

# A Genuine Random Sequential Multi-signature Scheme

**Yonglong Tang**

School of Mathematics and Statistics, Jishou University  
email: tytyls@163.com

## **Abstract**

*The usual sequential multi-signature scheme allows the multi-signers to sign the document with their own information and sequence, and the signature is not real random and secure. The paper analyzes the reasons for the insecurity of the previous multi-signature scheme, and puts forward a Genuine Random Sequential Multi-signature Scheme based on The Waters signature scheme, and the experiment proves that this scheme is a good scheme suitable for the practical application with high computing efficiency.*

**Keywords:** *bilinear maps, the waters signature scheme random, ordered multi-signature schemes*

## **1. Introduction**

The paper analyzes the reasons for the insecurity of the previous multi-signature scheme, and puts forward a Genuine Random Sequential Multi-signature Scheme based on The Waters signature scheme, and the experiment proves that this scheme is a good scheme suitable for the practical application with high computing efficiency

A multi-signature scheme allows  $n$  different Signers to jointly sign the same document, yielding a multi-signature of roughly the same size as a standard signature. A verifier is convinced that each signer participated in signing. By transmitting a multi-signature instead of  $n$  individual signatures, multi-signature schemes can greatly save on communication costs.

Mihir Bellare and Gregory Neven[1] for an Identity based multi-signature scheme from RSA. Craig and Zulfikar Ramzan[2] also for an identity-based multi-signature scheme. Alexandra Boldyreva et al.[5] propose a new primitive that they call ordered multi-signatures and a formal security model for ordered multi-signatures. "The ordered multi-signatures produces a compact multi-signatures, uses constant-size keys, is 'sequential' in that signers sign one after another and no further interaction among

Comparing to the "structured" signatures [13], the ordered multi-signatures scheme in [5] is in the noninteractive setting. It is practical. Provable security is the basic requirement for signature schemes. Alexandra Boldyreva et al.[5] proved ordered multisignature scheme secure in the random

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Commitment scheme is a basic building block and has diverse applications to cryptographic proto-cols, especially to zero-knowledge proofs[14]. Informally, a commitment scheme is a two-stage protocol between a sender and a receiver. In the first stage, the sender commits to a value  $b$ , and in the second, the sender 'reveal' this value to the receiver. We want two security properties from a commitment scheme. The hiding property says that the receiver does not learn anything about the value  $b$  during the commit stage. And the binding property says that after the commit stage there is at most one value that the sender can successfully

open. According to the computational power of senders and receivers, commitments can be classified into several possible types [15]. In this paper, we mainly focus on construction of perfectly hiding and computationally binding commitment (PHCBC).

Some works about perfectly hiding commitment: Construction of PHCBC is an attractive problem. PHCBCs with a constant number of rounds were shown to exist based on specific number-theoretic assumptions (or, more generally, based on any collection of claw-free permutations with an efficiently-recognizable index set), and collision-resistant hash functions [16]. Protocols with higher round complexity were shown to exist based on different types of one-way functions. Protocols with  $O(n/\log n)$  rounds were based on one-way permutations [17] and regular one-way functions [18]. Finally, a protocol with a polynomial number of rounds was based on any one-way function [19]. In [20],  $O(n/\log n)$  rounds were shown to be the tight lower bound on the round complexity of PHCBC.

The Problem: It is not known whether PHCBC in constant rounds constructed can be achieved with one-way function or one-way permutation. There are many so-called "atomic" ZK protocols for NP that achieve constant error-probability in constant (three or four) moves [22]. Serial repetition lowers the error and preserves ZK, but at the cost of increasing the number of rounds to non-constant. So we would like to do parallel repetition. However, this is ruled out: first, we have the above-mentioned results of [23]; second, the latter also showed that in general parallel repetition does not preserve ZK. So one must build low error ZK protocols directly [24].

