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Image Super-Resolution Reconstruction Based On $L_{1/2}$ Sparsity

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Abstract

Based on image sparse representation in the shearlet domain, we proposed a $L_{1/2}$ sparsity regularized unconvex variation model for image super-resolution. The $L_{1/2}$ regularizer term constrains the underlying image to have a sparse representation in shearlet domain. The fidelity term restricts the consistency with the measured imaged in terms of the data degradation model. Then, the variable splitting algorithm is used to break down the model into a series of constrained optimization problems which can be solved by alternating direction method of multipliers. Experimental results demonstrate the effectiveness of the proposed method, both in its visual effects and in quantitative terms.

Keywords*: Super-Resolution, $L_{1/2}$ regularizer, Shearlet, variable splitting*

1. Introduction

Super-resolution (SR) technique offers a possibility to produce an image with a high-resolution from a set of images with low-resolution [1].With the theory of signal estimation, it offers the promise of overcoming some of the inherent resolution limitations of low-cost imaging sensors allowing better utilization of the growing capability of high-resolution displays, and remedies the shortage of the hardware. The super-resolution image reconstruction algorithm mainly includes four categories, algorithm of bi-cubic interpolation, reconstruction based method, study based algorithm and sparse-representation based approach. The algorithm of bi-cubic interpolation method is the simplest techniques. The development of reconstruction based method is more mature, but when with less low-resolution image prior knowledge cannot reconstruct image. Reconstruction based method and sparse-representation based algorithm are currently a very active area of research. The sparse-representation based method including Yang [2] proposes a new SR method under the condition of over-complete dictionary, Wang [3] presents a SR algorithm based on iterative learning redundant dictionary, Sun [4],[5] proposes sparsity regularized image super-resolution by forward-backward operator splitting method and multimorphology sparsity regularized image super-resolution algorithm, Adler [6] raises a shrinkage learning approach for single image super-resolution with overcomplete representations.

In sparse-representation based super-resolution methods, we often use L_0 and L_1 as sparsity regularized term. For L_0 regularization is a NP-hard problem and solving it generally is intractable, and L_1 regularization produces more sparse solution just need to be solving a convex optimization problem, but the sparseness of solution and robustness is not good. Hence, a new regularization emerges as the times require, that is the $L_{1/2}$ regularization, it easier to be solved than L_0 regularization, at the same time it has better sparse and robustness than L_1

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regularization. So the model of super-resolution based on shearlet $L_{1/2}$ regularizer was proposed in this paper. Using Variable Splitting method decoupling the original problem, and introducing alternating direction method of multipliers solves the simple constrained optimization problems.

2. Sparsity regularized SR model

2.1. Degradation model

The SR task is cast as the inverse problem of recovering the original high-resolution image by fusing the low-resolution image y, based on reasonable assumptions or prior knowledge about the observation model that maps the high-resolution image to the low-resolution ones.

The reconstruction of high-resolution images is sometimes modeled by

$$y = DBFx + n \tag{1}$$

where y is the observed low-resolution input image, x is the high-resolution image, F is a relative motion matrix, B is a blurring filter convolution resolution matrix and D is a down-sample operator matrix, n is the random additive noise. There is a bounded operator H, let $H = DBF$ .Then Eq. (1) amounts to

$$y = Hx + n \tag{2}$$

Super-resolution image reconstruction remains extremely ill-posed, since for a given low-resolution input y, infinitely many high-resolution images x satisfy the above reconstruction constraint.

2.2. Sparsity regularized SR

Recently years, sparse representation has been a popular area of research. The ideal image sparse representation in shearlet domain, and then the problem (2) can be written as following

$$y = HS\beta + n \tag{3}$$

where S stands for the shearlet -based, β represents sparse coefficient.

Various regularization methods have been proposed to further stabilize the inversion of this ill-posed problem. We are asked to recover x from observation y such that x is of the sparsest structure (that is, x has the fewest nonzero components). Thus, the sparsity problems can be modeled as the following so called L_0 regularization problem

$$\min_{\beta} \|\beta\|_0 \ s.t. \ \ y = HS\beta + n \tag{4}$$

where $\|\beta\|_0$ formally called L_0 norm, is the number of nonzero components of β.

