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Cost Allocation of Reactive Power Using Matrix Methodology in Transmission Network

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ABSTRACT

In the deregulated market environment as generation, transmission and distribution are separate entities; reactive power flow in transmission lines is a question of great importance. Due to inductive load characteristic, reactive power is inherently flowing in transmission line. Hence under restructured market this reactive power allocation is necessary. In this work authors presents a power flow tracing based allocation method for reactive power to loads. MVAr-mile method is used for allocation of reactive power cost. A sample 6 bus and IEEE 14 bus system is used for showing the feasibility of developed method.

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1. INTRODUCTION

A fair transmission pricing methodology should recover all the cost of the transmission system and provide profit to the transmission utility. So many methodologies have developed in past years for transmission cost allocation in transmission system.

The active power production capability of generator will reduced due to more reactive power. Hence, provision of pricing of reactive power becomes an important issue to be addressed in electricity market as similar to real power pricing [1]. The voltage of the system must be controlled as an component of reactive power supports so, the reliability can achieved but more pricing options required due to unrecovered obtained cost with inclusion of capital cost under scheme proposed in [2]. Reactive power transaction depends on indefinite sources such as susceptance of line, capacitor banks rating, generator capacity, installed FACT devices capacity and so on while real power flows depends on source and direction, Reactive power flow is continuously changing due to variable system operating condition. Further transmission of reactive power does not carry over longer distances because it needed to fulfill local requirements. So while locating reactive sinks, sources of reactive power identification became a big challenge. The scheme based on proportional sharing principal [3-4] offer an effectual computational tool but that concept neither discoverable nor verifiable for loss allocation. Power factor based reactive power costing methods are in traditional use but these methods are inappropriate for the restructured power systems, because they separately charged the cost of reactive power support. In addition, the tariff in current scenario only consider local charges and consumption of reactive power calculated with respect to those variables which do not judge the complete customer's usage [3-6]. Relative electrical distance (RED) idea for transmission charge allocation based on nodal pricing method influences the operation condition and system variable has been discussed in [7]. The majority of the above referred solutions [3-7] show that transmission usage charge also having cost of losses in transmission line as in form of their integral part so it does not required separate calculation. The non-acceptability of these methods is due to the long computational time and nonlinearity towards convergence. The Z-bus matrix and modified Y-bus matrix methods treated as circuit-based allocation methods, all the computation in these methods are based on admittance matrix to solved power flow [8], [9].The cost allocation towards line losses based on complex power injection has been addressed in [10]. The virtual flow methodology for assessment of flow of reactive power in transmission network due to different sources and particular load involvement with consideration of counter and loop flows without any difficulty has been addressed in [11]. Flow of electrical power based on tracing approach shows their importance due to its explanatory and comprehensibility in transmission network process. A method based on tracing of electrical power has been reported in [12], [13] which, having assumption that outflow and inflow on nodes are proportionally shared. This permits one to outline power flow in meshed structure. A tracing based reactive power flow is reported by Bialek with upward and downward looking principle. The upward looking principle look at the balancing of incoming flows towards the nodes and the downward looking principle look at the balancing of outgoing flows from the nodes , then compute the power spread among different loads [15].Power flow tracing methods dominate marginal participation method as there is full recovery of cost by tracing flow [16]. It also depends on the Kirchhoff current laws and easy to implements on larger power systems. Moreover it has very less volatility as compared to marginal participation methods. It also provides uniformity and fairness in charge allocation due to depends on actual usage of system [17]. The locational marginal pricing for congestion cost with FACT controller by real power rescheduling for pool based transaction has been discussed in [18]. Determination of generator contribution can be used for congestion management as proposed in [19].

In this paper a reactive power flow allocation method has been proposed. After allocation of reactive power, the total cost to be recovered from individual participant towards transaction of reactive power is also allocated to different participant. For allocation power flow tracing technique is used while for cost allocation MVAr-Mile method is used. Results are shown for 6 bus system and IEEE 14 bus system.

2. DEVELOPED METHODOLOGY

2.1. Model for Reactive Power Flow Allocation

The electrical power system network consists with different component so their behaviors towards tracing of power flow become topological, so power flow by tracing theory is based on true flows in transmission system with consideration of proportional sharing principle. It handles the common issue regarding distribution of VAR (reactive power) flows in a meshed system [12-14]. To determine electricity at the nodes, the nodal power flow based on tracing which generally use implementation of the KCL (Kirchhoff’s current law).

