Algorithm Development to Predict the Dynamic Characteristic of a Multi Rotor System

Amir Zaki Mubarak

[1]Mechanical Engineering Department, Syiah Kuala University, Jl.Tgk. Syech Abdurrauf, No.7, Darussalam, Banda Aceh 23111. Corresponding Author: amir_zaki_mubarak@yahoo.com

Abstract. Rotor systems have been widely used in mechanical applications in industry and even in daily live equipments. The motion effects of the rotor in certain cases can cause discomfort to the user even can cause of failure to the system itself. Therefore the dynamic characteristic analysis of the system is very essential to be analysed in designing a rotor system. By the development of computer technology, the analysis can be straightforwardly performed through numeric method by utilizing existing computational software, i.e. MATLAB. Existing researchers have developed the algorithm to analyse the dynamic characteristic of Jeffcot rotor and dual rotor systems by using MATLAB. The objective of this research is to conduct further investigation in developing algorithm to predict the dynamic characteristic of a multi rotor system. The analysed rotor system consists of a rotor shaft and several disks. The analysis is conducted for a variation number of disk. The equation of motion of the system is derived based on the potential and kinetic energy of the system. Lagrange's equation is then applied to the equation of motion that it can be solved numerically. Based on the equation of motion, a computational algorithm is then developed to determine the natural frequency of the rotor shaft which is the main objective of this research. The algorithm is developed and simulated in MATLAB. The simulation result is closed with theoretical analysis.
Keywords: dynamic characteristic, natural frequency, Lagrange's equation, rotor dynamic.

Introduction

Dynamic analysis is very essential to be considered in designing a rotor system. A variety of methods has been developed to analyse the dynamic of a rotor system. The researches have been conducted in many mechanical applications, such as in turbine drive for a hybrid vehicle vibration system (Thelen *et al.*, 2007) and in flywheel energy storage for a fuel cell powered transit bus (Hearn et al 2007).

Among the components of the rotor system, failure is more likely to occur at the rotor shaft and its bearing which is mostly caused by the vibration of the system. The investigation about this has been carried out to improve the performance. Analytical approach for estimating the shaft temperature difference magnitude in a journal bearing has been developed (Murphy *et al.*, 2009). The rotor transient response on rolling element bearings was also investigated (Fleming *et al.*, 2006).

The failure of the rotor system can be avoided by knowing the natural frequency. Based on the natural frequency, the critical speed of the rotor system is then predicted. The critical speed area is the concern in designing a rotor system. The critical speed solutions have been developed to predict the behaviour of the rotor system. The analysis requires the accuracy and a long time to get the best result. However, by the development of knowledge and computer technology, the analysis can be straightforwardly performed by numerical method. By this method, the eigen value solution can be simply carried out by the help of computer.

The presence of numerical software also simplified the analysis process. The use of finite element method in analysing the dynamic characteristic of a rotor system can be easier with the help of the numerical software. It has been developed a computer program to determine the natural frequency and mode shape of a simple mono rotor (Mubarak et al 2007). A computer program to resolve the natural frequency of a dual rotor system has been developed. (Mubarak *et al.*, 2012). This article describes the research result which is the extent of the dual rotor investigation. The objective of this research is to determine the natural frequency of a multi rotor system. The analysis is performed on multi rotor systems experiencing free vibration without damping.

Matarials and Methods

Modelling of rotor system

The rotor system to be analysed is a multi-rotor system that consists of a shaft with several disks attached to it. The derivation of the equation of motion of the multi rotor system is started by building the analytical model. The system is modelled as a flexible shaft with a disk on it as can be seen in figure 1. The equation of motion of the system is derived numerically that it can be solved by finite element method. In the finite element model, the shaft is divided into several line elements and each node has four digits of freedom: u and w which are translational displacements and θ and ψ which are rotational displacements.