The L_0 regularization can be understood as penalized least squares with penalty $\|\beta\|_0$. The complexity of the model is proportional with the number of variables, and solving the model generally is intractable. In order to overcome such difficulty, many researchers have suggested to relax the L_0 regularization and instead, to consider the following L_1 regularization [7]

$$\min_{\beta}\|\beta\|_1 \;\; s.t. \;\; y = HS\beta + n \tag{5}$$

where $\|\beta\|_1$ is the L_1 norm. The L_1 regularization is a convex quadratic optimization problem, and therefore, can be very efficiently solved. However, the L_1 regularization cannot handle the collinearity problem, and cannot recover a signal or image with the least measurements when applied to compressed sensing. Thus, a further modification is required. Among such efforts, a very natural improvement is the suggestion of the $L_{1/2}$ regularization.

3. $L_{1/2}$ sparsity regularized SR model

We have conducted a continuation study of a new regularization framework, $L_{1/2}$ regularization, for better solution of sparsity problem [8]. Then Eq. (5) amounts to

$$\min_{\beta}\|\beta\|_{1/2}^{1/2} \;\; s.t. \;\; y = HS\beta + n \tag{6}$$

where $\|\beta\|_{1/2}^{1/2}$ is the $L_{1/2}$ norm.

According to the theory of $L_{1/2}$ regularization, $L_{1/2}$ sparsity regularized super-resolution reconstruction can be modeled as a variational problem as follows

$$\hat{\beta} = \min_{\beta}\|\beta\|_{1/2}^{1/2} + \frac{1}{2}\|HS\beta - y\|_2^2 \tag{7}$$

where $\hat{\beta}$ is the estimation of the coefficient β. It has been shown in the previous studies [9], that $L_{1/2}$ regularization provides a potentially powerful new approach for sparsity problems which is capable of yielding more sparse solutions than L_1 regularization, and it performs best among all L_q regularizations with q in $(0,1]$. The $L_{1/2}$ regularization, however, leads to a nonconvex, non-smooth and non-Lipschitz optimization problem difficult to be solved fast and efficiently. Accordingly, we propose the algorithm of variable splitting to solve $L_{1/2}$ regularization, can get optimal and the only solution efficiently.

Afonso [10] proposes a new algorithm of variable splitting to deal with image inverse problem. Variable splitting (VS) is a simple procedure that consists in creating a new variable, and then addressed using the alternating direction method of multipliers (ADMM). We now return to the unconstrained optimization formulation of regularized image recovery in (7). By the theory of variable splitting, with

$$f_1(\beta) = \frac{1}{2}\|HS\beta - y\|_2^2 \tag{8}$$

$$f_2(\beta) = \|\beta\|_{1/2}^{1/2} \tag{9}$$

$$G = I \tag{10}$$

The constrained optimization formulation is thus

$$\min_{\beta,\alpha}\frac{1}{2}\|HSa-y\|_2^2+\|\beta\|_{1/2}^{1/2} \quad s.t.\beta = \alpha \tag{11}$$

Using variable splitting addresses the constrained optimization formulation (11) leads to

$$\beta_{k+1}=\left((HS)^H HS+\mu I\right)^{-1}\left((HS)^H y+\mu\beta_k^{'}\right) \tag{12}$$

$$\alpha_{k+1}=\begin{cases}\frac{2\alpha}{3}\left(1+\cos\left(\frac{2\pi}{3}-\frac{2\psi(\alpha)}{3}\right)\right) & |\alpha|>\frac{\sqrt[3]{54}}{4}\lambda^{\frac{2}{3}}\\ 0 & otherwise\end{cases} \tag{13}$$

where $\beta_k^{'}=\alpha_k+d_k$, μ is the multipliers parameter, $\psi(\alpha)=\arccos\left(\frac{\lambda}{8}\left(\frac{|\alpha|}{3}\right)^{-3/2}\right)$. We can drive the optimal and the only solution by cycling iterative to solve the formulations (12) and (13), that is high-resolution image $\hat{x}=S\hat{\beta}$.

From what has been discussed above, the approach of process of using variable splitting solve the $L_{1/2}$ sparsity regularized super-resolution model as follows.

