis was done by data reduction, data presentation, and conclusion. The data obtained was then triangulated by time and theory.

Each subject was interviewed, and the problem was resolved immediately before the posttest. Test the credibility of data on any subject by triangulating time was done by sending problems that are comparable to the problem of posttest on any subject at different times. The results obtained at the conclusion of the triangulation consistency answer each subject in completing the posttest and issue a problem that is equivalent to posttest. Therefore, the ability of mathematical reasoning can be analyzed using the issue of posttest or the equivalent problem posttest. In this study, the data analyzed was the data of each subject in solving posttest. After each data subject was analyzed, the conclusion then carried out the triangulation theory. Moreover, the student response was the answer that represents each category mathematics of high, medium and low abilities.

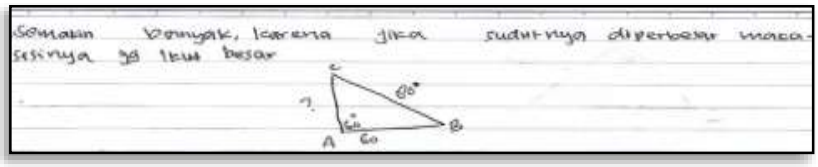
Results and Discussion

Research shows that students with high, medium, low math skills, and not being able to do mathematical reasoning were because students with high and intermediate math

skills mastered only one of six an indicator of the ability of mathematical reasoning. While the students with low ability did not overwhelm a single mathematical reasoning ability indicator, the following are the description of mathematical reasoning abilities of students.

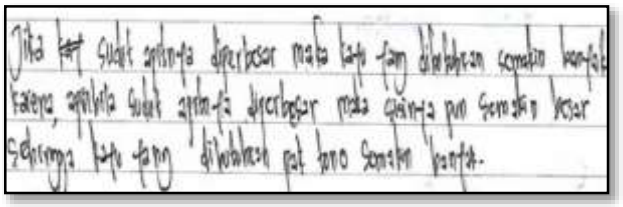
1) Mathematical reasoning ability on the alleged filing indicators
Question:
Mr. Tono wanted to create a garden picket fence in the shape of a triangle, with sides of 80 meters and 60 meters with an included angle of 60°.
If the included angle is enlarged, whether the wood needed to make the garden fence more or less? And what if the included angle is reduced? Please explain!

Mathematical reasoning abilities of students can be seen in Figure 1, 2, and 3.



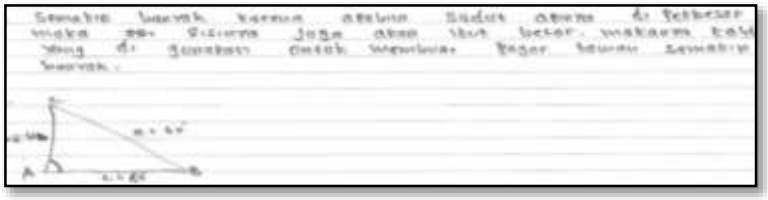
Translation: More and more. Because if the angle is then enlarged his side also gained great.

Figure 1. The answer to high ability students



Translation: If the included angle is getting bigger than the wood is needed more and more, because if the included angle is getting bigger, then his side also gets bigger so that wood required Mr. Tono more and more.

Figure 2. The answer to medium ability students



Translation: more and more. Because when the angle is enlarged so the sides will also join the great. Hence the wood used to make the garden fence more and more.

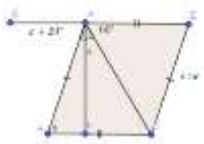
Figure 3. The answer to low ability students

Based on Figure 1, 2, and 3, it can be known that high and medium ability students were capable in filing alleged limited in the ability to estimate the answer accompanied the right reasons but cannot show proof of his righteousness. That was because students did not understand the problem well. This was in accordance with the results of interviews with students of high ability who said: "Which is already known, hypotenuse is 80, and the sides next to it will be 60". Sought means hypotenuse when the angle was enlarged. Similar with it, students with the capabilities of the medium also said: "If the angle is enlarged, hypotenuse is larger too." While the students with low ability could not afford in filing alleged evidenced through the following interviews:
Q: can you prove which side of that be great if the angle is enlarged?
S: (students thinking and finally shook his head)
Q: cannot? Roughly what formula use to prove it?

S: use cos.
Q: the ratio of cosines or rules of cosines?
S: I do not know

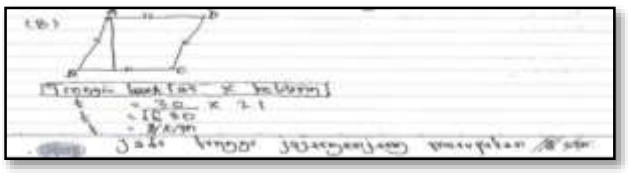
The conversation above noted that students do not understand the problem and the answer given try it.