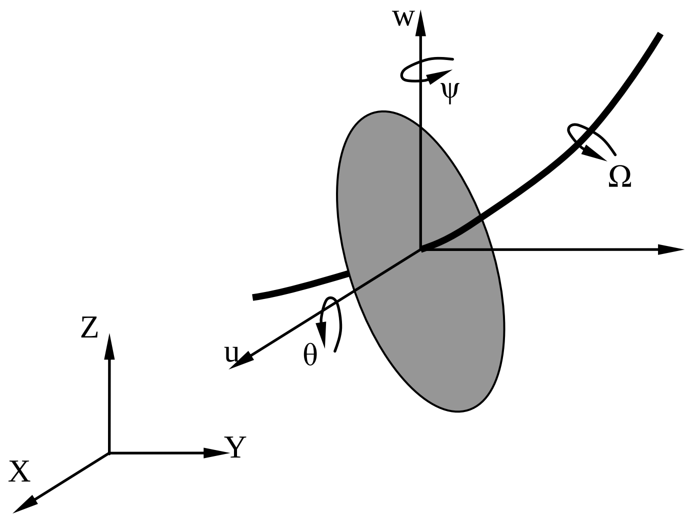

Figure 1 Rotor model.

The derivation of equation of motion of the disk and the shaft for a multi rotor system is identical with the derivation for a dual rotor system as briefly described by Mubarak et al, 2012. Therefore, the equation of kinetic energy of the disk is:

$$T_D = \frac{1}{2} m_d (\dot{u}^2 + \dot{w}^2) + \frac{1}{2} I_{dx} (\dot{\theta}^2 + \dot{\psi}^2) + \frac{1}{2} I_{dy} \Omega^2 \tag{1}$$

The equation of kinetic energy of the rotor shaft is:

$$T_s = \frac{\rho S}{2} \int_0^L (\dot{u}^2 + \dot{w}^2) dy + \frac{\rho I}{2} \int_0^L (\dot{\psi}^2 + \dot{\theta}^2) dy + \rho I L \Omega^2 \tag{2}$$

Furthermore the potential energy equation of the shaft is:

$$U_s = \frac{EI}{2} \int_0^L \left[\left(\frac{\partial^2 u}{\partial y^2} \right)^2 + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 \right] dy + \frac{F_0}{2} \int_0^L \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] dy \tag{3}$$

The equation of motion of the system is derived by substituting the kinetic energy of the shaft and the disk and also the potential energy of the shaft to the Lagrange's equation. By the application of Lagrange's equation, the classical mass matrix, rotary inertia matrix and the classical stiffness matrix are obtained as part of the equation of motion of the system.

Having the inertia matrices and the stiffness matrix determined, the dynamic analysis of the rotor system can be directly resolved by using finite element method. The program algorithm is developed with the flow as be seen in Figure 2. The challenge in developing the algorithm for multi rotor, compared with the developing for mono rotor or dual rotor, is to make the programme as interactive as possible. This is to accommodate the usage of the programme that the user should be able to use the programme for any kind of multi rotor they wish. It is especially related to the disk placement and element meshing.

