Problem-Based Learning Associated by Action Process Object Schema Theory in Mathematics Instruction

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Abstract

Problem-Based Learning (PBL) Associated by Action Process Object Schema (APOS) Theory were implemented to 26 prospective mathematics teachers in the mathematics for senior high school students course. It was hypothesized that the six steps of PBL are appropriate with mental constructions of APOS that make students can reach the objectives of the course. Five main questions on recorded semi-structured interviews were applied to get the information about the implementation of PBL associated by APOS theory and students' ability to create mathematics instruction in senior high school by using PBL associated by APOS theory. The data strongly supported this model in confirmatory factor analyses. The data also revealed the hypothesized differential relations between teacher cognitions and teacher education.

Key words: Problem-Based Learning; action process object schema; Mathematics instruction.

Introduction

In an effort to develop education towards a better quality, Indonesia has set 2013 curriculum as the curriculum that should be implemented in every level of education. There are principles emphasized in order to achieve the quality that has been designed in curriculum documents (2014). The principles presented entirely associated by the facility to be provided by a teacher against students in the learning process. Overall this principle should be a guideline in implementing the learning process including mathematics learning.

Teachers, as person who perform the learning activities, are expected to apply these principles using appropriate models of learning. One model of learning that is recommended in the above regulation is a problem-based learning model. In addition, the learning mode used in the 2013 curriculum is directed learning through a scientific approach and indirect learning. Directed learning is done to produce an immediate impact in the form of knowledge and skills, while indirect learning to produce companions impact that develop values and attitudes.

On the basis of the implementation of the curriculum, teachers need to anticipate it by implementing quality learning through their attention to various aspects, primarily related to students. The main supporting aspects related to students in mathematics learning, in terms of how mathematical concepts can be obtained and understood by the students.

Initial studies with high school math teachers in Bandung Regency and the math teachers of vocational schools in Karawang show that math teachers tend to better prepare students only in answering exam questions. They are less concerned about the efforts to convey mathematical concept to students as a whole. This is evident when teachers were asked how their efforts to teach the concept of limit function and derivative function. The teachers tend to teach the concepts separately without any connection. This has resulted in lack ability to understand the concept of limit function and derivative function as a whole of the students and even the teachers themselves.

The existence of various policies in the field of mathematics education would certainly impact on efforts to prepare prospective teachers of mathematics itself. One of the courses that prepare mathematics teachers to teach mathematics in high school is mathematics for high school course. Prospective mathematics teachers through this course is expected to be early to know the strategic efforts that must be implemented to improve the quality of mathematics learning in line with expectations in the curriculum document.

Literature Review

Problem-Based Learning

Problem-Based Learning (PBL) is a learning approach that can challenge the students to make them think visible. PBL is known as a progressive active-learning and learner-centered approach where unstructured problems (or simulated real-world complex problems) are used as the starting point and anchor for the learning process (Tan, 2004, p. 7). The process of learning in PBL according to Delisle (1979) is organized into the following steps: connecting with the problem, setting up the structure, visiting the problem, revisiting the problem, producing a product or performance, and evaluating performance and the problem. Through these steps, PBL can make the students find themselves, asking questions and answering others posed by their peers.

Research on problem-based learning at all educational levels have been carried out and managed to prove that this model runs quite effective. Padmavathy and Mareesh (2013) states that by adopting PBL in teaching mathematics, teacher can create a number of students who are creative, critical decision makers, problem solvers, entirely is needed later in the world of competition. PBL teaching strategies according to Padmavathy and Mareesh (2013) had an impact on the content of the knowledge presented in the more widely opportunities. The students learn the content with more engagement and increase their active participation, motivation and their interest in mathematics. This of course can lead the students have positive attitudes towards mathematics.

Abdullah, Tarmizi and Abu (2010) stated that the group of students who learn mathematics using a PBL models can use heuristic problem solving of Polya more effectively. As a result they were able to demonstrate mathematical communication skills better, and can work together better than students who study mathematics with conventional learning.

Meanwhile, a similar thing was also expressed by Fatade, Mogari and Arigabu (2013) which states that PBL can make students more creatively, act in a focused, rational thinking and dealing effectively with a friend in their group in mathematics. The use of PBL can also make the teachers know when and how they use scaffolding for learning.
The results of the above study reinforce earlier findings about the use of PBL in teaching mathematics. Schmidt, Loyens, Van Gogg and Paas (2007) stated that PBL was compatible with the person's cognitive architecture that is being studied, because the elements of learning in PBL allows for flexible adaptation of coaching and cognitive call. Meanwhile Hmelo-Silver, Duncan and Chinn (2007) states that the PBL approach involving students, with the application of appropriate scaffolding, in doing exercises and concepts that will increase the understanding of the construction of knowledge, known as a learning process.

