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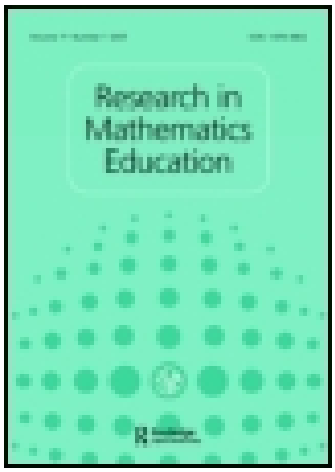

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THE NATURE OF PARTICIPATION AFFORDED BY TASKS, QUESTIONS AND PROMPTS IN MATHEMATICS CLASSROOMS

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8 THE NATURE OF PARTICIPATION AFFORDED BY TASKS, QUESTIONS AND PROMPTS IN MATHEMATICS CLASSROOMS

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This paper reports on the development of an analytical instrument which identifies mathematical affordances in the public tasks, questions and prompts of mathematics classrooms. The aim is to become more articulate about mathematical activity. I have explored the use of several frameworks which identify learning outcomes, structures of knowledge, mental actions, teaching actions and intentions and found that none of them give me access to the detail of what makes one mathematics lesson different from another for learners. From the experience of using these I devised a new analytical tool which unfolds patterns of participation afforded in mathematics lessons. This tool has been tested on several videos of lessons, and has been used by pre-service teaching students to analyse their own lessons.

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DEVELOPMENT

This paper is a contribution to ongoing work in which I have come to view teaching as the creation, with learners, of micro-cultures in which mathematical activity is afforded and constrained, and to view learning partly as the shifts, changes and developments which take place in and through participation in those activities. In this paper I describe my recent efforts to become more articulate about the mathematical content of mathematics lessons in ways which give insight into those micro-cultures which might explain differences in learning between classes.

In Watson (2004) I wrote about how mathematical micro-cultures can be described in terms of the activities they afford, the anticipations which might be structured by enabling constraints, and the attunement of learners towards patterns of participation - an ecological environment in which the presence of explicit variation and invariance in mathematical objects and signs contributes to the structure of activity. I queried whether affordances and constraints (Greeno, 1994) were inherent properties of systems separate from the perceptions of individuals within them, or whether they are perceptual, arising from the inherent and inherited ways of viewing and reacting to the symbols, words and objects which are introduced into the classroom, usually by the teacher or other authority. A further possibility is that they are emergent, arising from interaction between students and teacher and their shared and different pasts. Greeno uses the idea of 'attunement' to describe patterns of participation, and I use this idea to liberate me from relativistic realities so that I can focus on the mathematical affordances initiated by teachers as a dominant source of activity in lessons. Focusing on affordances can be a powerful method for analysing how some teaching might be differently effective than some other teaching, and how learning can be understood by examining ways in which learners might participate in what is

available in the learning environment. This view does not imply that learning is predictable and determined by teaching, but it offers insight into the possibilities for action in a situation – actions which are what the teacher is trying to shape, and those which are spontaneous and unexpected.

Analysis of classroom incidents using these ideas allows the discipline of mathematics to contribute to the analysis in a central way, alongside socio-cultural perspectives. To exemplify this, consider this task: given a rectangle, adapt it through cutting and/or pasting to make new rectilinear shapes such that comparisons of their areas and perimeters to those of the original rectangle exemplify every possible combination of: more, same and less area; more, same and less perimeter[1]. The task affords opportunity to exemplify rectilinear shapes, to describe qualities of area and perimeter, to compare quantities, to consider different ways to alter shapes and so on. These choices are constrained by the cutting and/or pasting relationship and the need for certain area and perimeter comparisons to be imagined and constructed. To take up the task requires searching and selecting, adapting, controlling alteration, comparing features, and so on. Some of these comparisons could be made easily by thinking of an example of a general class, whereas for others very particular examples had to be constructed before general classes could be identified. Some of them could be made by applying knowledge of how to calculate area and perimeter, but others needed theorems about optimisation of area. Learners who are attuned to shifting between generalities and particular cases can participate in the task, whereas others not so attuned are afforded the opportunity to shift towards this kind of mathematical participation. Over time, repeated patterns of affordance and constraint lead to the development of patterns of participation, so learners who are often given exemplification and experimentation as tasks are more able to take part than those for whom these expectations are new.

