International Journal of Advances in Applied Sciences (IJAAS)
Vol. 2, No. 1, March 2013, pp. 41~50
ISSN: 2252-8814

Dispersion of Thermo Elastic Waves in a Rotating Cylindrical Panel

R .Selvamani[1], P. Ponnusamy[2]
[1] Department of mathematics Karunya University, Coimbatore, Tamil Nadu, India
[2] Department of mathematics, Govt Arts College (Autonomous), Coimbatore, Tamil Nadu, India

Article Info

Article history:

Received Oct 9, 2012
Revised Jan 13, 2013
Accepted Jan 27, 2013

Keyword:

Bessel function solution
Dispersion analysis
Rotating cylindrical panel
Thermo elasticity

ABSTRACT

The three dimensional dispersion of thermo elastic waves in a homogeneous isotropic rotating cylindrical panel is investigated in the context of the linear theory of thermo elasticity. Three displacement potential functions are introduced to uncouple the equations of motion**.** The frequency equations are obtained for traction free boundary conditions using Bessel function solutions.In order to illustrate theoretical development, numerical solutions are obtained and presented graphically for a zinc material. In this study we found that the wave characteristics are more stable and realistic in the presence of thermal and the rotation parameters.

Corresponding Author:

R .Selvamani,
Departement of Mathematics,
Karunya University,
Coimbatore, Tamil Nadu, India, 64114.
Email: selvam1729@gmail.com

1. INTRODUCTION

The dispersion of displacement, temperature change in a rotating cylindrical panel is plays a vital role in smart material applications and rotating gyroscope. This type of model analysis is very important in bio sensing applications in nuclear magnetic resonance (NMR), magnetic resonance imaging (MRI) and echo planar imaging (EPI). The analysis of thermally induced vibration of rotating cylindrical panel is common place in the design of structures, atomic reactors, steam turbines, supersonic aircraft, and other devices operating at elevated temperature. At the present time applied mathematicians are exhibiting considerable interest in dynamical methods of elasticity, since the usual quasi static approach ignores certain very important features of the problems under consideration. That approach is based on the assumption that the inertia terms may be omitted from the equations of motion. This assumption holds good only when the variations in stresses and displacements, but there arise number of problems in engineering and technology, when this assumption may not hold good and the inertia terms in the equations of motion may have lead to cases of considerable mathematical complications. In the field of nondestructive evaluation, laser-generated waves have attracted great attention owing to their potential application to noncontact and nondestructive evaluation of sheet materials. The high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses, reducing the strength of the aircraft structure. In the nuclear field, the extremely high temperatures and temperature gradients originating inside nuclear reactors influence their design and operations. Moreover, it is well recognized that the investigation of the thermal effects on rototing elastic wave propagation has bearing on many structural applications.

