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Mathematical Model of Linear Switched Reluctance Motor with Mutual Inductance Consideration

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Article Info	ABSTRACT
	This paper presents developing an mathematical model for linear switched reluctance motor (LSRM) with account of the mutual inductance between the phases. Mutual interaction between the phases of LSRM gives the positive effect, as a rule the power of the machine is increased by 5-15%.

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1.   INTRODUCTION

A linear switched reluctance drive with a large air gap is developed for railway vehicles [1], [2]. The possibility to apply this type of electric machines is defined by the great value of normal force component between stator and rotor, which can be used for generation of levitation and assurance of guidance system.

Such kind of electric machine obtains the passive rotor consisted of ferromagnetic elements located along track structure. Rotors elements have great mechanical strength, which eliminates restrictions for transmission of mechanical traction force and suspension and gives the possibility to create the passive discrete track structure with reduced materials consumption, and at the same time, the design of stator winding with concentrated coils is extremely simple.

However, the large air gap (air gap12 mm) reduces the efficiency of the drive system (efficiency 76%).The design and control objectives are to maximize the efficiency at the given motor dimensions and the output power. The purpose of this paper is the design rationale for the choice of the linear motor, which increases efficiency.

Switched reluctance machines (SRM) are designed as a high-quality type of electromechanical energy converter and can be applied to the industrial transport. The main distinguishing feature of SRM is the lack of winding at the toothed rotor. In comparison with electrical machines of another types, SRM is structurally simpler and technologically effective, it has less copper and insulating materials consumption when almost identical masses of electrical steel. As a result it makes possible to achieve higher energy and weight-size parameters, to reduce the cost of electrical machines and expenses for their operation.

During SRM modelling and designing it is usually assumed that all machine's phases are independent both in electrical and magnetic relation. It is accepted that mutual inductance is low in these machines and it can be ignored. However, at the present time researchers are having tendency to take into account the mutual influence of phases for classical SRM [3] – [11]. This mutual interaction gives the

positive effect, as a rule the power of the machine is increased by 5-15%.

Mathematical model of linear SRM (LSRM) is presented in the paper. Configuration of LSRM is similar with rotating motor which has 18 stator teeth and 15 rotor teeth. This configuration provides a strong magnetic coupling between phases of the machine that must be considered when modelling. The model allow to research electromechanical and electromagnetic processes in motor.

2.   MATHEMATICAL MODEL

For magnetic characteristics, calculation of LSRM configuration [6, 7] Finite Element Method Magnetics (FEMM) package was used [10]. LSRM equation, taking into account the mutual inductance of each phase, is presented below:

$$u = R \cdot i + \frac{d\psi(\sum\limits_{k=1}^{m} i_k, \theta)}{dt} \tag{1}$$

where u is the phase voltage, R is the phase resistance, i is the phase current, ψ is the phase flux linkages, (in our case it is a function of seven variables), m is the number of phases, θ is the expressed in electrical degrees linear shift between translator and stator, k is the phase number of electrical machine.

The distribution pattern of the magnetic field lines to the LSRM configuration is shown in figure 1.

It is obvious that all coils within one phase are connected in opposite direction. Magnetic flow generated by phase A is completely run through adjacent phases (*B, C, D, E, F*).
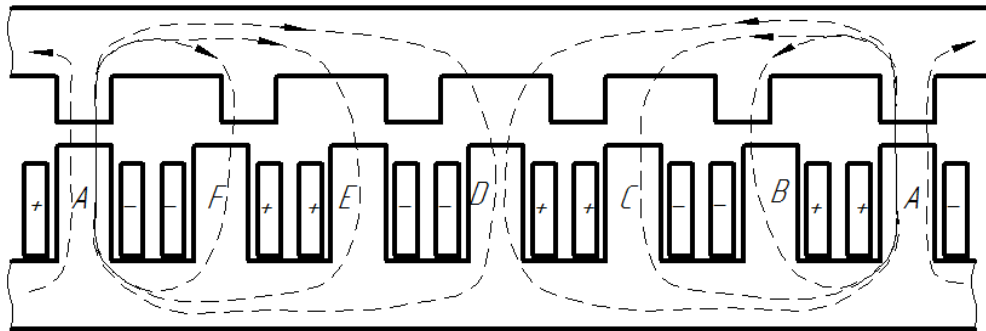

Figure 1. Layout of coils and distribution of magnetic field lines in LSRM under operation of phase A

LSRM is provided with three coils in a phase generated opposite magnetic flows closing via adjacent phases. Therefore, the modelling of this machine should take into account the interaction between adjacent phases.

