A Physiologic Model for the Problem of Blood Flow through Diseased Blood Vessels

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Article Info	ABSTRACT
	This study focuses on the behavior of blood flow through diseased artery in the presence of porous effects. The laminar, incompressible, fully developed, non-Newtonian in an artery having axially non-symmetric but radially symmetric stenosis is numerically studied. Here blood is represented as Herschel-Bulkley fluid model and flow model is shown by the Navier-Stokes and the continuity equations. Using appropriate boundary conditions, numerical expression for volumetric flow rate, pressure drop and wall shear stress have been derived. The expressions are computed numerically and results are presented graphically. The effects of porous parameter on wall shear stress, stenosis length, stenosis size and stenosis shape parameter are discussed. The wall shear stress increases as the porous parameter, stenosis size and stenosis length increases, but as the stenosis shape parameter increases, the wall shear stress decreases. The work shows that the results obtained from the porous wall model are significantly different from those obtained by the rigid wall model.

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1.   INTRODUCTION

Atherosclerosis is a slow disease in which arteries become clogged and hardened. It leads to cardiovascular disease, which is the leading cause of death in all over the world. Aherosclerotic Figure (1.a) artery is very important because of the fact that the cause and development of many cardiovascular diseases are related to the nature of blood movement and the mechanical behavior of the blood vessel walls. It is defined as a partial occlusion of the blood vessels due to the accumulation of cholesterols and fats and the abnormal growth of tissue. Atherosclerosis is one of the most frequently anomaly in blood circulation. Once the constriction is formed, the blood flow is significantly altered and fluid dynamical factors play important roles. The exact mechanism for the development of this vascular disease is unclear. Various investigators emphasized that the formation of the intravascular plaques and the impingement of ligaments and spure on the blood vessel wall are some of the major factors for the initiation and development of this vascular disease. In 1968 Young analyzed the effect of stenosis in circular tube. Shukla et al. (1980) used several different non-Newtonian models for simulations of blood flow in large arteries and they observed that there is no effect of the yield stress of blood on either the velocity profiles or the wall shear stress. Cavalcanti (1995) has discussed the inadequacies of such studies for the determination of the model for plaque growth. These observations provide future direction for research. Quarteroni et al. (2000), Neofytou (2003) founded the important hemo- dynamical characteristics like the wall shear stress, pressure drop and frictional resistance in catheterized coronary arteries under normal as well as the pathological conditions due to stenosis being present. Leuprecht (2001) measured wall shear stress downstream of axi-symmetric stenoses in the presence of hemo-dynamics forces acting on the plaque, which may be responsible for plaque rupture. Dwyer et al.

(2001) and Abraham et al. (2004) founded the influence of stenosis morphology on flow fields and on quantities such as wall shear stress among stenotic vessels with very mild stenosis. Sharma et al. (2001), Daripa et al. (2002) considered a mathematical analysis of blood flow through arteries using finite element Galerkin approaches. Pontrelli (2000, 2001) and Sharan and Popel (2001) utilized the Brinkman model to investigate the two-dimensional interstitial flow through the tunica media of an artery wall in the presence of an internal elastic lamina.
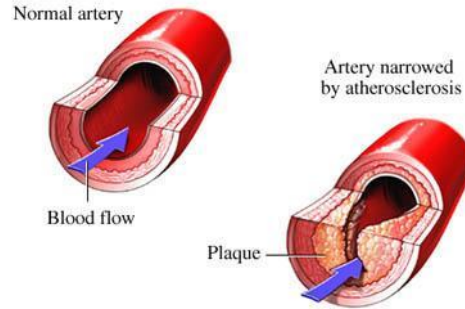

Figure 1a. Atherosclerosis

Yakhot (2004) studied the pulsatile flow of blood under the influence of body acceleration treating blood as a third grade fluid. Sarojani and Nagarani (2008) studied the flow of a casson fluid in a tube filled with a porous medium under periodic body acceleration with application on artificial organs. Johnston (2004), Grigioni et al. (2002) studied the effect of body acceleration on pulsatile flow of non-Newtonian fluid through a stenosed artery and observed that all the instantaneous flow characteristics are affected by the application of body acceleration. Sanjeev et al. (2009) worked on the Pulsatile flow of blood in a constricted artery with body acceleration and observed that the velocity and flow rate increases but effective viscosity decreases, due to a slip wall. The aim of the present investigation has been to study the effect of porous parameter on flow of blood considered as blood as a Herschel-Bulkley fluid model. In this model problem has been sorted out numerically. Numerical expressions are shows the variation of the velocity profile volumetric flow rate, wall shear stress.