It is with these understandings, which depend heavily on a view of mathematics as a disciplined collection of ways of thinking, participating and being (Freudenthal, 1991), that I approached the task of analysing a set of videos. Is there a perspective which is independent of subject content, teaching style, lesson structure or classroom culture which can be used to compare the nature of mathematical participation in lessons?

BACKGROUND

In the IAMP project (Watson, DeGeest and Prestage, 2003) we took a phenomenographic approach to charting the practices of teachers who deliberately worked to counteract disadvantage and underachievement. We found that, apart from a few features (such as giving learners space to learn, and maintaining the complexity of mathematics), belief, persistence and courage seemed to be more important than specific teaching tactics (Watson and DeGeest, 2005). However, simultaneous work which focused on the design of tasks (Watson and Mason, 2006) suggested that a

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closer analysis of the affordances and constraints of mathematical activity would be useful.

In a current three-year project with Els De Geest, Changes in Mathematics Teaching (CMTP), the target students are those who enter secondary school below national target levels of achievement but the central unit of analysis has shifted from individual teachers to mathematics departments. We are chronicling the stories of three teams of teachers who deliberately set out, in September 2005, to rescue a significant number of such students. Eventually we are going to describe their practices, and identify factors which appear to contribute to, or hinder, success. We have interviews with teachers and teaching assistants; interview and test data (where available) from some students; documents; observation notes of department meetings; copies of resources; videos of lessons; and so on.

For each of these data-types we first do a content analysis, followed by categorising the content using a range of perspectives. Much of the data from teachers is coded using third generation activity theory (Engestrom, 1998), seeing interaction between departmental and classroom activity as a site for identifying parameters of change. Activity theory allows us to 'lay out' the stories of the departments, and identify common and particular tensions between teachers, between schools and for each teacher and department over time, but this approach leaves important details in the background. For example, we know from earlier projects that readers will ask 'but what did they actually do?'

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LESSON VIDEOS

The problem of analysing videos from the first year of the project is the focus of this paper. The purpose of videoing lessons was to collect a sample of classroom practices over the duration of the project to get some sense of the range and of any similarities and differences, or patterns, between and within schools. It is important to note that the departments appear to espouse similar overarching interests in the development of mathematical thinking. None of them chose 'drill and skill' as an approach to rescuing learners. How were we to analyse the videos to produce a full description of the range of practices in classrooms, at a level of detail which is informative for the research schools and more widely, especially as observing individual lessons gives little insight into how learners gradually become enculturated, over time, into the practices of a particular classroom?

My role in the project included analysis of the video, but I did not have an analytical tool to hand that I thought would be effective. The first stage of analysis of videos was straightforward, which was to produce an account of what I could hear and see which related to the unfolding mathematical story of the lesson. In other words, what utterances, actions and interactions between the teacher and others were publicly available to structure the mathematical activity? While making these accounts I had to work quickly and openly so that I could send them quickly back to the teachers to indicate the nature of the interest researchers were going to take in their teaching. Just

as third-generation activity laid out the parameters of the systems within which teachers were working, so these analytical accounts laid out the public discourse of each lesson but did little else.

FRAMEWORKS

To situate my work in the literature I looked in a variety of places for suitable frameworks to inform the next stage of analysis. There are five main kinds of focus for these:

- learning outcomes
- structures of knowledge
- mental actions students might undertake
- teaching intentions
- teachers' actions.

Existing analytical frames can shape what we look at, but may mask detail, yet by using several frames successively, and reflecting on how they foreground and background aspects of the data, I become more articulate about the fine grain of commonalities, differences, and relationships between teachers, tasks and classroom practices and, more importantly, developed research questions.

Teachers expect that what they say, and the tasks they set, will help learners achieve certain learning objectives. It seemed sensible to start with Bloom's (1956) taxonomy as this is currently 'around' in schools which are focusing on 'learning' as a whole-school issue.