To determine the correlation in conjunction with incoming and outgoing flows the proportional sharing and nodal method is adopt. Hence this principle is similar for the validation of true power and reactive power flows. The model proposed and implemented in this paper considered network is as lossless [15], [16].

Let $\mathrm{ln} = 1 \ldots\ldots\ldots \mathrm{e}$ shows entire transmission line in the power system structured, $\mathrm{G_n} = 1 \ldots\ldots\ldots \mathrm{g}$ is entire quantity of generating units and $\mathrm{D} = 1 \ldots\ldots \mathrm{d}$ is the entire quantity of users in the structure. Again $\mathrm{P_{GG}} = \mathrm{diag}\ (\mathrm{P_{G1}}, \mathrm{P_{G2}}, \ldots, \mathrm{P_{Gg}}\ )$ represents generation in diagonal matrix. Thus from [16]

$$\mathrm{U} = \mathrm{K_m}^{-1}\mathrm{P_L} \tag{1}$$

$$\mathrm{U^T P_{GG}} = (\mathrm{P_G})^\mathrm{T} \text{ or } \mathrm{P_G} = \mathrm{P_{GG}U} \tag{2}$$

By combining equation (1) and (2)

$$\mathrm{P_G} = \mathrm{P_{GG}K_m}^{-1}\mathrm{P_L} \tag{3}$$

Obtained matrix $\mathrm{P_{GG}K_m}^{-1}$ is called generation production matrix. The generation production matrix is indicated by GPM $= (\mathrm{t_{ij}})$, i.e., Where,

$$\mathrm{GPM} = \mathrm{P_{GG}K_m}^{-1} \tag{4}$$

$$\mathrm{R_{i\to j}} = \mathrm{t_{ij}R_{Lj}} \tag{5}$$

Here $\mathrm{t_{ij}R_{Lj}}$ represent the reactive flow contribution of generator situated at bus i to the load at bus j.

Reactive power allocated to generator placed at bus i share the line $\mathrm{s-b}$ can be calculated by,

$$\mathrm{RP}_{\mathrm{i\to s-b}} = \mathrm{t}_{\mathrm{is}}\mathrm{rf}_{\mathrm{s-b}} \tag{6}$$

To obtaining the contribution of reactive power by loads similar procedure is repeat.
Where the diagonal matrix $\mathrm{P}_{\mathrm{LL}} = \mathrm{diag}\,(\mathrm{P}_{\mathrm{L1}}, \mathrm{P}_{\mathrm{L2}}, \ldots\ldots, \mathrm{P}_{\mathrm{Ld}})$ and $\mathrm{EFM} = \mathrm{P}_{\mathrm{LL}}({\mathrm{K}_{\mathrm{m}}}^{-1})^{\mathrm{T}}$ is the extraction factor matrix of loads to generators [16].

2.2. Cost Allocation Model for Reactive Flows

For allocation of reactive power cost following algorithm is developed. For this purpose MVAr-mile method is used. In this model, reactive power charge is allocated with respect to the reactive power base capacity of the transmission line.
If the cost of the line is denoted as $\mathrm{TC}_{\mathrm{s-b}}$ (in Rs/hr) then Reactive power cost allocated to users is given by:
For generator G_i full transmission usage cost allocation is given by, $\mathrm{FTRC}_{\mathrm{s-b}}^{\mathrm{G_i}}$

$$\mathrm{FTRC}_{\mathrm{s-b}}^{\mathrm{G_i}} = \frac{\mathrm{RP}_{\mathrm{i\to s-b}}}{\mathrm{rf}_{\mathrm{base\ s-b}}} \times \mathrm{TC}_{\mathrm{s-b}} \tag{7}$$

Total transmission Reactive power cost by ${\mathrm{TRC}_{\mathrm{f}}}^{\mathrm{G_i}}$ allocated to generator G_i is given by:

$${\mathrm{TRC}_{\mathrm{f}}}^{\mathrm{G_i}} = \sum_{\mathrm{ln}=1}^{\mathrm{e}} \mathrm{FTRC}_{\mathrm{ln}}^{\mathrm{G_i}} \tag{8}$$

Similarly for Load $\mathrm{L_h}$ full transmission usage cost allocation is given by, $\mathrm{FTUC}_{\mathrm{s-b}}^{\mathrm{L_h}}$

$$\mathrm{FTRC}_{\mathrm{s-b}}^{\mathrm{L_h}} = \frac{\mathrm{RP}_{\mathrm{j\to s-b}}}{\mathrm{rf}_{\mathrm{base\ s-b}}} \times \mathrm{TC}_{\mathrm{s-b}} \tag{9}$$

Total transmission Usage cost ${\mathrm{TRC}_{\mathrm{f}}}^{\mathrm{L_h}}$ allocated to Load $\mathrm{L_h}$

$${\mathrm{TRC}_{\mathrm{f}}}^{\mathrm{L_h}} = \sum_{\mathrm{ln}=1}^{\mathrm{e}} \mathrm{FTRC}_{\mathrm{ln}}^{\mathrm{L_h}} \tag{10}$$

2.3. Partial Recovery Model

Partial recovery model provide cost recovery with respect to rated reactive power capacity of transmission line.