## 2. Preliminaries

### 2.1. NP Relations

NP relations We say that a binary relation

$$R \subseteq \{0,1\}^* \rightarrow \{0,1\}^*$$

is an NP relation if there exists a polynomial  $p(\cdot)$  such that for any

$$(x, w) \in R,$$

$$|w| \leq p(|x|)$$

and in addition there exists a polynomial time Turing machine for deciding membership in  $R$ . We denote by  $L_R$  the following:

$$L_R = \{x \mid \exists w \text{ s.t. } (x, w) \in R\}$$

We say that

$$L \in \text{NP} \text{ if } L = L_R$$

for some NP relation  $R$ .

A negligible function is a function that grows slower than the inverse of any polynomial. That is,

$$\nu : \mathbb{N} \rightarrow \mathbb{N}^0$$

is negligible if for any positive polynomial  $p(\phi)$  there exists a number  $n_0$  such that

$$\nu(n) < 1/p(n)$$

for all

$$n > n_0$$

One-Way Function A function

$$f: \{0,1\}^* \rightarrow \{0,1\}^*$$

is called one-way if the following conditions hold:

1. There exists a deterministic polynomial-time algorithm  $A$  such that on input  $x$ ,  $A$  outputs  $f(x)$ ;
2. For every non-uniform probabilistic polynomial-time algorithm  $A'$  there exists a negligible function  $\nu$  such that for all sufficiently large  $k$ , it holds that

$$\text{Prob}(x \leftarrow \{0,1\}^* \\ A'(f(x)) \in f^{-1}(f(x)) < \nu(k)$$

$f$  is a one-way permutation if it is a 1-1 and length preserving one-way function.

## 2.2. Commitment scheme

Definition 1 ( $\text{Gen}, \text{Com}, \text{Ver}$ ) is a commitment scheme[11] if:

- a. Efficiency:  $\text{Gen}, \text{Com}$  and  $\text{Ver}$  are polynomial-time algorithms;
- b. Completeness: for all  $m$  it holds that

$$\text{Prob}(\text{crs} \leftarrow \text{Gen}(1^k); \\ (\text{com}, \text{de c}) \leftarrow \text{Com}(\text{crs}, m); \\ \text{Ver}(\text{crs}, \text{com}, \text{de c}, m) = 1) = 1$$

- c. Binding: for any polynomial-time algorithm sender there is a negligible function  $\nu$  such that for all sufficiently large  $k$  it holds that

$$\text{Prob}(\text{crs} \leftarrow \text{Gen}(1^k) \\ (\text{com}, m_0, m_1, \text{de } c_0, \text{de } c_1) \leftarrow \text{Sender}(\text{crs}) \\ m_0 \neq m_1 \text{ and} \\ \text{Ver}(\text{crs}, \text{com}, \text{de } c_0, m_0), \text{Ver}(\text{crs}, \text{com}, \text{de } c_1, m_1) = 1) \leq \nu(k)$$

- d. Hiding: for any adversary receiver there is a negligible function  $\nu$  such that for all  $m_0, m_1$  where

$$m_0 \neq m_1$$

and all sufficiently large  $k$  it holds that

$$\text{Prob}(\text{crs} \leftarrow \text{Gen}(1^k); \\ b \leftarrow \{0,1\}; (\text{com}, \text{de c}) \leftarrow \text{Crs}(\text{crs}, m_b); \\ b \leftarrow \text{receiver}(\text{com})) < 1/2 + \nu(k)$$

The "hiding" and "binding" of the above definitions are "computationally hiding" and "computationally binding" respectively. A commitment is perfectly hiding if for any computationally unbounded adversary receiver for all  $m_0, m_1$  where

$$m_0 = m_1$$

and all sufficiently large  $k$  it holds that

$$\text{Prob}(\text{crs} \leftarrow \text{Gen}(1^k));$$

$$b \leftarrow \{0, 1\}; (\text{com}, \text{de } c) \leftarrow \text{Crs}(\text{crs}, m_b)$$

$$b \leftarrow \text{receiver}(\text{com}) < 1/2$$

A commitment is perfectly binding for any computationally unbounded sender for all sufficiently large  $k$  it holds that

$$\text{Prob}(\text{crs} \leftarrow \text{Gen}(1^k);$$

$$(\text{com}, m_0, m_1, \text{de } c_0, \text{de } c_1) \leftarrow \text{Sender}(\text{crs});$$

$$m_0 \neq m_1$$

and

$$\text{Ver}(\text{crs}, \text{com}, \text{de } c_0, m_0), \text{Ver}(\text{crs}, \text{com}, \text{de } c_1, m_1) = 1 = 0$$