The static analysis cannot predict the behavior of the material due to the thermal stresses changes very rapidly. Therefore in case of suddenly applied load, thermal deformation and the role of inertia are getting more important. This thermo elastic stress response being significant leads to the propagation of thermo elastic stress waves in solids. The theory of thermo elasticity is well established by Nowacki [1]. Lord and Shulman [2] and Green and Lindsay [3] modified the Fourier law and constitutive relations, so as to get hyperbolic equation for heat conduction by taking into account the time needed for acceleration of heat flow and relaxation of stresses. A special feature of the Green–Lindsay model is that it does not violate the classical Fourier's heat conduction law. Vibration of functionally graded multilayered orthotropic cylindrical panel under thermo mechanical load was analyzed by X.Wang et.al [4]. Hallam and Ollerton [5] investigated the thermal stresses and deflections that occurred in a composite cylinder due to a uniform rise in temperature, experimentally and theoretically and compared the obtained results by a special application of the frozen stress technique of photo elasticity. Noda [6] has studied the thermal-induced interfacial cracking of magneto electro elastic materials under uniform heat flow. Chen et al [7] analyzed the point temperature solution for a pennay-shapped crack in an infinite transversely isotropic thermo-piezo-elastic medium subjected to a concentrated thermal load applied arbitrarily at the crack surface using the generalized potential theory. Abouhamze [8] discussed a multi objective optimization strategy for optimal stacking sequence of laminated cylindrical panels is presented with respect to the first natural frequeny and critical buckling load using the weighted summation method. He used the trained neural network to evaluate the fitness function in the optimization process and in this way increasing the procedure speed. Chadwick [9] studied the propagation of plane harmonic waves in homogenous anisotropic heat conducting solids. Sharma [10] investigated the three dimensional vibration analysis of a transversely isotropic thermo elastic cylindrical panel. The application of powerful numerical tools like finite element or boundary element methods to these problems is also becoming important. Prevost and Tao [11] carried out an authentic finite element analysis of problems including relaxation effects. Eslami and Vahedi [12] applied the Galerkin finite element to the coupled thermo elasticity problem in beams. Huang and Tauchert [13] derived the analytical solution for cross-ply laminated cylindrical panels with finite length subjected to mechanical and thermal loads using the extended power series method. Ponnusamy and Selvamani [14] investigated the wave propagation in a generalized thermol elastic plate embedded on elastic medium. Ponnusamy and Selvamani [15] have studied the dispersion analysis of generalized magneto-thermo elastic waves in a transversely isotropic cylindrical panel using the wave propagation approach.Later,Selvamani and Ponnusamy [16] studied the damping of generalized thermo elastic waves in a homogeneous isotropic plate using the wave propagation approach and obtained the numerical result for Zinc plate. Since the speed of the disturbed waves depend upon rotation rate, this type of study is important in the design of high speed steam, gas turbine and rotation rate sensors.Loy and Lam [17] discussed the vibration of rotating thin cylindrical panel using Love first approximation theory. Bhimaraddi [18] developed a higher order theory for the free vibration analysis of circular cylindrical shell. Zhang [19] investigated the parametric analysis of frequency of rotating laminated composite cylindrical shell using wave propagation approach. Body wave propagation in rotating thermo elastic media was investigated by Sharma and Grover [20]. The effect of rotation, magneto field, thermal relaxation time and pressure on the wave propagation in a generalized visco elastic medium under the influence of time harmonic source is discussed by Abd-Alla and Bayones [21].The propagation of waves in conducting piezoelectric solid is studied for the case when the entire medium rotates with a uniform angular velocity by Wauer [22]. Roychoudhuri and Mukhopadhyay studied the effect of rotation and relaxation times on plane waves in generalized thermo visco elasticity [23]. Gamer [24] has discussed the elastic-plastic deformation of the rotating solid disk. Lam [25] has studied the frequency characteristics of a thin rotating cylindrical shell using general differential quadrature method.

In this paper, the three dimensional dispersion of thermo elastic waves in a homogeneous isotropic rotating cylindrical panel is discussed using the linear three-dimensional theory of thermo elasticity. The frequency equations are obtained using the traction free boundary conditions. The Bessel function with complex argument is directly used to find the solutions and are studied numerically for the material Zinc. The computed non-dimensional phase velocities are plotted in the form of dispersion curves.

2. FORMULATION OF THE PROBLEM

Consider a cylindrical panel as shown in Fig.1 of length L having inner and outer radius a and b with thickness h and uniform angular velocity $\vec{\Omega}$. The angle subtended by the cylindrical panel, which is known as center angle, is denoted by α. The cylindrical panel is assumed to be homogeneous, isotropic and linearly elastic with Young's modulus E, Poisson ratio ν and density ρ in an undisturbed state.

In cylindrical coordinate the three dimensional stress equation of motion, strain displacement relation and heat conduction in the absence of body force for a linearly elastic rotating medium .

$$\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + \sigma_{rz,z} + r^{-1}\left(\sigma_{rr} - \sigma_{\theta\theta}\right) + \rho(\vec{\Omega}\times(\vec{\Omega}\times\vec{u}) + 2\vec{\Omega}\,\vec{u}_{,t}) = \rho u_{,tt}$$

$$\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + \sigma_{,rzz} + \sigma_{\theta z,z} + 2r^{-1}\sigma_{r\theta} = \rho v_{,tt}$$

$$\sigma_{rz,r}+r^{-1}\sigma_{\theta z,\theta}+\sigma_{zz,z}+r^{-1}\sigma_{r\theta}+\rho(\vec{\Omega}\times(\vec{\Omega}\times\vec{u})+2\vec{\Omega}\,\vec{u}_{,t})=\rho w_{,tt}$$

$$K\left(T_{,rr}+r^{-1}T_{,r}+r^{-2}T_{,\theta\theta}+T_{,zz}\right)=\rho c_v T_{,t}+\beta T_0\left(u_{,rt}+r^{-1}\left(u_{,t}+v_{,\theta t}\right)+w_{,tz}\right) \tag{1}$$