Bloom's taxonomy of learning objectives
Knowledge
Comprehension
Application
Analysis
Synthesis
Evaluation

When applied to mathematics, Bloom's taxonomy seems very crude. For example, it does not provide for post-synthetic mathematical actions, such as abstraction and objectification, although it could be argued that the reflection involved in evaluation might contribute to these taking place. However, in classrooms it is more likely that 'evaluation' derived from this model would be an affective and/or target-accounting process rather than a reflection on emergent learning which might encapsulate recent experience as a new mathematical conceptual entity. Bloom's taxonomy also underplays knowledge and comprehension in mathematics, both of which are multi-

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layered and require successive experiences in different mathematical contexts. 'Comprehension' can mean anything from 'understands how to do it' to 'understands its place in some overarching unifying theory'. 'Knowledge' can refer to results, techniques, concepts or behaviours. For example, what does it mean to have knowledge of equations? Knowing what an equation is, knowing how to work out what it represents, recognising one in unfamiliar contexts and knowing how to solve it are very different kinds of knowledge.

Another possible contender arises in the SOLO (Structure of Observed Learning Outcomes) taxonomy from Biggs and Collis (1982):

Biggs and Collis' SOLO taxonomy
Pre-structural
Unistructural
Multistructural
Relational
Extended abstract

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This offers plausible links between what teachers offer and what learners might perceive: if learners are only offered unistructural situations (simple and obvious relations) they are less likely to develop multistructural understandings. This approach is more promising than Bloom's for my aim to analyse micro-differences in teaching, in that it enumerates input and output variables, it prioritises relationships, and it allows for abstraction. These possible learning outcomes can be used to devise questions which make finer distinctions than the vague notions of 'lower order' and 'higher order' which are often found in the literature on questioning. Translated from a model of learning to a model of teaching, however, this taxonomy does not allow for the interplay between simple and complex examples, between symbols and images, and between examples and generalisation, which characterise mathematical activity. It is also true of any such taxonomies that it matters whose view you are taking – what may seem multistructural to a teacher may be treated as unistructural by a student, and *vice versa*. This central problem, disagreement between the teacher's intentions and learners' perceptions, confounds any attempt to use 'learning outcome' taxonomies to categorise teaching, and yet without complex articulation of learning, teachers cannot sensibly create or select tasks.

Several frameworks describe structures of mathematical knowledge. I will not rehearse them here, but in general they describe initial activity with mathematical objects and tools, then subsequent generalisation and abstraction of ideas at a more formal level. At an advanced level mathematics is seen to involve successive cycles of reification, use and manipulation, leading to further experience and further abstraction and reification (Floyd, 1981; Dubinsky, 1991; Tall, 1995). These models, especially those which include interaction as a means to shift between perceptual,

spontaneous and formal, scientific conceptualisations, link an epistemology of mathematics to a constructivist psychology of logical learning.

This is well-illustrated in the Van Hiele model of geometric understanding (Usiskin, 1982) which describes the human activity of working mathematically by characterising visualisation, analysis, informal deduction, formal deduction and becoming rigorous as levels at which you can structure tasks for students, as well as levels of understanding. If we treat the van Hiele model as a complex web, rather than as a linear hierarchy, we begin to get some sense that different kinds of mathematical action could be triggered by different kinds of task, prompts and questions, and that learners might use different 'levels' of understanding as tools for different aspects of their work. A development from the van Hiele model to a more comprehensive relationship between teachers' offerings and students' attention is Mason's work on the structure of attention (for example, Mason *et al.*, 2005, p.291) in which he extends Van Hiele's ideas to other areas of mathematics and presents them as different kinds of focus rather than as a hierarchy of understanding. Engagement in mathematics includes awareness of whole objects, discerning details, seeing similarities and relationships, focusing on properties, and seeing properties anew as definitions or axioms.

Another non-hierarchical description of learning activity is developed in the Pirie-Kieren model of mathematical understanding (1994), which attempts to relate different kinds of mathematical engagement and allows for 'folding-back' to earlier levels with fresh insight rather than assuming a monotonic outward movement. However, their 'layers' do not always closely match teachers' utterances. For example, one layer is described as 'image-having' and, while 'having' an image associated with a concept is a powerful tool for learning, there are limits to what a teacher can do to bring that about. As with frameworks for learning outcomes, awareness of the importance of concept images can lead teachers to provide opportunities for their development but cannot link teaching to learning in a deterministic sense.