### 3. Digital Signature Schemes and Security Model

#### 3.1. Digital Signature Schemes

A signature scheme consists of the following three oracle model [6] algorithms: a key generation algorithm Key Gen, a signature generation algorithm Sign and a signature verification algorithm Verify [7].

Key Gen, which on input  $1^k$ , where  $k$  is the security parameter, outputs a pair  $(p_k, s_k)$  of matching public and private keys. Algorithm Key Gen is probabilistic.

Sign, which receives a message  $m$  and the private key  $s_k$ , and outputs a signature

$$\theta = \text{Sign}(m, s_k).$$

The signing algorithm might be probabilistic. Verify, which receives a candidate signature  $\theta$ , a message  $m$  and a public key  $p_k$ , and returns an answer  $\text{Verify}(p_k, m, \theta)$  as to whether or not  $\theta$  is a valid signature of  $m$  with respect to  $p_k$ . In general, the verification algorithm need not be probabilistic.

Existential unforgeability under an adaptive chosen message attack introduced by Goldwasser, Micali and returns an answer  $\text{Verify}(p_k, m, \theta)$  as to whether or not  $\theta$  is a valid signature of  $m$  with respect to  $p_k$ . In general, the verification algorithm need not be probabilistic. valid signature of  $m$  with respect to  $p_k$ . In general, the verification algorithm need not be probabilistic.

Existential unforgeability under an adaptive chosen message attack introduced by Goldwasser, Micali and Rivest [9], has become the standard notion of signature security. The security model of existential unforgeability (in the random oracle model) is defined using the following game between a challenger and an adversary A [5].

### 3.2. Security Model

Setup the challenger runs algorithm Key Gen to obtain a public  $p_k$  and private key  $s_k$ . The adversary A is given  $p_k$ . Queries proceeding adaptively, A requests signatures with  $p_k$  on at most  $q_s$  messages of his choice

$$N_1, N_2, \dots, N_{q_s} \in \{0, 1\}^*.$$

The challenger responds to each query with a signature  $\theta_i = \text{Sign}(s_k, M_i)$ . Algorithm A also adaptively asks for at most  $q_H$  queries of the random oracle H.

Output : Eventually, A outputs a pair  $(M, \theta)$  and wins the game if

- 1) M is not any of  $N_1, N_2, \dots, N_{q_s}$ .
- 2)  $\text{Verify}(p_k, M, \theta) = \text{valid}$ .

## 4. Preliminaries

### 4.1. Bilinear Maps [13].

G and  $G_T$  are multiplicative cyclic groups of order p. The group action on G and  $G_T$  can be computed efficiently; g is a generator of G ;

$$e: G \times G \rightarrow G_T$$

is an efficiently computable map with the following properties:

- 1) Bilinear: for all  $u, v \in G$  and  $a, b \in \mathbb{Z}$ ,

$$e(u^a, v^b) = e(u, v)^{ab};$$

- 2) Non-degenerate:

$$e(g, g) \neq I_{G_T},$$

where  $I_{G_T}$  is the identity of  $G_T$ . We say that G is a bilinear group if it satisfies these requirements.