The interaction between phases can be classified into two categories:

1) mutual inductance influence,

2) mutual saturation influence.

Mutual inductance is due to the field overlap through another phase. Mutual saturation is the impact of magnetizing force of one phase on the saturation of the other one. The level of saturation impacts on flux linkages and torque at the shaft of electrical machine.

We can write down the system of equations for six-phase machine. Let us assume that the adjacent phases have significant impact on the considered phases and the rest of phases do not have any strong influence:

$$
\begin{cases}
u_A = R_A \cdot i_A + \dfrac{d\psi_A}{dt} \\[2mm]
u_B = R_B \cdot i_B + \dfrac{d\psi_B}{dt} \\[2mm]
u_C = R_C \cdot i_C + \dfrac{d\psi_C}{dt} \\[2mm]
u_D = R_D \cdot i_D + \dfrac{d\psi_D}{dt} \\[2mm]
u_E = R_E \cdot i_E + \dfrac{d\psi_E}{dt} \\[2mm]
u_F = R_F \cdot i_F + \dfrac{d\psi_F}{dt}
\end{cases}
\tag{2}
$$

where

$$
\begin{aligned}
\psi_A &= \psi_{FA} + \psi_{AA} + \psi_{BA} \\
\psi_B &= \psi_{AB} + \psi_{BB} + \psi_{CB} \\
\psi_C &= \psi_{BC} + \psi_{CC} + \psi_{DC} \\
\psi_D &= \psi_{CD} + \psi_{DD} + \psi_{ED} \\
\psi_E &= \psi_{DE} + \psi_{EE} + \psi_{FE} \\
\psi_F &= \psi_{FE} + \psi_{FF} + \psi_{AF}
\end{aligned}
\tag{3}
$$

Write (3) through inductance

$$
\begin{aligned}
\psi_A &= M_{FA} \cdot i_F + L_A \cdot i_A + M_{BA} \cdot i_B \\
\psi_B &= M_{AB} \cdot i_A + L_B \cdot i_B + M_{CB} \cdot i_C \\
\psi_C &= M_{BC} \cdot i_B + L_C \cdot i_C + M_{DC} \cdot i_D \\
\psi_D &= M_{CD} \cdot i_C + L_D \cdot i_D + M_{ED} \cdot i_E \\
\psi_E &= M_{DE} \cdot i_D + L_E \cdot i_E + M_{FE} \cdot i_F \\
\psi_F &= M_{FE} \cdot i_E + L_F \cdot i_F + M_{AF} \cdot i_A
\end{aligned}
\tag{4}
$$

where Mjk is the mutual inductance between phases, Lk is the phase inductivity.

Since all phases of the machine consist of identical coils, it can be assumed that the resistance of each phase is the same:

$$
R_A = R_B = R_C = R_D = R_E = R_F = R .
$$

Given the facts that overlap in the phase operation is 120 deg and the area of generator and traction modes is 180 deg, then simultaneously not more than three phases of machine will operate in nominal mode.

Thus,

$$\begin{cases} u_Z = R \cdot i_Z + \dfrac{d\psi_Z}{dt} \\[2mm] u_X = R \cdot i_X + \dfrac{d\psi_X}{dt} \\[2mm] u_Y = R \cdot i_Y + \dfrac{d\psi_Y}{dt} \\[2mm] u_{Y+1} = \dfrac{d\psi_{Y+1}}{dt} \\[2mm] u_{Y+2} = \dfrac{d\psi_{Y+2}}{dt} \\[2mm] u_{Y+3} = \dfrac{d\psi_{Y+3}}{dt} \end{cases}$$

(5)