2) Mathematical reasoning ability on indicators do mathematical manipulation



Question:
Observe the following parallelogram pictures!
If known $\angle BAE = x + 20^{\circ}, \angle BAC = x^{\circ}, \angle CAD = 60^{\circ}$ and $\angle ABC = y^{\circ}$, length CD = 8cm. Estimate height of paralthe lelogram ABCD if known parallelogram area of ABCD is 21 cm^2 and circumference = 30 cm!

Mathematical reasoning abilities of students can be seen in Figure 4 and 5.



Translation:
Height = Base x circumference
t = 30 x 21 → t = √630 → t = 8 cm
so the higher the parallelogram is 8 cm

Figure 4. The answer to medium ability students

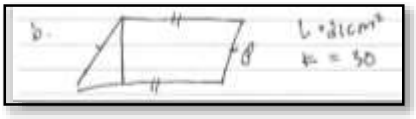


Figure 5. The answer to high ability students

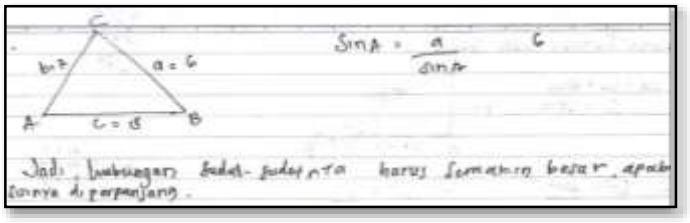
Based on Figure 4 and 5, it is known that high, medium and low ability students were unable to perform mathematical manipulations because students did not understand the problem correctly. It was known from the inability of students to link the rectangular shape with the rules of sinus and cousins. As a result, students did not know the fundamental values that could be obtained through the value of parallelogram around so that students apply the wrong procedure in solving the problem given. As shown in the following interview:

Q: So, how to find high parallelogram?
A: Ummm ... I don't know.

From the above conversation may note that students did not know how to find high-value parallelogram because students did not understand the problem given.

3) Mathematical reasoning ability on the indicator concludes, compiling evidence, provide a reason or proof of the truth of solutions.
Question:
In Δ ABC is known to side a = 6 cm, side b = 7 cm, c = 8 cm. If the length of each side in a triangle made of two times the length of the sides of a triangle are known, how a great relationship between the angles of triangle ABC with a large angle of a triangle that is new? Please explain!

Mathematical reasoning abilities of students can be seen in Figure 6 and 7.

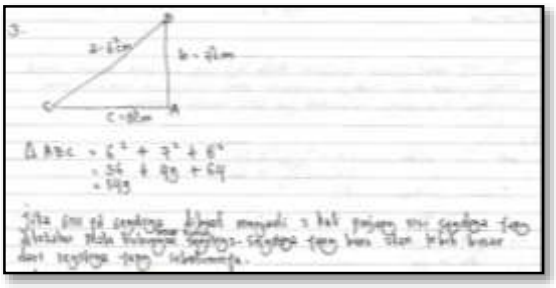


Translation:

$$\sin A = \frac{A}{\sin A}$$

So the relationship of the corners must be getting bigger in its extended

Figure 6. The answer to high ability students



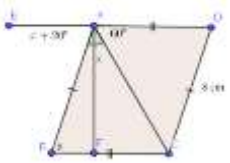
Translation:
ΔABC = $6^2 + 7^2 + 8^2 = 36 + 49 + 64 = 149$.
If the sides of a triangle are made into two times the length of the sides of a triangle are known then the large triangular corner of the relationship-the new triangles will be bigger than the previous triangle.

Figure 7. The answer to medium ability students

Based on Figure 6 and 7 in mind that students which capability of high and medium did not use data that support in explaining how to use. The explanation given was using the inappropriate relationship between the two triangles — medium ability students using sketch drawing a right triangle. Also, the data that was used to describe it was not correct even the researchers did not know the meaning of the formula being used. This was because students did not know the concept that could be used to support data to describe how to use. This statement was in accordance with the results of the interview. Students say "if its sides are made of 2 times the length of the sides is known, then a great relationship the new triangle angle will be greater than the previous triangle". As for the low-skilled students did not answer the question because students did not understand the problems of the given problem. As the conversation in the interview below:

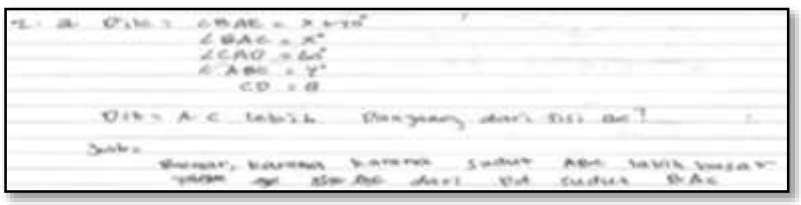
Q: How do you understand the question?
S: (students fell silent and thought) I don't understand.