### 4.2. The Waters Signature Scheme

The messages will be assumed to be bit strings of the form  $\{0,1\}^k$ . In practice, a collision-resistant hash function  $H_k$  :

$$\{0, 1\}^* \rightarrow \{0,1\}^k$$

can be used to create message of the desired length. The scheme first choose groups G and  $G_T$  of prime order p such that an admissible pairing

$$e: G \times G \rightarrow G_T$$

can be constructed and choose a random Generator

$$g \in G, k+1$$

additional random generators

$$u', u_1, u_2, \dots, u_k \in \square G.$$

The Waters signature scheme consists of three algorithms as follows. Key Gen. Pick random

$$\alpha \leftarrow_R Z_p$$

and set

$$A \leftarrow e(g, g)^\alpha.$$

The public key  $p_k$  is

$$A \in G_T.$$

the private key  $s_k$  is  $\alpha$ . Sign( $s_k$ ,  $m$ ). Parse the user's private key  $s_k$  as

$$\alpha \in Z_p$$

and the message  $m$  as a bitstring

$$(n_1, n_2, \dots, n_k) \in \{0,1\}^k,$$

Pick a random  $r \in_R Z_p$  and compute

$$\theta_1 \leftarrow g^\alpha \left( u' \prod_{i=1}^k u_i^{n_i} \right)^T,$$

$$\theta_2 \leftarrow g^T$$

The signature is

$$\theta = (\theta_1, \theta_2) \in G^2$$

Verify ( $p_k$ ,  $m$ ,  $\theta$ ). Parse the user's public  $p_k$  as

$$A \in G_T,$$

the message  $m$  as a bitstring

$$(n_1, n_2, \dots, n_k) \in \{0,1\}^k,$$

and the signature  $\square \theta$  as

$$(\theta_1, \theta_2) \in G^2 \square.$$

Verify that

$$e(\theta_1, g) \bullet e\left(\theta_2, u' \prod_{i=1}^k u_i^{n_i}\right)^T = A$$

holds if so, output valid ; if not, output invalid. This signature is existentially unforgeable under an adaptive chosen message attack if the Computational Diffie-Hellman (CDH) problem[13] in bilinear groups is hard.

#### 4.3. Ordered multisignature schemes and their security

An ordered multi-signature scheme (OMS) consists of the following four algorithms[6]. A parameter generation algorithm Opg that returns some global information  $l$  for the scheme. This algorithm can be run by a trusted third-party or standards bodies. A key generation algorithm Okg run by a user that on the input global information  $l$  returns a publicprivate key-pair  $(p_k, s_k)$ .

A signing algorithm OSign run by a user on inputs its secret key  $s_k$ , a message  $n \in \{0, 1\}^*$ , a list of  $i-1$  public keys

$$L = (pk_1, \dots, pk_{i-1}),$$

and an OMS-so-far  $\theta'$ . It returns a new OMS  $\theta$ , or  $\perp$  if the input is deemed invalid. A deterministic verification algorithm OVf that on inputs a list of public keys  $(pk_1, \dots, pk_n)$ , a publicprivate key-pair  $(p_k, s_k)$ . A signing algorithm OSign run by a user on inputs its secret key  $s_k$ , a message

$$n \in \{0, 1\}^*,$$

a list of  $i-1$  public keys

$$L = (pk_1, \dots, pk_{i-1}),$$

and an OMS-so-far  $\theta'$ . It returns a new OMS  $\theta$ , or  $\perp$  if the input is deemed invalid. A deterministic verification algorithm OVf that on inputs a list of public keys  $(pk_1, \dots, pk_n)$ , a message  $n$ , and an OMS  $\theta$  returns valid or  $\perp$ . The security model of OMS in [6] extends the notion of security for multi-signatures in [8] to also ensure authenticity of the signing order. Similarly to the model of [8], the users are required to prove knowledge of their secret keys during public-key registration with a CA. For simplicity, this is modeled by requiring an adversary to hand over secret keys of malicious signers. This is known as the registered-key or certified-key model. The security model of existential unforgeability for OMS is defined using the following game associated to OMS and a forger  $A$  with access to an oracle. The game runs in three stages[11]:

Setup the game first runs Opg to obtain output  $l$  and then generates a challenge key-pair  $(pk, sk)$  by running Okg on input  $l$ [3]. Attack:  $A$  runs on inputs  $l, pk$ .  $A$  may query a key registration oracle with a key-pair  $(pk', sk')$  and coins  $c$  used for key generation, which records  $pk'$  as registered if  $Okg(l, c) \Rightarrow (pk', sk')$ . (This is a simplified model of a possibly more-complex key registration protocol with a CA that involves proofs of knowledge of secret keys.)  $A$  also has access to a signing oracle OSign, which on inputs  $m, \theta, L$  returns  $\perp$  if not all public keys in  $L$  are registered and  $OSign(s_k, m, \theta, L)$  otherwise. Forgery: Eventually,  $A$  halts with outputs a list of public keys

$$L^* = (pk_1^*, \dots, pk_n^*),$$

a message  $n^*$ , and a purported OMS signature  $\theta$ . This output is considered to be a forgery if it holds that [11]

1)  $\text{OVf}(L^*, n^*, \theta) = \text{valid}$ ;

2)  $pk_{i^*} = pk$

for some

$$i^* \in \{1, \dots, n\};$$

3) All public keys in  $L^*$  except  $pk$  are registered;

4)  $A$  did not query  $n^*$ ,  $\theta$ ,  $L'$  to its signing oracle where

$$|L'| = i^* - 1$$

for any

$$\theta' \in \{0, 1\}^k. [12]$$

We define that an ordered multi-signature is  $(t, q_c, q_s, N, \epsilon)$  unforgeable if not  $t$ -time adversary making  $q_c$  certification queries and  $q_s$  signing queries can win the above game with advantage more than  $\epsilon$ , where  $N$  is an upper bound on the length of the sequential signatures involved [4].

## 5. New Scheme

### 5.1. Construction

We construct an ordered multi-signature WOMS from the Waters signature. Our scheme is defined by the following algorithms. The messages will be assumed to be bit strings of the form  $\{0, 1\}^k$ . Parameter generation algorithm and key generation algorithm were produced as the Waters signature scheme. Parameter generation Opg: The algorithm first choose groups  $G$  and  $G_T$  of prime order  $p$  such that an admissible pairing  $e$ :

$$G \times G \rightarrow G_T$$

can be constructed and choose a random generator

$$g \in G, k+1$$

additional random generators

$$u', u_1, u_2, \dots, u_k \in G.$$

Key generation Okg: Pick random

$$\alpha \leftarrow_R \mathbb{Z}_p$$

and set

$$A \leftarrow e(g, g)^\alpha.$$

The public key  $pk$  is

$$A \in G_T.$$



The private key  $s_k$  is  $\theta$ . Signing OSign: On inputs  $s_k$ ,

$$n = (n_1, n_2, \dots, n_k) \in \{0,1\}^k,$$

$$L = (pk_1, \dots, pk_{i-1}),$$

and an OMS-so-far  $\theta'$ , the algorithm first verifies that Ovf

$$(L, m, \theta') = \text{valid},$$

as defined below and if not, outputs  $\perp$  and halt. For a first signer,  $\theta'$  is defined as  $(I_G, I_G, I_G)$ . Then parse  $\theta'$  as

$$(S', R', T') \in G3.$$

Choose random

$$r_i, t_i \in Z_p,$$

and compute

$$S = S' g^{\alpha_i + it_i} \left( u' \prod_{j=1}^k u_j^{n_j} \right)^{T_j},$$

$$R = R' g^{T_i},$$

$$T = T' g^{it_j}$$

the signature is

$$\theta = (S, R, T).$$

Verification Ovf: On inputs

$$(pk_1, \dots, pk_n),$$

$$n = (n_1, n_2, \dots, n_k) \in \{0,1\}^k, \theta,$$

the algorithm first checks that all  $pk_1, \dots, pk_n$  are distinct, if not, it output  $\perp$  and halt. Then parse  $\theta$  as  $(S, R, T)$  and verify if

$$e(S, g) = e \left( R, u' \prod_{j=1}^k u_j^{n_j} \right)^{-1}$$

$$= A_1 A_2 \dots A_n e(T, g)$$

if so, output valid ; if not, output  $\perp$ . An ordered multisignature in our scheme has the form

$$S = g^{\sum_{i=1}^n \alpha_i + it_i} \left( u' \prod_{j=1}^k u_j^{n_j} \right)^{\sum_{j=1}^n r_j},$$