If the cost of the line is denoted as $\mathrm{TC}_{\mathrm{s-b}}$ (in Rs/hr) then reactive power cost allocated to users is given by:
For generator $\mathrm{G_i}$, partial transmission usage cost allocation is given by $\mathrm{PTRC}_{\mathrm{s-b}}^{\mathrm{G_i}}$

$$\mathrm{PTRC}_{\mathrm{s-b}}^{\mathrm{G_i}} = \frac{\mathrm{RP}_{\mathrm{i\to s-b}}}{\mathrm{rf}_{\mathrm{rcap\ s-b}}} \times \mathrm{TC}_{\mathrm{s-b}} \tag{11}$$

Total transmission reactive power cost by partial recovery model ${\mathrm{TRC}_{\mathrm{p}}}^{\mathrm{G_i}}$ allocated to generator $\mathrm{G_i}$

$${\mathrm{TRC}_{\mathrm{p}}}^{\mathrm{G_i}} = \sum_{\mathrm{ln}=1}^{\mathrm{e}} \mathrm{PTRC}_{\mathrm{ln}}^{\mathrm{G_i}} \tag{12}$$

Similarly for load $\mathrm{L_h}$, partial transmission reactive power cost allocation is given by$\mathrm{PTRC}_{\mathrm{s-b}}^{\mathrm{L_h}}$

$$\mathrm{PTRC}_{\mathrm{s-b}}^{\mathrm{L_h}} = \frac{\mathrm{RP}_{\mathrm{j\to s-b}}}{\mathrm{rf}_{\mathrm{rcap\ s-b}}} \times \mathrm{TC}_{\mathrm{s-b}} \tag{13}$$

Total transmission reactive power cost ${\mathrm{TRC}_{\mathrm{p}}}^{\mathrm{L_h}}$ allocated to load $\mathrm{L_h}$

$${\mathrm{TRC}_{\mathrm{p}}}^{\mathrm{L_h}} = \sum_{\mathrm{ln}=1}^{\mathrm{e}} \mathrm{PTRC}_{\mathrm{ln}}^{\mathrm{L_h}} \tag{14}$$

The mathematical formulation in Equation 4 shows the contribution of active power of generator's to load in network. Contribution of reactive flow in line by generator can be obtained by mathematical formulation given in Equation 5.

3. RESULTS AND ANALYSIS

The presented model is implemented on standard 14 buses IEEE network and 6 bus sample network shown in Fig.1 to test their feasibility and effectiveness. The programming code is developed in MATLAB tool and results are obtained. Under MATLAB tool firstly power system components such as generator, transmission line and loads are modeled for the test system. Newton-Raphson method is used to determine power flow. The line flow limits were also checked out during the power flow. As the restructuring process in power system going on from a decade, trading of real power in power market are carried out while as a responsibility of system operator toward maintaining system security, stability and reliable operation play an important role. To achieve system operator importance, voltage and reactive power comes to the picture. In this paper, the reactive power contributions of demands have been determined using power flow tracing methods. In this perspective, the influence of total reactive power flow through the line is taken for the analysis.

The proposed model is implemented on test network with 6 buses and 14 buses to show their feasibility. First of all reactive power flows are allocated to loads at normal power flow condition by using modified Kirchhoff matrices methodology given in Table I for 6 bus system. For this purpose equation 5 is used. Allocation of the cost to be recovered from individual participant toward reactive power flow through the transmission network under normal operating condition is also done.

3.1. Sample 6 Bus System

The single line diagram of the sample 6 bus system is shown in Figure 1. It contains 3 generator buses and 3 load buses. The data is at 100 MVA base. Table 1 shows about line flows and cost for 6 bus system. Table 2 shows about allocated reactive power of different loads for 6 bus system. Table 3 shows about cost allocated to different loads for 6 bus system.