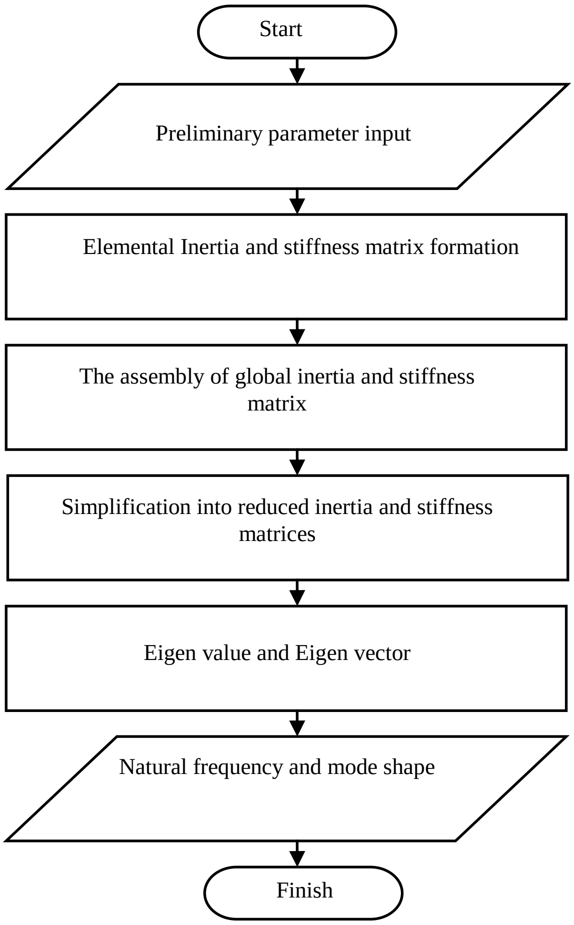

Figure 2 Programming flowchart

Input parameter of the plant

The program is designed to be able to determine the natural frequency of all kind of one shaft multi rotor systems. In this research, the program is tested for several cases with a variety number of disk. In addition, the input parameter of the plant is similar for all the cases:

1. Shaft density (ρ_p) = 7800 kg/m^3
2. Shaft modulus elasticity (E_p) = 2 x 10^{11} N/m^2
3. Shaft length (L_p) = 0.4 m
4. Shaft cross-sectional diameter (d_p) = 0.02 m
5. Shaft poisson's ratio (v) = 0.3
6. Disk inner diameter (d_1) = 0.02 m
7. Disk outer diameter (d_2) = 0.3 m
8. Disk thickness (h) = 0.03 m
9. Disk density (ρ_d) = 7800 kg/m^3

The program is tested for the cases with different number of disk. For finite element modelling, the shaft is divided into eight equal line elements for all cases. In case 1, the rotor system modelled as Jeffcot rotor with one disk in the centre. In addition, there are three disks in case 2. It is located in node 3, 5 and 7. Moreover, the number of disk in case 3 is five disks which are located at node 2, 4, 5, 6 and 8. In case 4, there are seven disks attached to the shaft which are located in node 2, 3, 4, 5, 6, and 7. The finite element modelling of the cases is shown in Figure 3. The result of test is validated with analytical calculation. In order to explore the effectiveness of the program, it is also tested to analyse the same cases with 16, 32, 64, and 128 elements model. The shaft is divided into the number of the elements for each case that the length of the elements is similar in each case but different among the cases.