For the use of PBL in college students' mathematics learning, Triantafyllou and Olga (2013) states that the use of PBL will enable teachers to combine constructionism pedagogical ideas with PBL, to teach mathematics in a wider context, and to adopt a student based strategy. On the other hand, it is expected to help students assimilate mathematical knowledge and Overcome Reviews their deficiencies regarding mathematics. Thus, the use of PBL could fit in college especially for prospective teachers of mathematics.

Action-Process-Object-Schema Theory

Talking about the theory of Action-Process-Object-Schema (APOS) will not be separated from Piaget's Reflective Abstraction as the underlying theory. Reflective Abstraction according to Piaget (in Arnon *et al.,* 2014) is: 1) the major mechanism of mental constructions in the development of thinking; and 2) the mental mechanisms that occur when the overall structures of mathematical logic developed in one's mind. Furthermore, according to Piaget (in Arnon *et al.,* 2014) Abstract Reflective contains two parts: 1) the reflection in the sense of a) awareness and thinking contemplative about content and operations on the content, b) recognized to reflect a variety of content and operation of a the cognitive level lower to a higher cognitive level; and 2) the higher cognitive level will be constructing and reorganizing content and operations acquired and will be content on the operations itself to be applied as new operations.

Thus, the development of knowledge about an object, either mental or physical, according to Piaget requires both the object and a subject who acts on the object. Someone will receive the properties of objects that are not derived from these objects, but derived from the way he or she acts to objects in the form of operations conducted against them. This suggests that abstract ideas are the most common and useful according to the concept of Reflective Abstraction, obtained from a variety of phenomena instead of taking a picture pickup.

Stages of construction and reorganization of content and operations on reflective abstraction has been raised by Dubinsky, being very close to the present mathematical ideas (Arnon *et al.,* 2014). Dubinsky's thought, related to Piaget's reflective abstraction, can be applied to understand the concept of limit function. When students, for the first time, construct knowledge about Δx that is read "delta *x*" with the meaning "change in *x*" and Δy that is read "delta *y*" with the meaning "change in *y*" constructed as operations that transforms the meaning of "changes in *x*" and "change in *y*". This transformation occurs when students are able to determine Δx and Δy on two points, say point A (x_1, y_1) and B (x_2, y_2) and determining the gradient of the line through A and B both for points A (x_1, y_1) and B (x_2, y_2) and at points A (2, -5) and B (1, 4). Then at a higher stage, the meaning of Δx and Δy will be content on the line gradient concept that will construct new operations and leads to an understanding of the concept of limit function, when "changes in *x*" increasingly reduced through the attribution of the concept on the bowstring a curve f (x).

From the example above, it can be seen that the concept of reflective abstraction, what is referred to as an object is the concept of "change in *x*" and "change in *y*". Objects will have the properties when given the command to determine the value of both the points yet to be determined and specified coordinates. These properties according to Piaget (in Arnon *et al.,* 2014) owned by the action is not just owned the object just so it certainly cannot be separated from the subject of the action.
While students are doing an action seeking a "change in *x*" and a "change in *y*" by the coordinates of two points that are known, then the action is termed by Piaget (in Arnon *et al.,* 2014) as a material action. Furthermore, students will try to determine whether the line gradient formed by two known points or lines will be as a bowstring or in the form of a tangent curve *f* (*x*). Continued activity is called by Piaget as the activity translates to the success of the actions of material determine "change in *x*" and "change in *y*", into understanding the operations seek gradient of the line and determine the basic concept of limit function. Understanding of the operations seek gradient of the line and determine the basic concept of limit function is called Piaget (in Arnon *et al.,* 2014) as a system of operations that have been interiorized.

In addition, students also acquire the rules to be able to acquire basic concepts limit function which will have implications for future activities. A translation of the material to the action of understanding of the operations and the understanding of the rules is what was intended as a Reflective Abstraction.