Where flux linkage with analogue of (4) will be the following:

$$\psi_Z = M_{YZ} \cdot i_Y + L_Z \cdot i_Z + M_{XZ} \cdot i_X$$
$$\psi_X = M_{ZX} \cdot i_Z + L_X \cdot i_X + M_{YX} \cdot i_Y$$
$$\psi_Y = M_{XY} \cdot i_X + L_Y \cdot i_Y + M_{ZY} \cdot i_Z$$
$$\psi_{(Y+1)} = M_{X(Y+1)} \cdot i_X + M_{Y(Y+1)} \cdot i_Y + M_{Z(Y+1)} \cdot i_Z$$
$$\psi_{(Y+2)} = M_{X(Y+2)} \cdot i_X + M_{Y(Y+2)} \cdot i_Y + M_{Z(Y+2)} \cdot i_Z$$
$$\psi_{(Y+3)} = M_{X(Y+3)} \cdot i_X + M_{Y(Y+3)} \cdot i_Y + M_{Z(Y+3)} \cdot i_Z$$

(6)

where indexes Z, X and Y correspond with combination of phase operation (F, A, B), (A, B, C), (B, C, D), (C, D, E), (D, E, F) и (E, F, A).

As was mentioned above, the considered machine has a strong mutual phase influence, it is therefore necessary to review the mutual inductance as a function of four variables:

$$M_{XY} = f(i_X, i_Y, i_Z, \theta)$$
$$M_{YX} = f(i_X, i_Y, i_Z, \theta)$$
$$M_{YZ} = f(i_X, i_Y, i_Z, \theta)$$
$$M_{ZY} = f(i_X, i_Y, i_Z, \theta)$$
$$M_{ZX} = f(i_X, i_Y, i_Z, \theta)$$
$$M_{XZ} = f(i_X, i_Y, i_Z, \theta)$$

As far as the magnetic system of the machine is symmetric and the return period is 60 deg., we can write the following:

$$L_X = f(i_X, i_Y, i_Z, \theta)$$
$$L_Y = L_X(i_X, i_Y, i_Z, (300 + \theta))$$
$$L_Z = L_X(i_X, i_Y, i_Z, (\theta + 60))$$
$$M_{XY} = f(i_X, i_Y, i_Z, \theta)$$
$$M_{YX} = M_{XY}(i_X, i_Y, i_Z, (300 + \theta))$$
$$M_{YZ} = M_{XY}(i_X, i_Y, i_Z, (\theta + 60))$$
$$M_{ZY} = M_{XY}(i_X, i_Y, i_Z, (120 - \theta))$$
$$M_{ZX} = M_{XY}(i_X, i_Y, i_Z, (\theta + 120))$$
$$M_{XZ} = M_{XY}(i_X, i_Y, i_Z, (360 - \theta))$$

Therefore, for complete computer simulation of this machine it is necessary to get additionally by means of Finite Element Method the relation of $\psi_X = f(i_X, i_Y, i_Z, \theta)$ or $L_X = f(i_X, i_Y, i_Z, \theta)$, $\psi_{XY} = f(i_X, i_Y, i_Z, \theta)$ or $M_{XY} = f(i_X, i_Y, i_Z, \theta)$. The application of FEMM package gives the possibility to get immediately the following relations: $\psi_X = f(i_X, i_Y, i_Z, \theta)$, $\psi_Z = f(i_X, i_Y, i_Z, \theta)$, $\psi_Y = f(i_X, i_Y, i_Z, \theta)$, $\psi_{Y+1} = f(i_X, i_Y, i_Z, \theta)$, $\psi_{Y+2} = f(i_X, i_Y, i_Z, \theta)$, $\psi_{Y+3} = f(i_X, i_Y, i_Z, \theta)$.

Let differentiate the equation (6):

$$\frac{d\psi_Z}{dt} = M_{YZ}\frac{di_Y}{dt} + \frac{\partial M_{YZ}}{\partial \theta}\omega i_Y + L_Z\frac{di_Z}{dt} + \frac{\partial L_Z}{\partial \theta}\omega i_Z + M_{XZ}\frac{di_X}{dt} + \frac{\partial M_{XZ}}{\partial \theta}\omega i_X$$

$$\frac{d\psi_X}{dt} = M_{ZX}\frac{di_Z}{dt} + \frac{\partial M_{ZX}}{\partial \theta}\omega i_Z + L_X\frac{di_X}{dt} + \frac{\partial L_X}{\partial \theta}\omega i_X + M_{ZX}\frac{di_Z}{dt} + \frac{\partial M_{ZX}}{\partial \theta}\omega i_Z$$

$$\frac{d\psi_Y}{dt} = M_{XY}\frac{di_X}{dt} + \frac{\partial M_{XY}}{\partial \theta}\omega i_{YX} + L_Y\frac{di_Y}{dt} + \frac{\partial L_Y}{\partial \theta}\omega i_Y + M_{ZY}\frac{di_Z}{dt} + \frac{\partial M_{ZY}}{\partial \theta}\omega i_Z$$

where $\omega = \dfrac{d\theta}{dt}$.