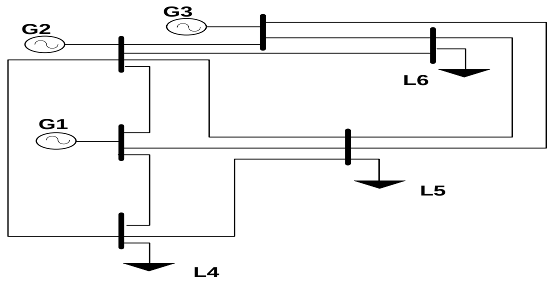

Figure 1. Single line diagram of sample 6 bus system

Table 1. Line Flows and Cost For 6 Bus System

Line	Flow(pu)	Cost (Rs/hr)
1-2	0.142	223.61
1-4	0.227	206.16
1-5	0.149	310.49
2-3	0.075	254.95
2-4	0.496	111.8
2-5	0.185	316.23
2-6	0.153	211.9
3-5	0.269	286.36
3-6	0.645	101.98
4-5	0.023	447.21
5-6	0.063	316.23
Total		2786.92

Table 2. Allocated Reactive Power of Different Loads for 6 Bus System

Allocated Reactive Power to Load4 (pu)	Allocated Reactive Power to Load5 (pu)	Allocated Reactive Power to Load6 (pu)
0.0846	0.0771	0.0169
0.1353	0.1232	0.0270
0.0888	0.0809	0.0178
0.0212	0.0130	0.0213
0.1403	0.0860	0.1412
0.0523	0.0321	0.0527
0.0433	0.0265	0.0435
0.0000	0.0793	0.1862
0.0000	0.1902	0.4465
0.0198	0.0012	0.0000
0.0000	0.0612	0.0015

Table 3. Cost Allocated to Different Loads for 6 Bus System

Charge allocated to Load4(Rs/hr)	Charge allocated to Load5(Rs/hr)	Charge allocated to Load6(Rs/hr)
133.2212	121.4108	26.61274
122.8786	111.8895	24.52123
185.0437	168.5815	37.09209
72.06587	44.19133	72.4058
31.62407	19.38468	31.82694
89.39908	54.87018	90.08282
59.96908	36.70163	60.24608
0	84.41765	198.2165
0	30.07224	70.59546
384.9895	23.3327	0
0	307.1949	7.529286

3.2. IEE 14 Bus System

The single line diagram of the IEEE 14 bus system is shown in Figure 2. It contains 2 generator buses and 12 load buses. The data is at 100 MVA base. Table 4 shows about line flows and cost for 14 bus system. Table 5 shows about allocated reactive power of different loads for 14 bus system. Table 6 shows about cost allocated to different loads for 14 bus system.

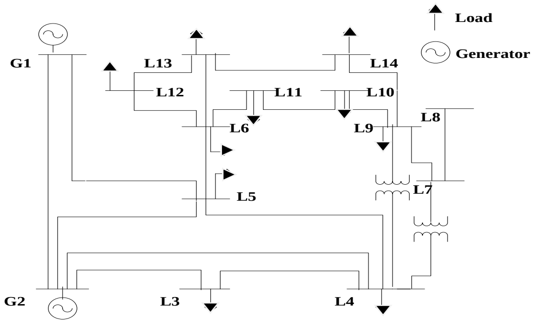

Figure 2. Single line diagram of IEE 14 bus system

Table 4. Line Flows and Cost for 14 Bus System

S.No.	Flow (MVAR)	Cost (Rs/hr)	S.No.	Flow (MVAR)	Cost (Rs/hr)
1	63.774	62.26	11	6.4	220.41
2	39.797	229.49	12	2.918	283.81
3	28.848	203.47	13	8.821	146.1
4	19.862	185.65	14	0	176.15
5	17.173	182.97	15	13.065	110.01
6	0.359	183.69	16	1.531	90.29
7	8.641	44.18	17	1.961	298.77
8	15.371	209.12	18	4.378	208.86
9	10.438	556.18	19	1.111	297.92
10	33.236	252.02	20	3.528	387.73