Step 1 Input an observed image y, sparse representation in shearlet domain, and change the priori constraints of ideal image to the sparse priori constraints of the coefficients.

Step 2 Initialize the number of iterations parameter k, multipliers parameter μ and the terminate parameter ε.

Step 3 Solve the model. Iterative solving formulations (12) and (13), until satisfy the stop criteria $\left\|\beta^{(k+1)}-\beta^{(k)}\right\|_2^2\le\varepsilon$.

Step 4 Outputs. Will be the optimal coefficient into the $\hat{x}=S\hat{\beta}$, that is SR reconstruction high-resolution image.

4. Experiment and analysis

We apply our methods to generic images such as Lena, Goldhill, Boat and Camera, and compared with bi-cubic interpolation [11] and the SALSA [10] (split augmented Lagrangian shrinkage algorithm). Experiments have same iterative times $k=150$ and the noise standard deviation $\sigma=0.47$. We evaluate the results of various methods three of the PSNR, SSIM and visually.

In our experiments, we magnify the input low resolution image by a factor of 2. Table 1 compares the performance of bi-cubic interpolation, SALSA and our algorithm, in terms of PSNR and SIMM. The proposed approach achieved an average gain of 1.577dB and 0.02809 over the better of bi-cubic interpolation and SALSA. Detail PSNR and SIMM are presented in Table 1 for all methods. We can conclude that our algorithm generates high-resolution images that are competitive or even superior in quality to images produced by other similar super-resolution methods.

Table 1. PSNR and SSIM comparisons of different methods

image	PSNR/dB		
	Bicubic	SALSA	Our Method
Lena	31.6612	31.6965	33.2548
Goldhill	30.6019	30.9542	31.3386
Boat	30.8374	31.2088	31.82
Camera	31.8414	31.6881	33.3851
image	SSIM		
	Bicubic	SALSA	Our Method
Lena	0.91446	0.91979	0.94788
Goldhill	0.82902	0.84544	0.8712
Boat	0.87011	0.88357	0.90263
Camera	0.95121	0.94318	0.97041

(a) LR (b) Bicubic (c) SALSA (d) Our Method

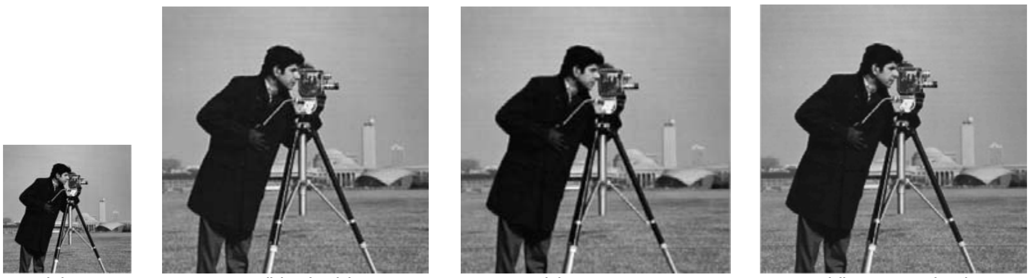
Figure 1. Super-resolution of the image Lena

(a) LR (b) Bicubic (c) SALSA (d) Our Method

Figure 2. Super-resolution of the image Goldhill

(a) LR (b) Bicubic (c) SALSA (d) Our Method

Figure 3. Super-resolution of the image Boats



(a) LR (b) Bicubic (c) SALSA (d) Our Method

Figure 4. Super-resolution of the image Camera

In Figure 1 - Figure 4, we compare our method with several more state-of-the-art methods on images of Lena, Goldhill, Boats and Camera, including bi-cubic interpolation and SALSA. The results of these Figures , which indicate that our method can generate much higher resolution images.

5. Conclusion

We have presented a new method toward image super-resolution sparse-representation based, that is $L_{1/2}$ sparsity regularized image super-resolution in shearlet domain. This super-resolution reconstruction model server shearlet as the image sparse representation, the sparsity as the regularization. Then, proposed variable splitting algorithm to solve the model, through alternating direction method of multipliers. Experiment results demonstrate the effectiveness of the proposed algorithm in this paper.

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