2. FORMULATION OF THE PROBLEM:

Consider the axisymmetric flow of blood in a uniform circular artery with an axially non-symmetric but radially symmetric mild stenosis. The geometry of the stenosis as shown in [Figure. (1.b)] Is assumed to be manifested as:

$$\frac{R(z)}{R_0} = 1 - A[L_0^{(m-1)}(z-d) - (z-d)^m], \qquad d \le z \le d + L_0 \qquad (1)$$

$$= 1, \qquad\qquad\qquad \text{otherwise,}$$

$$A = \frac{\delta}{R_0 L_0^m} \frac{m^{m/(m-1)}}{(m-1)},$$

Figure 1b. Stenotic Artery

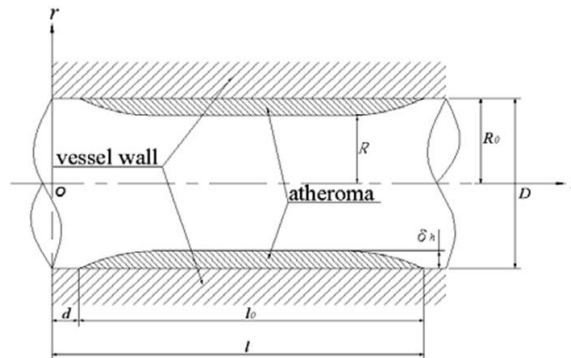*A Physiologic Model for the Problem of Blood Flow through Diseased Blood Vessels (Sapna Ratan Shah)*

where

R$_0$: Radius of normal tube
R(z) : Radius of stenotic region
L : The length of the artery
L$_0$: The length of the stenosis
d : Distance between equispaced points
δ : Maximum height of stenosis (δ << R$_0$)
m : Parameter determining the shape of stenosis (m ≥ 2)

Herschel-Bulkley fluid model: For Herschel-Bulkley fluid model, the relationship between stress and strain is given by:

$$f(\tau) = \left(-\frac{du}{dr} \right) = \frac{1}{\mu} (\tau - \tau_0)^n, \qquad\qquad \tau \geq \tau_0$$

$$f(\tau) = \left(-\frac{du}{dr} \right) = 0, \qquad\qquad \tau \leq \tau_0 \qquad\qquad (2)$$

$$\text{where} \quad \tau = \left(-\frac{dp}{dz}\frac{r}{2} \right), \qquad \tau_0 = \left(-\frac{dp}{dz}\frac{R_c}{2} \right),$$

$$e = \left(-\frac{du}{dr} \right)$$

where,

τ$_0$ = Measure of yield stress
τ = Stress tensor
e = Strain rate (-du/dr)
u = Velocity of fluid
μ = Viscosity of blood (Casson's viscosity coefficient)
r = radius of the artery

Darcy flow model: The Darcy model of flow through a porous media is,

$$U = \left(-\frac{kp}{\mu} \right) \qquad\qquad (3)$$

where

k = Porous parameter
μ = Viscosity of blood
p = Pressure gradient

Conservation equations: In the present investigation the flow formulation for Navier-Stokes equations for incompressible non-Newtonian fluids in the porous effect has been considered. The basic equations of motion in cylindrical polar coordinates are:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} + v\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial^2 r} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} \right) \qquad\qquad (4)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial^2 r} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial r^2} - \frac{v}{r^2} \right) \qquad\qquad (5)$$

$$\frac{\partial u}{\partial z} + \frac{1}{r}\frac{\partial(vr)}{\partial r} = 0 \qquad\qquad (6)$$