Dubinsky (in Arnon *et al.,* 2014) interpreted "material actions" as actions undertaken by a subject and an external part of the subject. While the "system" in the phrase "system operations" that have been interiorized as a scheme in this case, is the concept of limit function as a gradient of a tangent curve. When this concept has been conceived so that students can later be applied to the higher operating as understand the concept of the derivative function, he will be transformed into an object. This transformation occurs through a mental mechanism known as encapsulation.

The general framework in the form of content and operations on this item by Arnon et al. (2014) directs the operations of their own to be the new content. These operations rests on a foundation for more differences subtle, such as the difference between the material acts and interiorization, which bears the distinction between mental structures of action and process as well as how the mental mechanisms, as interiorization and encapsulation, The overall lead to the formation of different load conception progress: Action→Process→Object→Schema known today as the APOS theory.

In APOS theory, interiorization in Piaget's abstractive reflection is called the "Process". "Shifting" the term interiorization into the mental mechanisms of APOS theory, by reconstructed an external thing, namely the physical form of the Action into the subject's mind into a Process (interiorized operation). The Process itself is an internal action in the form of mental construction that similar with Action, but overall occurs more in mind than external subject (Arnon *et al.,* 2014).

Detailed information about how the APOS theory emerged based on Piaget's reflective abstraction and the development of intelligence that pay attention to the statements about the schema, thematization of schemas, and coordination of the schemas, also described by Arnon *et al.* (2014). This was done by having regard to the concept of positive integers which is described by Piaget throughout the period of development of this concept.

In applying this schema, a child using a schema of 1-1 correspondence (constructed before or simultaneously) to explain that the two sets of a particular set has the same unit or use a set of schema inclusion, to explain that a set have more (or less) members than the other set. The final understanding lay on the foundation of the schema development of seriation by the student's ability to imagine a sequence of sets such as: {1}, {1 + 1 + 1}, {1 + 1 + 1 + 1}, ...

With these constructions, the child called the sets as one, two, three, four, and so on. And also call their position in seriation as the first, second, third, fourth, and so on. Finally, the two schemas namely classification and seriation are being thematized and then coordinated in the formation of a new schema. A key step in the coordination emerge, when the child realizes that set with four members on the row above is also the fourth set on the line. The resulting scheme is the concept of positive integers. Piaget noticed throughout constructing this to be used as examples of reflective abstraction.

In the example above, the Actions are: a) transforming the physical objects by taking two sets of small objects, b) calculate numbers in one set, then the other set, and c) summing up the two results to obtain the total number of objects; and d) repeating the action in reverse order from two sets to see that the same of total amount. Meanwhile, their objects are: numbers (integers that are represented by a set of physical objects). The action applied to these objects is the summation, and the properties (operation, not a number) are commutative properties.

Coordination according to Piaget is an action of the two schemas. This is a very common usage of the term coordination includes various construction using two schemas, such as the one following the other, or back and so forth between the two schemas to use part of that one and then the other. In an effort to do this, schemas must be thematized in advance, which means made into objects (as the processes being encapsulated into objects) on the state of the action of coordination can then be applied.

The statement of a mentally transfer of an individual from the Actions to Processes, and from Processes to Objects, emerges clearly in the Piaget's discussion (in Arnon *et al.,* 2014) concerning the development of cognitive functions. Piaget's comments could be interpreted as referring to the function as a mapping that is originally an action and then processes and further to the objects. Dubinsky (in Arnon *et al.,* 2014) interpreted the types of these steps as a description of cognitive development that began with the Actions that interiorized into Processes and then encapsulated into Objects to the condition of actions will be applied. This is an example of Piaget's reflective abstraction, about the development of the Action to Process and then to Object and then to the Schema, the progress of which is the main part of the APOS theory.

According to Arnon *et al.* (2014), Piaget's reflective abstraction formed the antecedents to APOS theory –the mental structures of Action, Process, Object, Schema, and the mental mechanisms of interiorization, coordination, reversal, encapuslation, and thematization– as well as their formation into the developmental of APOS progression. This progression is illustrated in Figure 1. The figure shows that Actions operate on Objects; Actions are interiorized into processes (internal action); Processes are encapsulated into Objects; and Objects will be de-encapsulated back on Processes whence they came. The entire system is part of a Schema.

SCHEMA OF FIGURE 1

Figure 1. APOS Theory (Arnon *et al.,* 2014)

Problem-Based Learning Associated by Action-Process-Object-Schema Theory

The principle of problem-based learning process that is based on APOS theory (Action-Process-Object-Schema) run on 5 steps that are part of the establishment of the action, process, object and schema, namely: (1) connecting with the problem, (2) setting up the structure, (3) visiting the problem, (4) revisiting the problem, (5) producing a product or performance, (6) evaluating performance and the problem.