$$R = g^{\sum_{i=1}^n t_i},$$

$$T = g^{\sum_{i=1}^n it_i}$$

Correctness: It is easy to see that the verification equation is satisfied:

$$\begin{aligned} e(S, g) &= e \left( R, u' \prod_{j=1}^k u_j^{n_j} \right)^{-1} \\ &= A_1 A_2 \dots A_n e(T, g) e \left( R, u' \prod_{j=1}^k u_j^{n_j} \right) e \left( R, u' \prod_{j=1}^k u_j^{n_j} \right)^{-1} \\ &= A_1 A_2 \dots A_n e(T, g) \end{aligned}$$

$$\text{Let } T_i = e(g, g)^{t_i},$$

Then

$$\begin{aligned} e(T, g) &= e \left( g^{\sum_{i=1}^n it_i}, g \right) \\ &= e(g, g)^{\sum_{i=1}^n it_i} \\ &= \prod_{i=1}^n T_i^i \end{aligned}$$

The equation above ensures authenticity of the signing order.

## 5.2. The security proof

The security analysis of our scheme is similar to the analysis presented in [13]. Theorem: The WOMS is  $(t, q_c, q_s, N, \epsilon)$ -unforgeable if the Waters signature scheme is  $(t, q_c, q_s, N, \epsilon)$ -unforgeable on  $G$ , where

$$t' = t + O(q_c + N_{q_s} + N),$$

$$q' = q_s,$$

$$\epsilon' = \epsilon.$$

Proof Suppose  $A$  is a forger algorithm that  $(t, q_c, q_s, N, \epsilon)$ -breaks our WOMS. We construct an algorithm  $B$  that  $(t', q', \epsilon')$ -breaks the Waters signature scheme. Algorithm  $B$  is given a public key of the Waters signature scheme,

$$A = e(g, g)^\alpha .$$

It interacts with A as follows. Setup algorithm B runs A supplying it with the challenge key

$$\begin{aligned} p_k &= A \\ &= e(g, g)^\alpha \end{aligned}$$

Certification Queries: A wish to certify some public key  $p_{k'}$ , providing also its corresponding private keys. A  $k'$  lgorithm B checks that the private key is correct and if so, registers  $(p_{k'}, s_{k'})$  in its list of certified key pairs. OMS Signature Queries: Algorithm A requests an OMS under the challenge key  $p_k$  on a message  $m$ . In addition, it supplies an OMS-so-far  $\theta'$ , a list of  $i-1$  public keys

$$L = (p_{k_1}, \dots, p_{k_{i-1}}).$$

The simulator B first checks that the signature  $\theta'$  is valid; that each key in L has been certified; that the challenge key does not appear in L; and that

$$|L| < N.$$

B returns  $\perp$  if any of these conditions does not hold. Otherwise, B queries its own signing oracle for key  $p_k$ , obtaining a signature  $\theta$  on message  $n$ .

$$\begin{aligned} \theta_1 &= g^{\alpha_j} \left( u' \prod_{j=1}^k u_j^{m_j} \right)^{T_j}, \\ \theta_2 &= g^{T_j} \end{aligned}$$

B parse  $\theta'$  as  $(S_{i-1}, R_{i-1}, T_{i-1})$ , B pick a random  $t_i \in \mathbb{Z}_p$ , and compute

$$\begin{aligned} S_i &= S_{i-1} \theta_1 g^{it_i} \\ &= g^{\sum_{h=1}^i \alpha_h + it_h} \left( u' \prod_{j=1}^k u_j^{m_j} \right)^{\sum_{h=1}^i r_h} \end{aligned}$$

$$\theta = (S_i, R_i, T_i)$$

is a OMS on message  $m$  under keys

$$L = (p_{k_1}, \dots, p_{k_{i-1}}, p_k)$$

Output Eventually, A halts, outputting a forgery

$$\theta^* = (S^*, R^*, T^*),$$

a message

$$n^* = (n_1^*, n_2^*, \dots, n_k^*) \in \{0, 1\}^k,$$

and a list of public keys

$$L^* = (pk_1^*, \dots, pk_n^*)$$

This forgery must verify as valid under Ovf; all public key in  $L^*$  except  $pk$  must have been certified;