Frameworks which described teachers' intentions were not helpful for this project, since in our teacher interviews intentions were only described in general terms, for example, 'get them to think' or were topic-specific, for example, 'I want them to get a feel for graphs'.

The analytical frame derived for video analysis in the METE international project is more promising as a tool for focusing on mathematical prompts (Andrews, Hatch and Sayers, 2005). This gets close to the intentions of teaching through classifying features of mathematical meaning and structure without assuming that learners necessarily do what is intended. Thus it categorises what might be afforded and constrained in the public mathematical discourse. Their framework looks firstly at whether teachers emphasise and encourage:

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- derivational knowledge
- structural knowledge
- procedural knowledge
- use of efficient methods
- problem-solving
- reasoning

Further, it focuses on pedagogic strategies which are exploited to work on these foci: activating prior knowledge, exercising, explaining, sharing, exploring, coaching, assessing and questioning.

A related resource is provided by Tanner *et al.* (2005) who focus on the mode of interaction: lecture; funnelling; probing; shifting focus; and collective reflection.

Both these latter models focus on a finer grain of detail than the analytical frames used in the TIMSS seven-nation video study (Hiebert *et al.*, 2003). In the TIMSS study, descriptions of typical national lesson types were constructed which enabled cross-national comparison. Further analysis which probed beneath superficial lesson characteristics (board use, nature of questions set, shape of lesson, proportions of teacher-student talk and so on) found that lessons in the more successful countries (as measured by international tests) were characterised by high content level, coherence, structured argument and many opportunities for students to think, whatever the lesson format (Hiebert *et al.*, 2003; Leung, 2006). These descriptions are simply too broad to be of use to us, but are a useful pointer towards the need to map mathematical development in lessons rather than only looking at behavioural, organisational and social norms. Closer to home, the Leverhulme project (Brown *et al.*, 2001) found that analysis of observable lesson structures, and even identification of teachers' orientation (connectionist, discovery or transmissional (Askew *et al.*, 1997)), did not show strong correlation to the success of the mathematics teaching. They produced a detailed instrument for evaluating lessons which reflects the kinds of dimensions of good teaching which were reported in our earlier study (Watson *et al.* 2003).

The models in the last paragraph synthesise categories of teaching so that comparisons can be made between teachers and lessons which might explain differences in engagement and learning. In our study, we aim to categorise teaching so as to reveal a wide range of possible pedagogical choices. If the categorisation is too compact, subtle differences which might lead to different kinds of learning are hidden. If the categorisation is too complex it is unlikely to be useful in informing teaching. Combinations of the models from Andrews *et al*. and Tanner *et al*. were applied to some of the videos and helped to 'lay out' the contents, but this process enabled me to see that what I needed was a way to describe not *whether* teachers did these things or not, but *how* they did them. The Leverhulme instrument required certain value judgements and assumptions about 'better' teaching which were irrelevant to our purposes, but was useful to aid thinking about the qualities of mathematical activity. For example, if a teacher is encouraging links to be made between mathematical entities, what is the significance of the links and how is that

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being done? Most importantly, I found that, with the possible exception of the Leverhulme instrument (depending on interpretation), the sense of conceptual construction that is evident in models of understanding and attention was not embedded in these methods for analysing teaching. The Leverhulme instrument included features such as whether students were encouraged to make connections, apply ideas, generalise from tasks and give extended explanations, and whether the teacher constructs several explanations and exposes relational understandings. I realised that what I needed was a way to search systematically for features like this, and more, but for descriptive rather than evaluative purposes.

IDENTIFYING THE MATHEMATICAL AFFORDANCES OF LESSONS

The process of trying to use existing frames helped me to pose this research question: what opportunities to act mathematically are afforded and constrained by the public tasks, questions and prompts in mathematics classrooms? In other words, to construct my analytical categories I start from mathematics rather than from teaching, or from learning outcomes. Furthermore, this question suggests that the aim of teaching mathematics is to enable learners to act mathematically, an aim which was stated strongly in each of our project schools as they attempted to 'rescue' low-achieving students. To do better at mathematics, most teachers stated explicitly that such learners need to expand their range of kinds of mental and emotional engagement, rather than learn more techniques, definitions, concepts and procedures. Indeed mathematical learning is hard to sustain without engaging in the mathematical practices by which such entities were originally created. The minimal assumption is that if such practices are explicitly encouraged, named, talked about and valued then learners are more likely to participate in them than if they are tacit. This does not imply that learners should each rediscover and reconstruct knowledge, nor that learners need to act like professional mathematicians; rather that the practice of being a mathematics student ought to include specifically mathematical activity.