Connecting with problems and structuring them (steps 1 and 2) were categorized as Action. This happens because when students are encountered to the steps 1 and 2, they still commit the material action to the object being characterized by these actions. Visiting the problem step (step 3) is categorized as an interiorization of mental mechanism, and in the condition that the Actions and Processes are happen. This step is categorized as interiorization because its results will be the understanding of the issues that are internal in students' mind. Revisiting the problem step as an attempt to complete problem that has been defined and the conclusion of the alternatives solutions (step 4) is a step of reversal or coordination. When it became reversal, it will go back into action and process, while when the definition has been refined and agreed, then the alternatives to solving the problems will be the encapsulation step.

In this encapsulation step, it is obtained a product or the ability and their abilities and the abilities and the problem (step 5 and 6) which will eventually become the object. This step will also be a de-encapsulation when the resulting product cannot be understood after the test. Cycle is ongoing in order to obtain a scheme of which it considers most appropriate solution and can be generalized. Thus it can be said that the Problem-Based Learning has a learning steps which can be associated with APOS theory.

Mathematics for Senior High School Students Course

Mathematics for Senior High School Students Course is the Course that has the purpose to deepen the mathematics material that will be taught to high school students while at the same time gave the students an understanding of how to provide understanding to the senior high school students. Problem-based learning that is associated by APOS theory is expected to be understood by students as one of the steps that can be used to understand the lecture material given. Besides that, it is expected the students are able to use PBL based on APOS theory related to the practice of teaching at the school. In general, this course will contain the following things:

1. Deepening the concepts that must be mastered by students and will be taught to high school students.
2. Searching the prerequisites material that must be owned by the students and creating concept maps relating to the material.
3. The problems typically encountered by teachers related to the concepts that will be given to students and the effort to solve them.
4. Determining learning scenarios that can solve problems by using problem-based learning approach through the activities that based on the APOS theory. In these activities, students are grouped by sub subject.
5. Delivering the presentation of results of group discussions.

Aim and Questions Addressed

Aim of this research is to know how far PBL associated by APOS theory can implemented in Mathematics for Senior High School Students Course. Besides that, it was needed to know about students' ability to create mathematics instruction in senior high school by using PBL that associated by APOS theory.

To reach the aims observed the information about: 1) how the students connecting and developing structure of the problem related to Action step in APOS theory; 2) how the students visiting the problem related to interiorization step in APOS theory; 3) how the students revisiting the problem, complete the defined problems, and conclude various alternative of problem solving related to reversal and coordination steps in APOS theory; 4) how does the students' encapsulation step was done when they obtain the capability and evaluating it; 5) how could the students be in the stage of having new object as well as a schema related to APOS theory.

Research Methods

Based on the aim of the research, students are given lectures of mathematics high school course by using PBL which is based on APOS theory. Furthermore, at the end of the mid semester, it was applied semi-structured interviews to 26 prospective mathematics teachers that will teach in secondary school. Interviews focused on five main questions: (i) what to do when going to solving the initial problem?; (ii) what is developed to understand the problem and relate it to the concept being studied?; (iii) when there is an error in concluding concept, what to do? (iv) how to conclude that the answer to the problem that posed is right? (v) Any phase obtained after completing a variety of questions? Answers were audio taped to guarantee a better and more reliable data transcription.

After the transcription, it was applied content analysis. The sample integrated 19 female and 7 male students for undergraduate degrees in mathematics education, in a Bandung West Java private university.

Results

After the content analysis of the five questions and considering the objectives of the study, the following results were obtained.

Table1. Students' answers to Q1: What have you done, when solving the initial problem?

Categories of answers	Frequency	(%)
Associating and structuring the problem on the known mathematical concept	18	69.23
Only associating the problem the problem on the known mathematical concept	5	19.23
Only structuring the problem	2	7.69
Unable to recognize the intended mathematical concept.	1	4.34

The analysis of the question Q1 (Table 1) reveals some interviewees who had not done two next steps of PBL. But, the majority of students (69.83 %) do so.

Table2. Students' answers to Q2: what have you developed to understand the problem and associate it with the mathematical concepts being studied?