$$pk_{i^*}^* = pk$$

for some

$$i^* \in \{1, \dots, n\};$$

$$|L^*| \leq N;$$

and A did not query  $m^*, \theta', L'$  to its signing oracle where

$$|L'| = i^* - 1$$

for any

$$\theta' \in \{0, 1\}^*$$

where  $S^*, R^*, T^*$  is as follows.

$$S^* = g^{\sum_{i=1}^n \alpha_i} \left( u' \prod_{j=1}^k u_j^{m_j^*} \right)^{\sum_{i=1}^n r_i},$$

$$R^* = g^{\sum_{i=1}^n r_i},$$

$$T^* = g^{\sum_{i=1}^n t_i}$$

Now, Algorithm B computes

$$\theta_1 = S^* g^{\alpha_j} \prod_{i \neq i^*} (g^{\alpha_i})^{-1} (T^*)^{-1},$$

$$\theta_2 = R^*$$

$$\alpha_i (i \neq i')$$

is the private key corresponding to each public key in  $L^*$ , B can know it by the certification procedure. We have

$$e(\theta_1, g)$$

$$\begin{aligned}
e\left(\theta_2, u' \prod_{j=1}^k u_j^{m_j^*}\right)^{-1} &= e(S^*, g) \cdot e\left(\prod_{i \neq i^*} g^{\alpha_i}, g\right)^{-1} \cdot e(T^*, g)^{-1} \\
e\left(R^*, u' \prod_{j=1}^k u_j^{m_j^*}\right)^{-1} &= e(S^*, g) \cdot e\left(R^*, u' \prod_{j=1}^k u_j^{m_j^*}\right)^{-1} \cdot e(T^*, g)^{-1} \prod_{i \neq i^*} e(g, g)^{-\alpha_i} \\
&= A_1 A_2 \dots A_n e(T^*, g) \cdot e(T^*, g)^{-1} \cdot \prod_{i \neq i^*} A_i^{-1} \\
&= A_{i^*}
\end{aligned}$$

So  $\theta_1, \theta_2$  is a valid Waters signature on  $n^*$  under the challenge key  $p_k = A_{i^*}$ , Since A did not make an OMS signing query at  $n^*$ , B did not make a signing query at  $n^*$ , so

$$\theta = \theta_1,$$

$\theta_2$  is a nontrivial Waters signature forgery. Algorithm B outputs

$$\theta = \theta_1,$$

$\theta_2$  and halts.

Algorithm B succeeds whenever A does. The running-time of B includes:

- 1) B's signing queries. B makes as many signing queries as A makes OMS signing queries
- 2) B handles A's certification queries. Each certification query can be handled in  $O(1)$  time
- 3) B handles OMS signing queries. Each OMS signing query can be handled in  $O(N)$  time
- 4) The other computations can be completed in  $O(N)$  time.

## 6. Conclusion

We will construct a PHCBC in two rounds from any one-way permutation, which is a negation of the result in [9].  $\Sigma$  protocol is our main tool to construct PHCBC.  $\Sigma$  protocol is a three-move interactive protocol between the prover and the verifier in which the verifier is only required to send random bits as a challenge to the prover. Based on  $\Sigma$  protocol, a new method to construct a commitment scheme was proposed in [10]. In this paper, we will use  $\Sigma$ -protocol on Hamiltonian-Cycle to construct PHCBC in constant rounds (i.e., two rounds) from any one-way permutation.

In this paper we gave an ordered multisignature scheme which is provably secure without random oracles. Our construction derives from the Waters signature scheme. It is also an interesting problem to find an ordered aggregate signature scheme provable secure without random oracles.

## Acknowledgment

A Project Supported by Scientific Research Fund of Hunan Provincial Education Department (No. 09C791). Thank Wei-hua Hou.

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