After this exploration of models, the analytical instrument which I devised and tested identifies dimensions of mathematical orientation and takes as given that the aim of teaching is to enable learners to act mathematically. This is not an instrument to analyse all classroom discourse or interactions; it is to analyse the teacher's contribution to shaping the content of the lesson. The focus, the unit of analysis, is the teacher's utterances or other expressions (such as through what is written on the board, or handed out on worksheets) which might be instructions about what to do, or demonstrations of what is possible, or other kinds of teacherly subject-focused instruction. Orientations, or directions of attention, are set up by the teacher performing certain acts, expressing certain things, initiating discussion about certain things, or asking learners to undertake certain acts. These could have been intentionally planned, in-the-moment micro-decisions, or unintentional. By 'acts' I mean ways in which individuals direct changes in objects, whether these be abstract ideas, or visible things, or symbolic constructions. These actions might be mental acts

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which are then manifested through talk, writing, movements of objects and other forms of mathematical representation[2].

Teacher makes or elicits declarative/nominal/factual/technical statements • Say what the lesson is about • Information giving • Define terms • Tell/know/ask facts, definitions, techniques • 'Research' facts, definitions, techniques *shift: Remember* **Learners are expected to** • Imitate method, copy object • Follow procedure • Find answer using procedure • Give answers *shift: fluency, report/record actions* **Teacher directs learner perception/attention** • Tell/show objects which are perceived as having a single feature • Tell/show objects which are perceived as having multiple features • Tell/show multiple objects • Indicate identification of characteristics/properties • Indicate classification • Indicate comparison • Indicate identification of variables and variation • Summarise what has been done *shift: public orientation towards concepts, methods, properties, relationships*	**Teacher asks for learner response** • Tells what to think about • Use prior knowledge • Find answer without known procedure • Visualise • Seek pattern • Compare, classify • Describe • Explore variation • Informal induction • Informal deduction • Create objects with one feature • Create objects with multiple features • Exemplify • Express in 'own words' *shift: personal orientation towards concepts, methods, properties, relationships* **Discuss implications** • Varying the variables deliberately • Adapting procedures • Identifying relationships • Explication/ Justification • Induction/ Prediction • Deduction *shift: analysis, focus on outcomes and relationships*	**Integrate and connect mathematical ideas** • Clarify • Associate ideas • Generalisation • Redescription • Summarise development of ideas • Abstraction • Objectification • Formalisation • New definition *shift: synthesis, connection* **Affirm/ act as if we know …** • Explore properties of new objects • Adapt/ transform ideas • Application to more complex maths • Application to other contexts • Evaluation of development of new idea • Prove *shift: rigour, objectification, use*

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This instrument needs some explanation. The bold headings are dimensions of mathematical pedagogic orientation, and classify the kinds of mathematical focus I identified in an initial scan of the videos. I have organised these with reference to hierarchical models of mathematical structure, but only because these have to be in some kind of order, not because this implies an ideal order for teaching, nor the order I saw in videos. Rather the order reflects the genesis of mathematical ideas as described by Dubinsky, Tall, Mason and others (above).

The words in normal text classify a range of public tasks and prompts within each overarching orientation. These were derived from watching the videos, informed by earlier work on mathematical activity (Watson and Mason, 1988), and by incorporating aspects of the models of mathematical knowledge outlined above, such as the SOLO focus on structure. I have also been influenced by theories of variation as described by Marton and others (for example, Marton *et al.*, 2004). I see these as the range of possibilities within each dimension. In particular, I tried to include the important shifts and relationships between the teacher showing something to learners, indicating aspects of it, and learners attending to these and finding aspects for themselves. The italicised words are a summary of the kinds of shift a learner might be hoped to make during mathematical activity. These reflect aspects of the Van Hiele model and Bloom's taxonomy, recast for the specific objects of mathematical knowledge. I have avoided assumptions about what students actually do learn.