Boundary conditions: The no slip condition on the stenosis surface gives the following boundary conditions

$$u = 0 \qquad at \qquad r = R_0$$
$$u = 0 \qquad at \qquad r = R(z) \tag{7}$$

3. SOLUTION OF THE PROBLEM:

Solving these equation, the velocity of blood, rate of flow and pressure in (7, 8, 9),

$$\frac{\partial u}{\partial r} = -(\frac{p}{2m})^{1/n} \ [r - R_c]^{1/n} \tag{8}$$

now total flow flux

$$Q = \int_0^R 2\pi u \, r \, dr = \pi \int_0^R r\left(-\frac{du}{dr}\right) dr, \tag{9}$$

by using equation (8) and equation (9), we have,

$$Q = \frac{\pi}{2}\left(\frac{P}{2\mu}\right)^{1/n} \frac{R^{(3+(1/n))}}{(1+(1/n))} f(y) \tag{10}$$

where $f(y) = [2(1 - \frac{R_c}{R})^{((1/n)+1)} - \frac{4}{((1/n)+2)}(1 - \frac{R_c}{R})^{((1/n)+2)} + \frac{4}{((1/n)+2)((1/n)+3)}$

$((1 - \frac{R_c}{R})^{((1/n)+3)} - ((-1)^{((1/n)+3)}(\frac{R_c}{R})))],$

$$\overline{y} = \frac{R_c}{R} << 1.$$

Using equation (8) we have,

$$P = \left(-\frac{dp}{dz}\right) = \frac{2\mu}{R^{(1+3n)}}\left(\frac{2Q}{\pi f(\overline{y})}(1 + \frac{1}{n})\right)^n \tag{11}$$

to determine λ, we integrate equation (11) for the pressure P_L and P_o are the pressure at $z = 0$ and $z = L$, respectively, where L is the length of the tube.

$$\Delta P = P_L - P_0 = \frac{2\mu}{\pi R_0^{1+3n}}\left(2Q\left(\frac{1}{n}+1\right)\right)^n \int_0^L \frac{dz}{\left(\frac{R(z)}{R_0}\right)^{(1+3n)} (f(\overline{y}))^n} \tag{12}$$

on using equation (12) and (10) gives,

$$\frac{dp}{dz} = P_L - P_0 = \frac{2\mu}{\pi R_0^{1+3n}}\left(2Q\left(\frac{1}{n}+1\right)\right)^n \varphi(z) \tag{13}$$

The pressure drop, p (= p at z = 0 and − p at z = L) across the stenosis in the tube of length, *L* is calculated from equation (9) as

$$\Delta p = \int_0^L \frac{2\mu}{\pi R_0^{1+3n}}\left(2Q\left(\frac{1}{n}+1\right)\right)^n \varphi(z)\,dz = \frac{2\mu}{\pi R_0^{1+3n}}\left(2Q\left(\frac{1}{n}+1\right)\right)^n \psi \tag{14}$$

Where $\quad \psi = \int\limits_{0}^{L} \varphi(z)dz = \int\limits_{0}^{d} [\varphi(z)]_{R/R_0 = 1} \, dz + \int\limits_{d}^{d+L_0} \varphi(z)dz + \int\limits_{d+L_0}^{L} [\varphi(z)]_{R/R_0 = 1} \, dz$

The first and third integrals in the expression for obtained above are straight forward whereas evaluation of the second integral is a formidable task and therefore will be evaluated numerically.

$$p = \frac{2\mu}{\pi\, R_0^{1+3n}} \left(2Q\left(\frac{1}{n}+1\right)\right)^n \int\limits_0^L \frac{dz}{\left(\dfrac{R}{R_0}\right)^{(3+(1/n))} \left(1-\dfrac{16}{7}\left(\dfrac{r}{R}\right)^{(1/n)}+\dfrac{4}{3}\left(\dfrac{r}{R}\right)-\dfrac{1}{21}\left(\dfrac{r}{R}\right)^{(1+(1/n))}\right)} \tag{15}$$