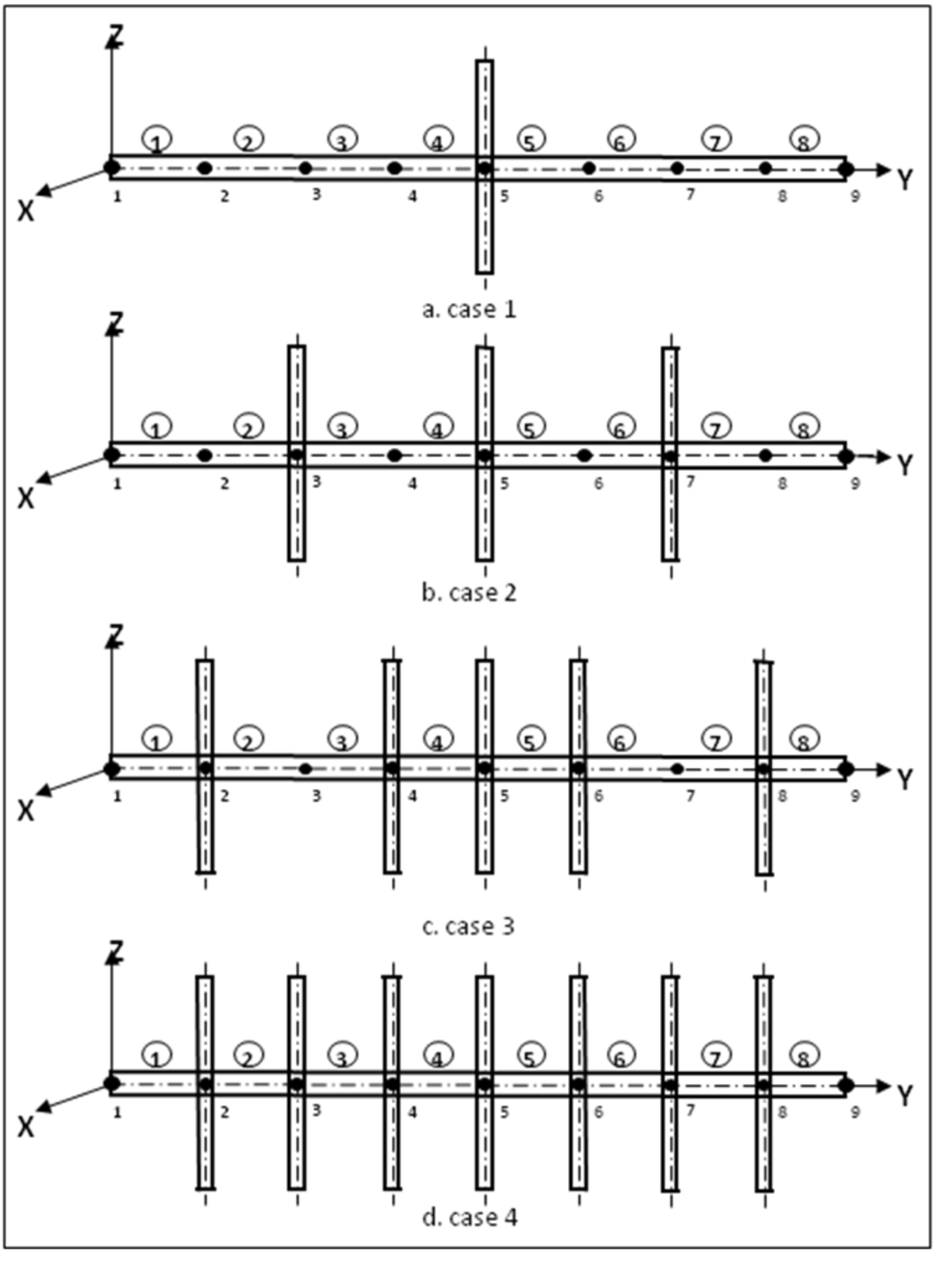

Figure 3 Finite element modelling.

Results and Discussion

The program is designed to be flexible for any cases related to the multi rotor system. The program is specialised for rotor system with one shaft. In this research, the program is tested to be able to solve the natural frequencies of the system effectively. The result of the program is validated with analytical calculation. Table 1 shows both results.

Table 1. Result of the program with eight elements model and result of analytical calculation

Cases	Result of Analytical Calculation		Result of The Program		
	First Natural Frequency (rad/sec)	First Natural Frequency (Hz)	First Natural Frequency (Hz)	Second Natural Frequency (Hz)	Third Natural Frequency (Hz)
Case 1	265.7169869	42.273157	41.8487	927.9462	1527.281
Case 2	174.6830794	27.79048991	30.0076	118.2583	247.5681
Case 3	147.9471378	23.53704464	24.5474	118.067	223.7627
Case 4	125.5619853	19.97577039	21.3021	84.5652	187.5597

Table 1 shows the first natural frequencies from the analytical calculation and three lowest natural frequencies resulted from the program for each case. By using the program it is easier to obtain all the natural frequencies, whether first, second, third and so on, as all the calculation is done by the computer that the result can be obtained easily in a short time.

The first natural frequencies resulted from the program is close enough to the result obtained from the analytical calculation. The result of the computer program is about two points higher than the one from analytical calculation. Only in case 1, the result of the program is very close to the analytical result, the difference is only 0.43 Hz.
Both results show the similar trend as can be seen in Figure 4. The more number of disks attached to the shaft, the lower the natural frequency of the rotor system. This trend is in agreement with the statement about a rotor system that the increase of the mass in small scale can significantly decrease the natural frequency value (Mubarak *et al*., 2012).