4) Mathematical reasoning ability in concluding the statement indicators



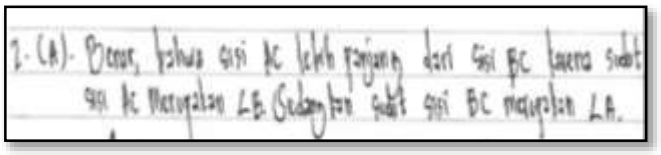
Question:
Observe the following parallelogram pictures!
If known $\angle BAE = x + 20^{\circ}, \angle BAC = x^{\circ}, \angle CAD = 60^{\circ}$ and $\angle ABC = y^{\circ}$, length CD = 8cm. Show that is it true the side AC is longer than on the side of BC? Give a reason!

Mathematical reasoning abilities of students can be seen in Figure 8 and 9.



Translation:
Correct. Because of the angle of ABC is greater than at the angle of BAC.

Figure 8. The answer to high ability students



Translation:
Correct. That the AC side is longer than the BC side. Because the angle of the AC side is $\angle B$. While the side angle of BC is $\angle A$

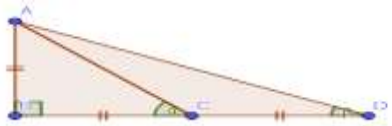
Figure 9. The answer to medium ability students

Based on figures 8 and 9, high-ability students are not able to draw conclusion statements correctly. Students' ability to show supporting evidence was limited to seeing images without doing in-depth analysis. Students did not know the concept of a straight angle and opposite angles found in the problem. As a result, students were not able to prove the truth of the answer. Students were capable in concluding answers to statements correctly, but they failed to show supporting evidence. While students with low abilities did not answer questions, after conducting the interview, it is known that students concluded using guesses on their observations through the images given to the problem. As in the following interview:

S: angles in AC is longer than BC. So, AC is longer than BC.
Q: how much does the angle of the sides AC and BC?
S: If the corner AC = 60° angle, if BC = 20°.

From the above statement in mind that students were only answering based on observations on the image. The right answer that the AC = 70^o and the BC = 50^o

5) Mathematical reasoning ability in indicator check the validity of an argument
Question:

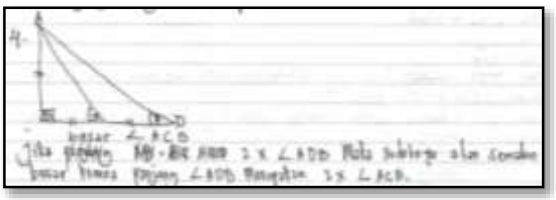


Observe the following picture!
If the length of AB = BC = CD, whether large ∠ACB = 2 × ADB? Give a reason!

Mathematical reasoning abilities of students can be seen in Figure 10 and 11.

Translation:
Correct. ∠ACB = 2 × ADB

Figure 10. The answer to high ability students



Translation: If ∠ACB = 2 × ∠ADB, it is then the angle will be even greater because the length ∠ADB is 2 × ∠ACB

Figure 11. The answer to high ability students

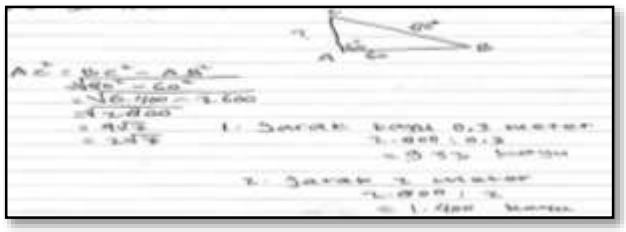
Based on Figure 10 and 11, it is known that students with high ability were able to find ACB angle values using the same triangle foot concept as supporting data. However, the students did not show evidence of the truth of the argument given because the students halved the ACB angle value to obtain an ADB angular value. In moderate ability students, they did not use supporting data in examining the validity of an argument. Students gave a statement that supports the truth of an argument

but showed no evidence of the answers correctly. While low-ability students did not answer because they did not understand the problem given, as it is known through the conversation of the interview below:

Q: How do you understand the question?
S: I don't understand.

6) Mathematical reasoning ability on the indicator to find patterns or the nature of the symptoms of mathematically to make generalizations
Question:
Mr. Tono wanted to create a garden picket fence in the shape of a triangle, with sides of 80 meters and 60 meters with an included angle of 60°.
Estimate how much wood is needed to be able to fence Pak Tono's park if:
1) The distance between the woods is 0.3 meters
2) The distance between the woods is 2 meters.
From point 1) and 2) make a formula (general form) to estimate how much wood is needed to be able to fence Pak Tono's park!