where ρ is the mass density, c_v is the specific heat capacity, $\kappa=K/\rho c_v$ is the diffusivity, K is the thermal conductivity, T_0 is the uniform reference temperature , the displacement equation of motion has the additional terms with a time dependent centripetal acceleration $\Omega\times(\Omega\times\vec{u})$ and $2\Omega\times\vec{u}_{,t}$ where, $\vec{u}=(u,0,w)$ is the displacement vector and $\vec{\Omega}=(0,\Omega,0)$ is a constant, the comma notation used in the subscript denotes the partial differentiation with respect to the variables. The stress strain relations are given as follows

$$\sigma_{rr}=\lambda(e_{rr}+e_{\theta\theta}+e_{zz})+2\mu e_{rr}-\beta(T)$$
$$\sigma_{\theta\theta}=\lambda(e_{rr}+e_{\theta\theta}+e_{zz})+2\mu e_{\theta\theta}-\beta(T) \tag{2}$$
$$\sigma_{zz}=\lambda(e_{rr}+e_{\theta\theta}+e_{zz})+2\mu e_{zz}-\beta(T)$$

Where e_{ij} are the strain components, β is the thermal stress coefficients, T is the temperature, t is the time, λ and μ are Lame' constants. The strain e_{ij} are related to the displacements are given by

$$\sigma_{r\theta}=\mu\gamma_{r\theta}\quad \sigma_{rz}=\mu\gamma_{rz}\quad \sigma_{\theta z}=\mu\gamma_{\theta z}\quad e_{rr}=\frac{\partial u}{\partial r}\quad e_{\theta\theta}=\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial\theta} \tag{3}$$
$$e_{zz}=\frac{\partial w}{\partial z}\quad \gamma_{r\theta}=\frac{\partial v}{\partial r}-\frac{v}{r}+\frac{1}{r}\frac{\partial u}{\partial\theta}\quad \gamma_{rz}=\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z}\quad \gamma_{z\theta}=\frac{\partial v}{\partial z}+\frac{1}{r}\frac{\partial w}{\partial\theta}$$

(4)

Where u,v,w are displacements along radial, circumferential and axial directions respectively, $\sigma_{rr},\sigma_{\theta\theta},\sigma_{zz}$ are the normal stress components and $\sigma_{r\theta},\sigma_{\theta z},\sigma_{zr}$ are the shear stress components , $e_{rr},e_{\theta\theta},e_{zz}$ are normal strain components and $e_{r\theta},e_{\theta z},e_{zr}$ are shear strain components.

Substituting the Eq. (3) and Eq. (2) in Eq. (1), gives the following three displacement equations of motion

$$(\lambda+2\mu)\left(u_{,rr}+r^{-1}u_{,r}-r^{-2}u\right)+\mu r^{-2}u_{,\theta\theta}+\mu u_{,zz}+r^{-1}(\lambda+\mu)v_{,r\theta}-r^{-2}(\lambda+3\mu)v_{,\theta}+(\lambda+\mu)w_{,rz}$$
$$-\beta\left(T_{,r}\right)+\rho\left(\Omega^2u+2\Omega w_{,t}\right)=\rho u_{,tt}$$
$$\mu\left(v_{,rr}+r^{-1}v_{,r}-r^{-2}v\right)+r^{-2}(\lambda+2\mu)v_{,\theta\theta}+\mu v_{,zz}+r^{-2}(\lambda+3\mu)u_{,\theta}+r^{-1}(\lambda+\mu)u_{,r\theta}+r^{-1}(\lambda+\mu)w_{,\theta z}$$
$$-\beta\left(T_{,\theta}\right)=\rho v_{,tt}$$
$$(\lambda+2\mu)w_{,zz}+\mu\left(w_{,rr}+r^{-1}w_{,r}+r^{-2}w_{,\theta\theta}\right)+(\lambda+\mu)u_{,rz}+r^{-1}(\lambda+\mu)v_{,\theta z}+r^{-1}(\lambda+\mu)u_{,z}$$
$$-\beta\left(T_{,z}\right)+\rho\left(\Omega^2w+2\Omega u_{,t}\right)=\rho w_{,tt}$$
$$(\lambda+2\mu)w_{,zz}+\mu\left(w_{,rr}+r^{-1}w_{,r}+r^{-2}w_{,\theta\theta}\right)+(\lambda+\mu)u_{,rz}+r^{-1}(\lambda+\mu)v_{,\theta z}+r^{-1}(\lambda+\mu)u_{,z}$$
$$-\beta\left(T_{,z}\right)+\rho\left(\Omega^2w+2\Omega u_{,t}\right)=\rho w_{,tt}$$
$$\rho c_v\kappa\left(T_{,rr}+r^{-1}T_{,r}+r^{-2}T_{,\theta\theta}+T_{,zz}\right)=\rho c_v T_{,t}+\beta T_0\left[u_{,tr}+r^{-1}(u_{,t}+v_{,t\theta})+w_{,tz}\right] \tag{5}$$