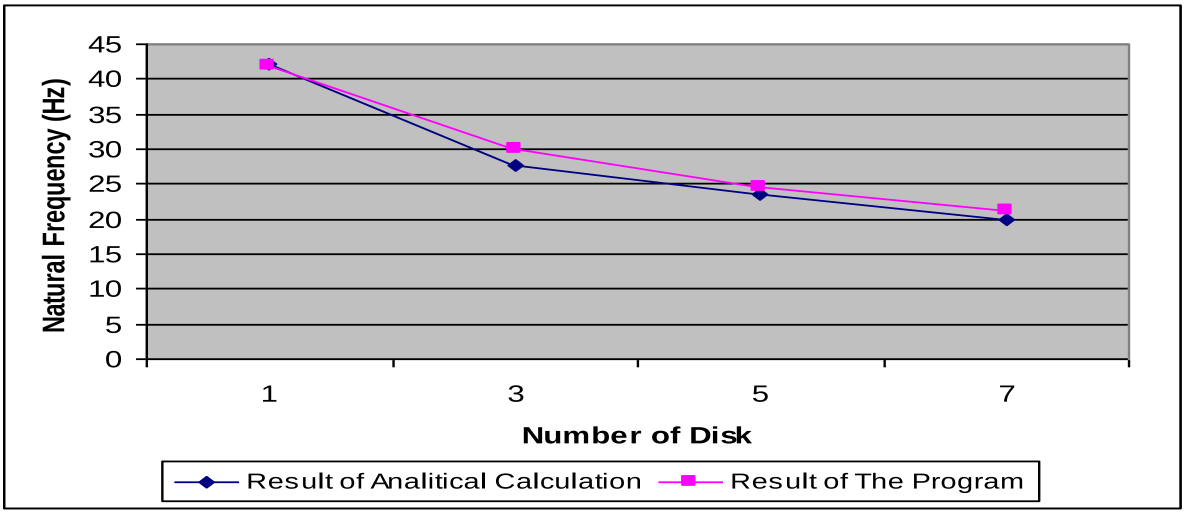

Figure 4. Comparison of the natural frequencies resulted from the program with the analytical calculation

The program is also run to analyse the natural frequencies with 16, 32, 64, and 128 finite elements model while the position of the disks are the same for every model. The result of the program is shown in Table 2, 3, and 4. The tables show the first, second, and third natural frequencies of the system for every cases.

Table 2. Result of the program with eight and 16 elements model

Cases	8 Elements			16 Elements		
	First Natural Frequency (Hz)	Second Natural Frequency (Hz)	Third Natural Frequency (Hz)	First Natural Frequency (Hz)	Second Natural Frequency (Hz)	Third Natural Frequency (Hz)
Case 1	41.8487	927.9462	1527.281	41.8487	969.0144	1525.7962
Case 2	30.0076	118.2583	247.5681	30.0124	118.3264	247.7323
Case 3	24.5474	118.067	223.7627	24.5527	118.2784	224.0895
Case 4	21.3021	84.5652	187.5597	21.3073	84.6455	187.9353

Table 3. Result of the program with 32,and 64 elements model

Cases	32 Elements			64 Elements		
	First Natural Frequency (Hz)	Second Natural Frequency (Hz)	Third Natural Frequency (Hz)	First Natural Frequency (Hz)	Second Natural Frequency (Hz)	Third Natural Frequency (Hz)
Case 1	41.8487	979.4602	1525.5809	41.8487	982.0788	1525.5371
Case 2	30.0136	118.3434	247.7732	30.0139	118.3477	247.7834
Case 3	24.554	118.3314	224.171	24.5543	118.3447	224.1914
Case 4	21.3086	84.6656	188.0294	21.3089	84.6706	188.0529

Table 4. Result of the program with 128 elements model

Cases	128 Elements		
	First Natural Frequency (Hz)	Second Natural Frequency (Hz)	Third Natural Frequency (Hz)
Case 1	41.8487	982.7338	1525.5268
Case 2	30.014	118.3487	247.786
Case 3	24.5544	118.348	224.1965
Case 4	21.309	84.6719	188.0588