Table 5. Allocated Reactive Power of Different Loads for 14 Bus System

L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14
24.0395	12.8888	2.185	3.22	0	0	7.9544	2.526	1.0205	1.776	3.9481	4.2157
15.0014	8.043	1.3635	2.0094	0	0	4.9638	1.5763	0.6368	1.1083	2.4637	2.6308
14.7957	5.5619	0.4933	0.727	0	0	3.4325	0.8517	0.2304	0.401	0.8914	1.4631
10.1869	3.8294	0.3397	0.5005	0	0	2.3633	0.5864	0.1586	0.2761	0.6137	1.0073
8.8078	3.3109	0.2937	0.4328	0	0	2.0434	0.507	0.1372	0.2387	0.5306	0.8709
0.359	0	0	0	0	0	0	0	0	0	0	0
1.7808	3.5804	0	0	0	0	2.2097	0.3799	0	0	0	0.6902
3.1677	6.369	0	0	0	0	3.9307	0.6758	0	0	0	1.2278
2.1511	4.325	0	0	0	0	2.6692	0.4589	0	0	0	0.8338
3.6432	7.325	2.2603	3.331	0	0	4.5207	1.9757	1.0557	1.8373	4.0843	3.2028
0	0	0	1.6032	0	0	0	0.5768	0.5081	0.8843	1.9657	0.8619
0	0	0	0.731	0	0	0	0.263	0.2317	0.4032	0.8963	0.393
0	0	0	2.2097	0	0	0	0.795	0.7003	1.2188	2.7093	1.1879
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	8.8022	1.5133	0	0	0	2.7496
0	0	0	0	0	0	1.0315	0.1773	0	0	0	0.3222
0	0	0	0	0	0	1.3212	0.2271	0	0	0	0.4127
0	0	0	0	0	0	0	4.378	0	0	0	0
0	0	0	0	0	0	0	0	0	0.8653	0.1708	0.0749
0	0	0	0	0	0	0	0	0	0	2.4527	1.0753

Table 6. Cost Allocated to Different Loads for 14 Bus System

L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14
23.468	12.582	2.133	3.143	0	0	7.765	2.466	0.996	1.733	3.854	4.115
86.505	46.380	7.862	11.587	0	0	28.623	9.089	3.672	6.391	14.206	15.170
104.356	39.229	3.479	5.127	0	0	24.210	6.007	1.625	2.828	6.287	10.319
95.216	35.793	3.175	4.678	0	0	22.089	5.481	1.482	2.580	5.736	9.415
93.842	35.27603639	3.129231293	4.611274442	0	0	21.77143761	5.401839516	1.461799569	2.543232924	5.653286089	9.279017819
183.69	0	0	0	0	0	0	0	0	0	0	0
9.104	18.305	0	0	0	0	11.297	1.942	0	0	0	3.528
43.096	86.649	0	0	0	0	53.476	9.194	0	0	0	16.704
114.619	230.453	0	0	0	0	142.226	24.452	0	0	0	44.4283
27.625	55.543	17.139	25.258	0	0	34.279	14.981	8.005	13.931	30.970	24.286
0	0	0	55.212	0	0	0	19.864	17.498	30.454	67.696	29.683
0	0	0	71.098	0	0	0	25.579	22.535	39.215	87.175	38.223
0	0	0	36.598	0	0	0	13.167	11.598	20.186	44.873	19.674
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	74.116	12.742	0	0	0	23.152
0	0	0	0	0	0	60.832	10.456	0	0	0	19.001
0	0	0	0	0	0	201.292	34.600	0	0	0	62.87729679
0	0	0	0	0	0	0	208.86	0	0	0	0
0	0	0	0	0	0	0	0	0	232.034	45.800	20.084
0	0	0	0	0	0	0	0	0	0	269.553	118.176

In Opportunity cost method the total cost of reactive power including capacitor cost is 7.31 $/hr or 467.84 Rs/hr and in case of Triangle method total cost of reactive power including capacitor cost is 267 $/hr or 16020 Rs/hr reported in [20] while the total cost allocated obtained from proposed model as shown in Table VI for IEEE 14 bus system is 69.21$/hr or 4152.94 Rs/ hr which is more acceptable to attract the investor in deregulated power market as compared to Opportunity cost method and Triangle method.

4. CONCLUSION

The main objective of the model proposed in this paper is to allocate reactive power for each load based upon the proportion of reactive power flow through the transmission line as per demand by the load. The reactive power flow tracing is done by constructing reactive power flow matrix.

The main reason behind the reactive power flow is the inductive loading at the load end; hence by using MVAr-Mile method the cost of this reactive power flowing is allocated to loads. Power system network is very large so this need to have additional information regarding reactive power injected by different sources as well as shunt admittance of the transmission line, the tracing of power flow scheme becomes as effective tool to achieve that. For reliable and stable operation of power system the reactive power economics play a vital role. By allocating the reactive power flow cost by proposed model, total embedded cost associated with the transmission line can be recovered and the size of reactive power sources installations such as capacitor bank, SVC and FACTS devices can be easily done.

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