Using equation (1) and equation (3) we have,

$$\tau = \left(\frac{U\mu R_0}{2k}\right)^{(3+(1/n))} \int\limits_d^{(d+L_0)} \begin{pmatrix} \left(1-\dfrac{16}{7}\dfrac{\delta}{R_0 L_0^m}\dfrac{m^{m/(m-1)}}{(m-1)}[L_0^{(m-1)}(z-d)-(z-d)^m]\right)(3+(1/n)) \\ +\dfrac{4}{3}\left(\dfrac{\delta}{R_0 L_0^m}\dfrac{m^{m/(m-1)}}{(m-1)}[L_0^{(m-1)}(z-d)-(z-d)^m]\right)^{(1/n)} \\ -\dfrac{1}{21}\left(\dfrac{\delta}{R_0 L_0^m}\dfrac{m^{m/(m-1)}}{(m-1)}[L_0^{(m-1)}(z-d)-(z-d)^m]\right)^{(1+(1/n))} \end{pmatrix} \tag{16}$$

4. RESULTS AND DISCUSSIONS

The motivation behind developing this mathematical model has been to study some aspects of blood flow through a stenosed artery in the presence of porous parameter having an axially non-symmetric but radially symmetric mild stenosis. Most of theoretical result such as velocity, volumetric flow rate, wall shear stress and pressure gradient are obtained in this analysis. Out of these results only the numerical solution of wall shear stress are shown. Wall shear stress is an important factor in the study of blood flow. Accurate predictions of the distribution of wall shear stress are particularly useful for the understanding of the effect of blood flow through stenosed artery in the presence of porous effects. In order to estimate the quantitative effects of porous parameter, stenosis size, stenosis length and stenosis shape parameter on wall shear stress, computer codes were developed and to evaluate the numerical results for wall shear stress for diseased system associated with stenosis due to the local deposition of lipids have been determine.

In order to understand the complete nature of blood flow in the presence of porous parameter, the computed numerical results by using the values of parameter based on experimental data in stenosed artery ($U = 50$cm/sec, $R_0 = 1.2$ cm and $\mu = 0.04$ dyne/cm^2) are plotted in Fig 2-4. In order to analyze the wall shear stress along the stenosed arterial segment under study, Fig.2 exhibits the variation of wall shear stress with porous parameter for different value of stenosis shape parameter. It is observed that in the stenosed portion of the artery, the wall shear stress increases with the increase in porous parameter while decreases as stenosis shape parameter increases. The result is consisting with the result of Shukla et al. (1980).
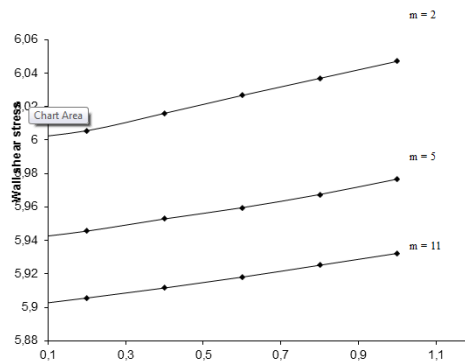

Figure 3. Porous Parameter, Variation of War Shear Stress with Porous Parameter for Different M
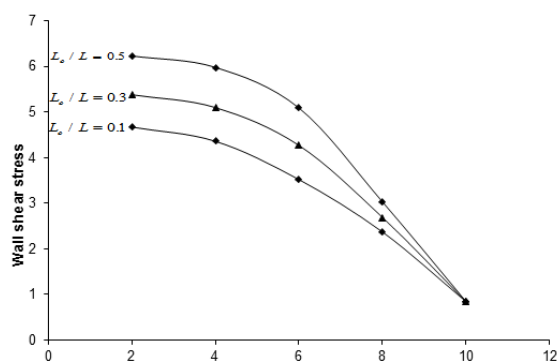
Figure 4. Stenosis Shape Parameter, Variation of Wall Shear Stress with Stenosis Shape Parameter

The graph for variations of wall shear stress with stenosis size for different value of stenosis shape parameter is shown in figure 3. It is evident that wall shear stress increases as stenosis size increases and wall shear stress decreases as stenosis shape parameter increases. As the stenosis grows, the wall shearing stress increases in the stenotic region. Our results are similar to those obtained by Sanjeev (2009). Fig.4 gives a comparison of our results with those reported by Johnston et al. (2004). It is shown that the wall shear stress decreases with increasing value of stenosis shape parameter and wall shear stress increases as stenosis length increases. In Fig.5 the variation of wall shear stress with stenosis shape parameter has been shown. The wall shear stress gives the reverse trend of stenosis size and stenosis length for increasing value of shape parameter. This figure illustrates that wall shear stress decreases as stenosis shape parameter increases, maximum wall shear stress occurs at (m = 2), i. e. in case of symmetric stenosis. These results are similar with the results of Pontrelli (2000).