The propulsion force depending on phase current and rotor position can be expressed in terms of coenergy. In our case the coenergy differential is:

$$dW_c(i_X, i_Y, i_Z, \theta) = \psi_X di_X + \psi_Y di_Y + \psi_Z di_Z + F_e d\theta \qquad (7)$$

where F_e is the propulsion force of machine.

The coenergy for the proposed machine can be found by integration (7) along the outline by analogy with [4]. The integration path is selected in the following way:

1) Integrate by rotation angle at zero current in all phases ($i_X = 0, i_Y = 0, i_Z = 0$);
2) Integrate by i_Z, keeping zero currents in the other two phases ($i_X = 0, i_Y = 0$), and rotation angle θ – as constant;
3) Integrate by i_Y, keeping zero current in phase X ($i_X = 0$), rotation angle θ and i_Z as constant;
4) Integrate by i_X, rotation angle θ, i_Y and i_Z are constant .

At the first stage of integration the torque integral is zero, since the torque is zero at zero phase currents ($i_X = 0, i_Y = 0, i_Z = 0$); at the following stages this integral is zero because the rotation angle θ is constant.

After integrating we receive the expression for coenergy of the considered machine when three phases operate simultaneously:

$$W_c(i_X, i_Y, i_Z, \theta) = \int_0^\theta F_e(0,0,0,\xi)d\xi + \int_0^{i_Z}\psi_Z(0,0,\xi,\theta)d\xi + \int_0^{i_Y}\psi_Y(0,\xi,i_Z,\theta)d\xi + \int_0^{i_X}\psi_X(\xi,i_Y,i_Z,\theta)d\xi =$$

$$= 0 + \int_0^{i_Z} L_Z\xi d\xi + \int_0^{i_Y}(L_Y\xi + M_{ZY}i_Z)d\xi + \int_0^{i_X}(L_X\xi + M_{YX}i_Y)d\xi =$$

$$= \frac{1}{2}L_X i_X^2 + \frac{1}{2}L_Y i_Y^2 + \frac{1}{2}L_Z i_Z^2 + M_{ZY}i_Y i_Z + M_{YX}i_X i_Y \quad,$$

where ξ – integration variable takes the following values θ, i_Z, i_Y, i_X in order for integrals.

Then for propulsion force calculation, we get the final expression:

$$F_e = \left.\frac{\partial W_c(i_X, i_Y, i_Z, \theta)}{\partial \theta}\right|_{i_X, i_Y, i_Z} =$$

$$= \frac{1}{2}\frac{\partial L_X}{\partial \theta}i_X^2 + \frac{1}{2}\frac{\partial L_Y}{\partial \theta}i_Y^2 + \frac{1}{2}\frac{\partial L_Z}{\partial \theta}i_Z^2 + \frac{\partial M_{ZY}}{\partial \theta}i_Y i_Z + \frac{\partial M_{YX}}{\partial \theta}i_X i_Y \qquad (8)$$

Therefore, the propulsion force of six-phase is expressed in terms of phase currents and linear shift between translator and stator. The effect of mutual inductance is considered in two last members of sum in the expression (8).

3.   RESULTS AND DISCUSSION

As the result of LSRM calculation it was received the relation of self and mutual flux linkages with other phases presented at figure 2. It is obvious that intensity of mutual flux linkages reaches 50% of its own.