Categories of answers	Frequency	(%)
Reviewing the problem to acquire mathematical concepts and how to teach it for high school students	15	57.69
Reviewing the problem only to acquire mathematical concepts	4	15.38
Reviewing the problem only to know how to teach mathematics for high school students	3	11.54
Directly making conclusions about solving problems	3	11.54
Unable to make a conclusion	1	4.34

In question Q2, 22 prospective teachers referred to review the problem to understand and associate it with mathematical concepts. As presented in table 2, 15 of them (57.69%) consider that they review the problem to acquire mathematical concepts and to know how to teach the concepts for high school students, 4 of them (15.83%) review the problem only to acquire mathematical concepts, and 3 of them (11.54%) review the problem only to know how to teach the mathematical concepts for high school students.

Table3. Students' answers to Q3: when there has been an error in concluding concept, what to do?

Categories of answers	Frequency	(%)
Revisiting the problem to find appropriate concept and alternative	14	53.84
Specify some other alternative solutions, without revisiting the problem	5	19.23
Revisiting the problem but did not have another alternative	3	11.54
Creating a new answer without alternative	3	11.54
Didn't do anything	1	4.34

In question Q3, 17 prospective teachers referred to revisit the problem. As presented in table 3, 14 of them (53.84%) consider that they revisited the problem to find appropriate concepts and alternative, 3 of them (11.54%) review the problem only to acquire mathematical concepts, and 3 of them (11.54%) review the problem only to know how to teach the mathematical concepts for high school students.

Table4. Students' answers to Q4: how do you conclude that your answers to the problems are proper?

Categories of answers	Frequency	(%)
After re-evaluation and believed to be true	16	61.54
After re-evaluation but not sure to be true	5	19.23
Immediately convinced without doing evaluation	4	15.38
Not evaluated and not convinced	1	4.34

In question Q4, 21 prospective teachers referred to reevaluate the answers to the problems. As presented in table 4, 16 of them (61.54%) consider that they reevaluated the answers and believed to be true and 5 of them (19.23%) reevaluated the answers but not sure to be true.

Table 5. Students' answers to Q5: What are the stages that you earn after solving the problem?

Categories of answers	**Frequency**	**(%)**
Knowing the problem based on the concept that once owned (Action); Relating the owned concept by the new concept to be understood; (Interiorization-Process); Testing owned new concept (coordination-reversal), Testing initial conclusion (encapsulation-de-encapsulation); and conclude a new concept (Object)	15	57.69
Knowing the problem based on the concept that once owned (Action); Relating the owned concept by the new concept to be understood; (Interiorization-Process); and conclude a new concept (Object)	5	19.23
Relating the owned concept by the new concept to be understood; (Interiorization-Process); Testing owned new concept (coordination-reversal); and conclude a new concept (Object)	3	11.54
Knowing the problem based on the concept that once owned (Action) and conclude a new concept (Object)	2	7.69
Conclude new concept (Object), without recognizing the problem, testing prior conclusion, and relating the owned concept by the new concept.	1	4.34

In question Q5, 20 prospective teachers said that they had taken the steps (Action – Process – Object), which are contained in the APOS theory. As presented in table 5, 15 of them (57.69%) consider that they knew the problem based on the concept that they owned (Action); They Related the owned concept by the new concept to be understood; (Interiorization-Process); they tested owned new concept (coordination-reversal), they tested initial conclusion (encapsulation-de-encapsulation); and then they concluded a new concept (Object). 5 of them (19.23%) have done all the steps that are part of APOS theory, except testing owned new concept (coordination-reversal).

Conclusion

With this research we may conclude that Indonesian prospective mathematics teachers acknowledge the importance of PBL associated by APOS theory to understand the concepts of mathematics as well as to teach them. They recognized that the mental mechanisms and the mental structures described in APOS theory, they have felt themselves. With convinced about this, they can better prepare themselves before actually practicing their profession as a mathematics teacher.

Teaching mathematics using the PBL approach will make teachers more focused in knowing the mental mechanisms and the mental structures mentioned in APOS theory, and predicted is happening to students in the classroom. As a result, the math teacher would be able to make the right decisions in solving problems that occur in mathematical instruction in the classroom. The prospective teachers who have known this would have a very complete provision to make them professional mathematics teachers.

The lecturer who teaches mathematics for high school students' course to the prospective mathematics teacher using PBL approach associated by APOS theory, would make the lecturers have two main targets, namely: 1) the prospective teacher students are able to understand mathematical concepts; and 2) the prospective teacher students are able to teach math concepts to students with appropriate learning steps.

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