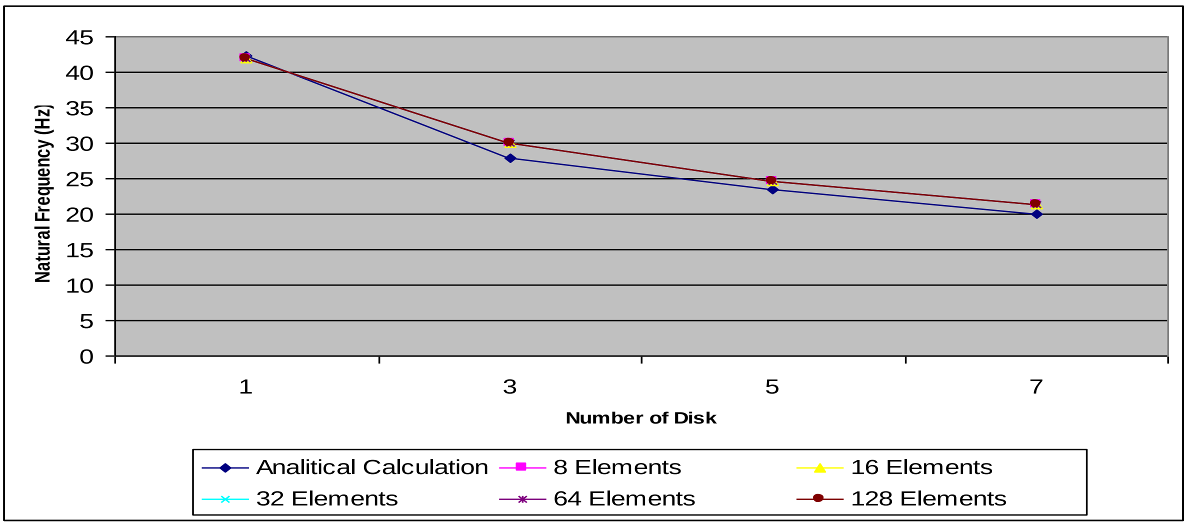

Figure 5. The comparison of the natural frequencies resulted from the program with different number of elements with the analytical calculation

Based on table 2,3 and 4, we can see that there's no significant different of the result of the first natural frequency. The different occurs at the second and third natural Frequency. The difference however, is small and acceptable. Based the tables we can conclude that for this program, the number of elements used has less effect to the result. This can be clearly seen in Figure 5, where all the results of the program with different number of elements are similar and they are just slightly different from the analytical calculation. This can be caused by the iteration process in MATLAB is quite good that a consistent result can be extracted. This phenomenon can also be caused by the use of line elements that the calculation process becomes simpler.

Conclusions

The use of MATLAB is very effective in analysing the natural frequency of a multi rotor system. The result of the program is closed to the result from the analytical calculation. The number of elements used in the programming process has less effect to the result. Thus the calculation can be effectively done with less number of elements.

References

Fleming D.P. 2006. Transient Response of Rotor on Rolling Element Bearings with Clearance, 7th IFToMM Conference on Rotor Dynamics. Vienna, Austria.

Hearn C.S. 2007. Low Cost Flywheel Energy Storage for a Fuel Cell Powered Transit Bus. Vehicle Power and Propulsion Conference, VPPC 2007, IEEE. ISBN: 978-0-7803-9760-6.

Mubarak A.Z., Balia F.N., Akhyar. 2007. Aplikasi Metode Elemen Hingga untuk Menentukan Frekuensi Pribadi dan Modus Getar Poros Rotor. SNTTM VI. Banda Aceh.

Mubarak. A.Z. 2012. Pengembangan Algoritma untuk Menganalisis Frekuensi Pribadi Poros dengan Rotor Ganda. Jurnal Teknik Mesin Unsyiah, Vol. 1, No. 1.

Murphy B.T., Lorenz J.A. 2009. Simplified Morton Effect Analysis for Synchronous Spiral Instability. Proceedings of PWR2009, ASME Power. Albuquerque, New Mexico.

Thelen R.F. 2007. Testing of A 3 MW High Speed Generator and Turbine Drive for A Hybrid Vehicle Propulsion System. Proceedings of GT2007, ASME Turbo Expo 2007: Power for Land, Sea and Air. Montreal, Canada.