a) Flux linkages of phase A

b) Flux linkages of phase A with phase B

c) Flux linkages of phase A with phase C

d) Flux linkages of phase A with phase D

e) Flux linkages of phase A with phase E

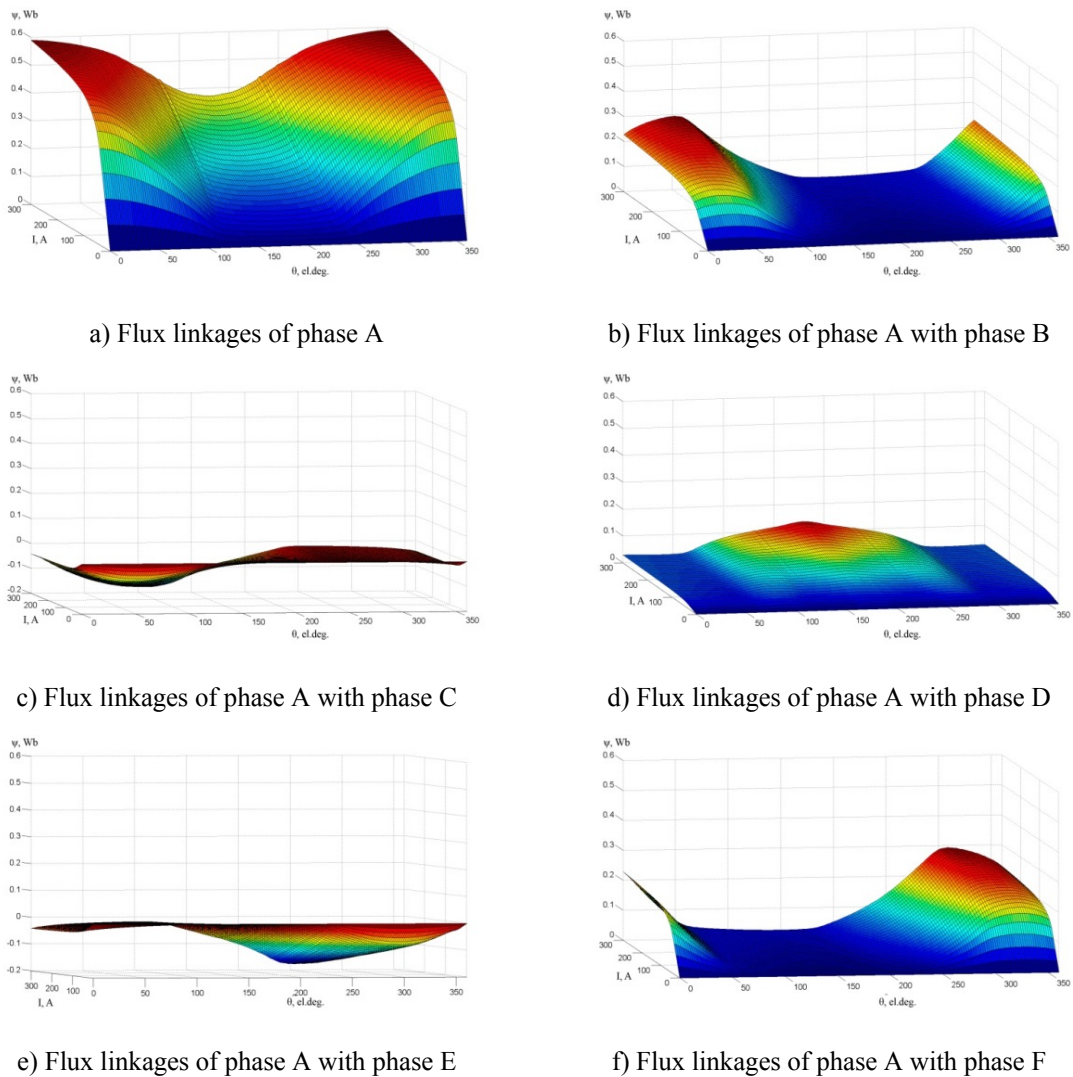f) Flux linkages of phase A with phase F

Figure 2. The relation of self and mutual flux linkages of LSRM

Based on diagrams analysis given above, it follows that the considered LSRM has intense mutual influence between phases. At the same time, as can be seen, there is a significant impact on the adjacent phases of the machine.

Presented results allow concluding that the effects of mutual inductances are important and should be considered in mathematical model of LRSM. The mutual inductances cannot be neglected in a switched reluctance machine design. To estimate of their influence on the machine performance preliminary calculations and analyses should be done.

4.   CONCLUSION

Expression analysis (8) shows the positive effect from the strong mutual inductance between the phases of LSRM. However, the strong mutual inductance has the impact on control parameters and this fact requires the application of more complicated control algorithms considering the processes occurring in all phases.

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