The above coupled partial differential equations is also subjected to the following non-dimensional boundary conditions at the surfaces $r=a,\,b$

(i) The traction free non dimensional mechanical boundary conditions for a stress free edge are given by

$$\sigma_{rr}=\sigma_{r\theta}=\sigma_{rz}=0, \tag{6a}$$

(ii). The non dimensional insulated or isothermal thermal boundary condition is given by

$$T_{,r}+hT=0 \tag{6b}$$

Where h is the surface heat transfer coefficient .Here $h\to0$ corresponds to thermally insulated surface and $h\to\infty$ refers to isothermal one.

To solve Eq. (5), we take [10]

$$u=\frac{1}{r}\psi_{,\theta}-\phi_{,r}\qquad v=-\frac{1}{r}\phi_{,\theta}-\psi_{,\varpi}\qquad w=-\chi_{,z}$$

Using Eq. (5) in Eq. (1), we find that ϕ, χ, T satisfies the equations.

$$((\lambda+2\mu)\nabla^2{}_1+\mu\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}+\rho\Omega^2)\phi-((\lambda+\mu)\frac{\partial^2}{\partial z^2}+2\Omega\frac{\partial^2}{\partial z\partial t})\chi=\beta\ (T) \tag{7a}$$

$$(\mu\nabla_1{}^2+(\lambda+2\mu)\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}+\rho\Omega^2)\chi-((\lambda+\mu)\nabla_1{}^2-2\Omega\frac{\partial^2}{\partial r\partial t})\phi=\beta(T) \tag{7b}$$

$$(\nabla_1{}^2+\frac{\partial^2}{\partial z^2}-\frac{\rho}{\mu}\frac{\partial^2}{\partial t^2}-\rho\Omega^2)\psi=0 \tag{7c}$$

$$\nabla_1{}^2T+\frac{\partial^2T}{\partial z^2}-\frac{1}{k}\frac{\partial T}{\partial t}+\frac{\beta T_0(i\omega)}{\rho C_V K}(\nabla_1{}^2\phi+\frac{\partial^2\chi}{\partial z^2})=0 \tag{7d}$$

Eq. (7c) in terms of ψ gives a purely transverse wave, which is not affected by temperature. This wave is polarized in planes perpendicular to the z-axis. We assume that the disturbance is time harmonic through the factor $e^{i\omega t}$.

3. SOLUTION TO THE PROBLEM

The Eqs. (7) are coupled partial differential equations of the three displacement components. To uncouple Eqs. (7), we can write three displacement functions which satisfies the simply supported boundary conditions followed by Sharma [10]

$$\psi(r,\theta,z,t)=\overline{\psi}(r)\sin(m\pi z)\cos(n\pi\theta/\alpha)e^{i\omega t}$$

$$\phi(r,\theta,z,t)=\overline{\phi}(r)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t} \tag{8}$$

$$\chi(r,\theta,z,t)=\overline{\chi}(r)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t}\mathrm{F}$$

$$T(r,\theta,z,t)=\overline{T}(r,\theta,z,t)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t}$$

Where m is the circumferential mode and n is the axial mode, ω is the angular frequency of the cylindrical panel motion. By introducing the dimensionless quantities

$$r^{'}=\frac{r}{R}\qquad z^{'}=\frac{z}{L}\qquad \overline{T}=\frac{T}{T_0}\qquad \delta=\frac{n\pi}{\alpha}\qquad t_L=\frac{m\pi R}{L}\qquad \overline{\lambda}=\frac{\lambda}{\mu}\qquad \in_4=\frac{1}{2+\overline{\lambda}}$$

$$C_1{}^2=\frac{\lambda+2\mu}{\rho}\qquad \varpi^2=\frac{\omega^2R^2}{C^2{}_1}\qquad \Gamma=\frac{\rho\Omega^2R^2}{2+\overline{\lambda}} \tag{9}$$