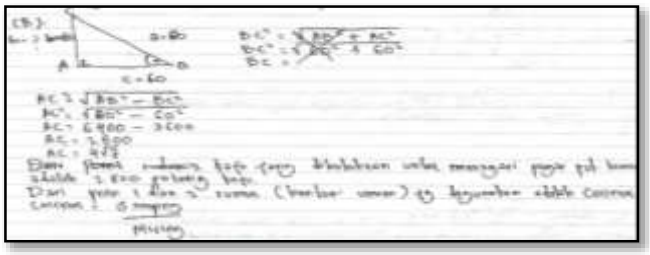
Mathematical reasoning abilities of students can be seen in Figure 12, 13 and 14 below.



Translation:
0.3 m wood distance
2800: 3 = 933 woods

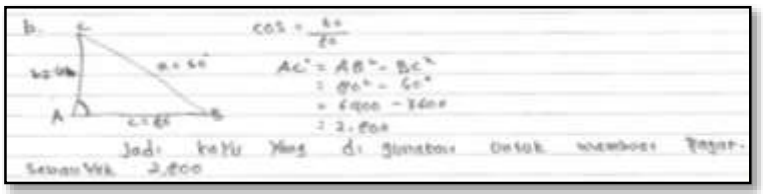
2 m wood distance
2800: 2 = 1.400 woods

Figure 12. The answer to high ability students



Translation:
The wood needed to fence the Pak Tono fence is 2800 pieces of wood.
From point 1 and two the formula (general form) used is cosine.
cosine = side / hypotenuse

Figure 13. The answer to medium ability students



Translation:
So, wood is needed to defend the fence as much as 2800.

Figure 14. The answer to low ability students

Based on Figure 12, 13, and 14 noted that students with high ability using the data support that is not correct, students used the concept of a right triangle in a settlement. Thus, students could not compose a pattern based on the concept of mathematics and in the process, students used value 2800 that was unnoticed from the source and the students were unable to devise common forms. Students with

medium ability were using the Pythagorean concept as supporting data, to be able to craft a pattern and a common form of mathematics. This was because the students understand that the problem was using a right triangle. While the low-ability students were not able to find the pattern or the nature of the symptoms of mathematical to make generalizations, this was because students did not understand the problem so they could not use data to support devise patterns and did not craft a common form correctly.

Based on the data analysis, students with high, medium and low math abilities who have learned through discovery learning models were not able to do mathematical reasoning. There were two factors that made students do not have mathematical reasoning skills, namely: 1) the implementation of learning through discovery learning, and 2) research instruments, in this case, a mathematical reasoning ability test.

First, weaknesses regarding implementing learning through discovery learning were found in the data collection and data processing phases. Problems experienced by students in the data collection and data processing phases were caused by students needing more time to find concepts about sine and cosine rules. The implementation of scientific activities through 5M activities that must be completed for each meeting results in the information obtained by students was not intact and seems rushed so that students did not have a strong and correct concept regarding the rules of sine and cosine. This result were gathered from the results of the students' mathematical reasoning tests.

Second, weaknesses regarding research instruments were mathematical reasoning test questions that tend to solve problems using evidence; these conditions were difficult things for students to do because students did not understand the concept well based on Santosa's research (2013) which states that imperfections in mathematical verification are related to lack of understanding of concepts, lack of knowledge of logic and methods of verification, limitations in understanding language and mathematical notation, lack of ability and knowledge in selecting facts and theorems to be applied, and lack of confidence in doing mathematical proof. Similarly, the results of Lestari's research (2015) revealed that the problems faced in conducting mathematical proof are problems in reading and understanding mathematical evidence, problems in presenting proof of truth in a mathematical statement, problems indirectly or indirectly proving, and problems in develop mathematical arguments to prove and deny a statement. From the statement, it is known that doing proof which is part of the reasoning is a difficult thing for students to do, especially if students are not accustomed to solving mathematical reasoning problems. As the results of Melin, Hadjar, and Sukayasa's research (2015) concluded that students' mathematical reasoning ability could not develop if mathematics is only as a series of procedures that must be followed and imitated through examples without knowing its meaning.

Conclusion

Based on the research results and discussion can be summed up that high ability students only mastered indicators filing alleged. The ability to ask the alleged extent on the ability to estimate the answer accompanied reasons correctly but the evidence given is not appropriate. For students with secondary capability only mastered indicators filing alleged. The ability to ask the alleged extent on the ability to estimate the answer accompanied reasons correctly but the evidence given is not appropriate. While the students with the low ability are incapable of mastering the mathematical reasoning ability of the indicators in the sixth, the indicator on the filing alleged the students estimate the answer accompanied reasons through the trials and the students do not know the concept of supporters that can be used in showing evidence.

The inability of students in mathematical reasoning because students do not master the concept correctly so not able to understand the problem and provide evidence for solving problems.

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