Although the list is hierarchical in terms of progress towards mathematical application and/or abstraction, it is intended to be complex rather than linear. It is not a model, as it does not have the essential connecting and relating features of a model. Rather it is the contents for a future model. Teaching and learning is not assumed to be unidirectional within it, nor should it be.

The meaning of 'object' is 'that which is being worked on' and includes mathematical statements, theorems, questions, examples, worked examples, exercises, illustrations, symbolic expressions, representatives of classes and so on. This decision alerts us to two further directions of development. The first would be to ask 'what kinds of objects does the teacher introduce into the lesson?'; a teacher who only ever offers worked examples is generating a very different micro-culture to the teacher who offers mainly conjectures and asks only for questions. The second would be to take the 'habits of mind' route (for example, Cuoco, Goldenberg and Mark, 1997) and ask 'what habits of mind does the teacher encourage?'; an example of this would be to have a questioning attitude to current understandings – something a teacher could enculturate over time in her classroom. I believe that both of these could provide profitable ways to look at lessons and classrooms, although the second is not observable in videos of individual lessons.

USING THE ANALYTICAL INSTRUMENT

Having constructed the instrument to provide detailed descriptions of micro-features of mathematics lessons I will illustrate its application to two examples. My purpose

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here is to show its power to produce generic maps of the development of mathematical ideas. The first lesson is a widely-distributed video clip of a lesson about factorising quadratics (NCETM, 2006. The initial focus in the lesson is on adding and multiplying given pairs of numbers: students are given examples to do this and there is a whole-class sequence in which a few students do these calculations, during which the usual problems with negative numbers crop up and are briefly corrected by the teacher. Students then work in pairs on similar calculations and are then given one number and a sum and asked to find the other number and the product; they are eventually asked to find two numbers given their sum and product. Thus there is a structured shift from being given numbers and asked to perform certain operations to being given the outputs of operations and asked to find the unknown inputs. This experience is then applied to finding factors of given quadratic expressions. This experience is then expressed by the teacher in the usual format for factorising quadratics and she draws attention to what they have managed to do. In my analysis, the sequence of activities afforded is:

> knowing facts, being offered objects with one feature, doing some calculations, telling facts, being offered objects with multiple features, identifying relationships, adapting procedures, creating objects with multiple features (pairs of numbers with a given sum and product), formalising, applying to more complex maths, being shown a representation, following a procedure, summarising the development of new ideas.

There is a sense of increasing complexity from simple algorithms to more mysterious 'find the numbers' tasks; each task is a transformation from previous tasks, building up to application within mathematics. Each new task makes use of experiences in the previous task. The lesson affords many dimensions of mathematical activity, specifically opportunities for learners gradually to adopt a personal orientation towards knowing how to find two numbers from their sums and products; this orientation would develop from analysing relationships within work done, which in turn is based on remembered multiplication facts; this new knowledge might be synthesised and applied to quadratic expressions, presented in a formal way. The analytical tool not only provided a structure with which to look closely at the kinds of mathematical activity afforded by the tasks, but also revealed a trajectory through types of mathematical engagement. The overall drift is from the top left to the bottom right of the instrument, but this is not a unidirectional path – there are jumps and zig-zags.

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In one of the videos from our own CMT project[3] a lesson started with the class being asked what they thought of when they saw the word 'algebra'. Much of the lesson then evolved from the nature of their contributions. In my analysis the sequence of kinds of activity afforded is:

> association of ideas, use of prior knowledge, exemplification, comparison, identifying relationships, new definitions, defining terms, copying, doing numerical examples, informal induction, formalising, creating objects with one feature, being offered objects with multiple features, classifying, explication, applying to other contexts.