After substituting Eq. (9) and Eq.8 in Eq. (7), we obtain the following system of equations

$$(\nabla_2^2+k_1{}^2)\overline{\psi}=0 \tag{10a}$$

$$(\nabla_2^2+g_1)\overline{\phi}+g_2\overline{\chi}-g_4\overline{T}=0 \tag{10b}$$

$$(\nabla_2^2+g_3)\overline{\chi}+(1+\overline{\lambda})\nabla_2^2\overline{\phi}+(2+\overline{\lambda})g_4\overline{T}=0 \tag{10c}$$

$$(\nabla_2{}^2-t_L{}^2+\in_2\varpi^2-i\in_3)\overline{T}+i\in_1\varpi\nabla_2{}^2\overline{\phi}-i\in_1\varpi t_L{}^2\overline{\chi}=0 \tag{10d}$$

where

$$\nabla_2^2=\frac{\partial^2}{\partial r^2}\frac{1}{r}\frac{\partial}{\partial r}-\frac{\delta^2}{r^2},\quad \in_1=\frac{T_0R\beta^2}{\rho^2C_VC_1K}\qquad \in_2=\frac{C_1{}^2}{C_VK}\qquad \in_3=\frac{C_1R}{K}$$

$$g_1=(2+\overline{\lambda})(t_L^2-\varpi^2+\Gamma)\qquad g_2=\in_4\left((1+\overline{\lambda})t_L+i\omega\right)t_L\qquad g_3=(\varpi^2-\in_4 t_L{}^2+\Gamma)$$

$$g_4=\frac{\beta T_0R^2}{\lambda+2\mu}\qquad g_5=\in_1\varpi$$

C_1 wave velocity of the cylindrical panel. A non-trivial solution of the algebraic systems (10) exist only when the determinant of Eqs. (10) are equal to zero.

$$\begin{vmatrix} (\nabla_2{}^2+g_1) & -g_2 & g_4 \\ (1+\overline{\lambda})\nabla_2{}^2 & (\nabla_2{}^2+g_3) & (2+\overline{\lambda})g_4 \\ ig_5\nabla_2{}^2 & -ig_5t_L{}^2 & (\nabla_2{}^2-t_L{}^2+\in_2\varpi^2-i\varpi\in_3) \end{vmatrix}(\overline{\phi},\overline{\chi},\overline{T})=0 \tag{11}$$

Eq. (11), on simplification reduces to the following differential equation:

$$\nabla_2^6+A\nabla_2^4+B\nabla_2^2+C=0 \tag{12}$$

Where,

$$A=-g_1+g_2(1+\overline{\lambda})+g_3-g_4g_5it_L{}^2+\in_2\varpi^2-i\in_3\varpi$$

$$B=-g_1g_3-g_1g_4g_5i-g_2g_4g_5i(2+\overline{\lambda})+t_l{}^2(g_1-g_2-g_3)+g_4g_5it_L{}^2+g_2\varpi^2\in_2(1+\overline{\lambda})-g_2i\in_3\varpi(1+\overline{\lambda})\theta$$
$$-g_2t_L{}^2\overline{\lambda}+g_3\varpi(\varpi\in_2-i\in_3)+g_1\varpi(i\in_3-\varpi\in_2)$$

$$C=g_1g_3(t_L{}^2+i\in_3\varpi-\in_2\varpi^2)+ig_3g_4g_5t_L{}^2(2+\overline{\lambda})$$

The solution of Eq. (11) is

$$\overline{\phi}(r)=\sum_{i=1}^{3}(A_iJ_\delta(\alpha_ir)+B_iY_\delta(\alpha_ir))$$

$$\overline{\chi}(r)=\sum_{i=1}^{3}d_i(A_iJ_\delta(\alpha_ir)+B_iY_\delta(\alpha_ir))$$

$$\overline{T}(r)=\sum_{i=1}^{3}e_i(A_iJ_\delta(\alpha_ir)+B_iY_\delta(\alpha_ir)) \tag{13}$$

Here, $(\alpha_ir)^2$ are the non-zero roots of the algebraic equation

$$(\alpha_ir)^6-A(\alpha_ir)^4+B(\alpha_ir)^2-C=0$$

The arbitrary constant d_i and e_i is obtained from

$$d_i=\left[\frac{(1+\overline{\lambda})\delta_i^2-(2+\overline{\lambda})\delta_i^2-g_1}{g_2(2+\overline{\lambda})-\delta_i^2-g_3}\right]$$

$$e_i=\left(\frac{\lambda+2\overline{\mu}}{\beta T_0R^2}\right)\left[\frac{\varepsilon_4\delta_i^2+\left(\varepsilon_4(g_1+g_3)+\varepsilon_4(1+\overline{\lambda})g_2\right)\delta_i^2+\varepsilon_4g_1g_3+\delta_i^2)-g_1g_3}{\varepsilon_4g_3+\varepsilon_4\delta_i^2-g_2}\right] \tag{14}$$