5. CONCLUSION

In this paper we used the laminar, incompressible, fully developed, non-Newtonian flow having axially non-symmetric but radially symmetric stenosis in the presence of porous effects. Here the blood is represented as Herschel-Bulkley fluid model and flow model is shown by the Navier-Stokes and the continuity equations. The advantage of this study is that here we calculated the effect of porous parameter on wall shear stress, stenosis shape parameter, stenosis size, stenosis length in an stenosed artery. It has been concluded that the wall shear stress increases as porous parameter, stenosis size and stenosis length increases but decreases as stenosis shape parameter increases. It has shown that the results were greatly influenced by the change of porous parameter and stenosis shape parameter. This model is able to predict the main characteristics of the physiological flows and would be helpful for the people working in the field of biomedical science as well as to the medical practitioners.

REFERENCES

[1] Abraham F., Marek B. and Matthias H, Shape optimization in steady blood flow: A numerical study of non-Newtonian effect. November 2004: 1-17.
[2] Daripa P., Ranjan K., A numerical study of pulsatile blood flow in an eccentric catheterized artery using a fast algorithm: J. Eng. Mathe. 2002: 42: 1-2.
[3] Dwyer H. A., Cheer A. Y., Rutaginire T. and Shahcheragin, Calculation of unsteady flows in curved pipes: ASME J. Fluids Eng. 2001: 123: 869-873.
[4] Grigioni M., Daniele C., and Davenio G., The role of wall shear stress in unsteady vascular dynamics: Progress in Biomed. Resea. 2002: 7.
[5] Johnston B., Johnston P. R., Corney S., and Kilpatrick D., Non-Newtonian blood flow in human right coronary arteries: Steady state simulation: J. Biome. 2004: 37: 709-720.
[6] Leuprecht A. and Perktold K., Computer simulation of non-Newtonian effects on blood flow in large arteries: Com. Meth. Biomec. & Biomed. Eng. 2001: 4: 149-163.

[7] Nagarani P. and Sarojamma G. Effect of body acceleration on pulsatile flow of casson fluid through a mild stenosed artery. Journal of Korea-Australia Reology. 2008: 20: 189-196.

[8] Neofytou P. and Drikakis D., Non-Newtonian flow instability in a channel with a sudden expansion: J. Non-Newto. Fluid Mech. 2003: 111: 127-150.

[9] Pontrelli G., Blood flow through a circular pipe with an impulsive pressure gradient. Math. Mod. Meth. Appl. Sci. 2000: 10: 187-202.

[10] Pontrelli G., Blood flow through an axisymmetric stenosis. Proc. Inst Mech. Eng, Part H, Eng Med. 2001: 215: 1-10.

[11] Quarteroni A, Tuveri M. and Veneziani A., Computational vascular fluid dynamics; Problems, models and methods: Compu. Visualization in Science. 2000: 2: 163-197.

[12] Sanjeev K. A mathematical model for Newtonian and non-Newtonian flow through tapered tubes. Int. Review of Pure and Applied Mathematics. 2010:5.

[13] Sharan M. and Popel A. S., A two-phase model for flow of blood in narrow tubes with increased effective viscosity near the wall, Biorhe. 2001: 38: 415-428.

[14] Shukla J. B., Parihar R.S. and Rao B. R. P., Effect of stenosis on non-Newtonian flow of the blood in an artery, Bull. Math. Biol. 1980: 42: 283-294.

[15] Yakhot A., Grinberg L. and Nikitin N., Modeling rough stenoses by an immersed-boundary method: J. Biomech. 2004.

[16] Young, D. F. Effects of a time-dependent stenosis of flow through a tube, Journal of Eng. Ind. 1968: 90: 248-254.