This lesson ranged across all parts of the list, the numerical examples being offered in the middle of the lesson as a key component of the development of new ideas, rather than as a precursor to, or a manifestation of, them. What kinds of learning shifts did the lesson afford? It started with an interplay between personal and public orientations until sufficient evidence was derived from comparing examples to synthesise a new (to them) definition of an aspect of algebra; then, after recording numerical examples, analysing and synthesising new ideas. These new ideas were subjected to a similar interplay of personal and public orientation with, finally, some examples of application in other contexts. Again, the general drift of the lesson is from left to right, although there is no use of factual or declarative knowledge in it. Within the general direction there are, as in the first example, jumps and zig-zags.

Although development of the analytical tool is work-in-progress, in the analysis of the first year's videos it enables us to be specifically mathematical in our descriptions of lessons, and to map patterns of opportunity for mathematical activity.

As part of the development work, a non-specialist researcher used the instrument to analyse three videos in detail and found that she could use it as a window on the mathematical content. To test it further, thirty PGCE mathematics students were asked to use it to 'map' the development of mathematical ideas in one of their recent lessons. They drew dots next to events in their lessons and joined them up with a directed path. Then they justified their choice of path to their neighbours – for example, what are the reasons for starting with some of the activities described on the left, or on the right? Teachers who found that they only used the earlier (leftmost) parts of the list could see easily that they had missed opportunities to prompt other kinds of mathematical thinking.

This kind of analysis enables us now to compare very different lessons in terms of the kinds of mathematical participation afforded, the kinds of shifts learners might make in the nature of their participation, and the ways in which these are developed during a lesson, through the affordances and constraints of the mathematical tasks and prompts.

The complex interplay and the emergent nature of the teaching-learning interaction demonstrated in the two lessons described above fits with what Vygotsky was aiming at with his notion of ZPD (Valsiner, 1988, p. 144), that is, the learner being supported, through interaction, to take over for herself the unfamiliar, more complex,

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thinking required to complete a task. In the first lesson, orientation towards devising personal ways of finding unknown inputs from given sums and products was supported through the use of structured questions given on worksheets rather than through public discussion; teaching-learning interaction focused on the shared development of demonstration. The second lesson demonstrates substantial use of learner exemplification to provide starting points for the development of new conceptual ideas, whereas learners in the first lesson constructed new procedures (Watson and Mason, 2005). In each lesson there was explicit progress from what is already known to new ideas, and in each lesson this progress culminated in application to a new context, but the parts of this progress which were explained by the teacher, or coordinated in public discussion, or carried out by learners, varied. In one lesson, the main task of learners was to exemplify while the teacher generalised; in the other, raw material was given and learners devised ad hoc methods, some of which could be generalised into algorithms for further application.

The importance (or otherwise) of the order of different features of lessons, and their placing in the sequence, can be conjectured through identifying how these influence what is afforded. For example, a lesson which finishes with definition of terms which are new in that lesson is very different from one which starts with definition of such terms. In the first, definition is part of the affirmation of new ideas; in the second, definition is an authoritative starting point. In the two lesson trajectories expressed in the boxes above, it can be seen that the first lesson requires students to trust that their initial factual knowledge and calculations will lead to something more interesting, whereas in the second students are engaged by using their own prior knowledge and generating their own examples at the start. However, comparisons suggest a need for value judgements. Evidence for valuing one over another is beyond the scope of this paper, even if it is desirable. Analysis of the whole dataset is ongoing, and the question of how to choose between the many comparisons which can be made is the next research problem to be solved.

It is important to recognise that none of this assumes that all students respond in intended ways; their response depends on past patterns of participation, personal disposition, interactions and all kinds of other factors. Attunements take time and multiple experiences to develop and require longitudinal research methods for a thorough understanding. But unless we can legitimately expect most students in a lesson to respond in hoped-for ways there is little point in teaching. This instrument allows me to say how mathematical activity is afforded through those things the teacher *can* control. For now, my claim is that the instrument and the processes of devising and using it have led, and are leading, to new insights about how mathematics lessons can be viewed.

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NOTES

[1] Grateful thanks go to Dina Tirosh and Ken Ruthven for suggesting this task, and to Maria Goulding for her insights about what it affords.

[2] I do not intend to examine here the ontological implications of this remark. Rather, I claim that continuing as if this is true allows me to focus on aspects which are under-researched but potentially significant.

[3] This was a lesson in which I had no other role than to video the teacher and have a brief discussion afterwards about intentions - not the focus of this paper.

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