Eq. (9a) is a Bessel equation with its possible solutions is

$$\overline{\psi}=\begin{cases} A_4J_\delta(k_1r)+B_4Y_\delta(k_1r), & k_1^2>0 \\ A_4r^\delta+B_4r^{-\delta}, & k_1^2=0 \\ A_4I_\delta(k_1'r)+B_4K_\delta(k_1'r), & k_1^2<0 \end{cases} \tag{15}$$

Where $k_1'^2=-k_1^2$ and J_δ and Y_δ are Bessel functions of the first and second kinds respectively while, I_δ and k_δ are modified Bessel functions of first and second kinds respectively. $A_{i,}B_i$ $i=1,2,3,4$ are the arbitrary constants. Generally $k_1^2\neq 0$, so that the situation $k_1^2\neq 0$ will not be discussed in the following . For convenience, we consider the case of $k_1^2>0$ and the derivation for the case of $k_1^2<0$ is similar.

The solution of Eq. (10a) is

$$\overline{\psi}(r)=A_4J_\delta(k_1r)+B_4Y_\delta(k_1r) \tag{16}$$

Where $k_1{}^2=(2+\overline{\lambda})\varpi^2-\left(t_L{}^2+\Gamma\right)$

4. SPECIAL CASES

4.1 Thermo elasticity

By taking $\Gamma=0$ the motion corresponding to the rotational mode decouple from the rest of motion and the various results reduces to the thermo elasticity

$$\phi(r)=\sum_{i=1}^{3}(A_iJ_\delta(\alpha_i r)+B_iY_\delta(\alpha_i r))\sin(m\pi)z\ \sin(n\pi/\alpha)\theta e^{i\omega t}$$

$$\chi(r)=\sum_{i=1}^{3}\ d_i(A_iJ_\delta(\alpha_i r)+B_iY_\delta(\alpha_i r))\sin(m\pi)z\ \sin(n\pi/\alpha)\theta e^{i\omega t} \tag{17}$$

$$T(r)=\sum_{i=1}^{3}\ e_i(A_iJ_\delta(\alpha_i r)+B_iY_\delta(\alpha_i r))\sin(m\pi)z\sin(n\pi/\alpha)\theta e^{i\omega t}$$

$$\psi(r)=(A_4J_\delta(k_1 r)+B_4Y_\delta(k_1 r))\sin(m\pi)z\cos(n\pi/\alpha)\theta e^{i\omega t}$$

With

$$g_1=(2+\overline{\lambda})(t_L^2-\varpi^2)\qquad k_1{}^2=(2+\overline{\lambda})\varpi^2-t_L{}^2 \tag{18}$$

Eqs. (17)& (18) constitute the solution for the homogenous isotropic cylindrical panel with traction free boundary conditions. It is noticed that Eq. (18) is similar to the particular case obtained and discussed by Sharma [10] in case of thermo elasticity.

4.2 Elastokinetic

In the present analysis if we take the coupling parameter for rotational and thermal field $\beta=\in_1=\Gamma=0$ then the equations will reduces to the classical case in elasto kinetic.

$$\begin{vmatrix}(\nabla_2^2+g_3) & -g_2\\ (1+\overline{\lambda})\nabla_2^2 & (\nabla_2^2-g_1)\end{vmatrix}(\overline{\phi},\overline{\chi})=0 \tag{19}$$

$$(\nabla_2^4+A_3\nabla_2^2+B_3)\overline{G}=0 \tag{20}$$

$$A_1=g_1+(c_3/c_2)g_2+g_3$$

$$B_1=g_1g_3$$

$$\overline{\phi}(r)=\sum_{i=1}^{2}[A_iJ_\delta(\alpha_i r)+B_iY_\delta(\alpha_i r)]$$

$$\overline{\chi}(r)=\sum_{i=1}^{2}d_i[A_iJ_\delta(\alpha_i r)+B_iY_\delta(\alpha_i r)] \tag{21}$$

$$d_i=\frac{(1+\overline{\lambda})(\alpha_i r)^2}{(\alpha_i r)^2+g_3}$$

Eqs. (19)& (20) constitute the solution for the homogenous isotropic cylindrical panel with traction free boundary conditions. It is noticed that Eq. (19) and Eq.20 are similar to one as obtained and discussed by Chen et al [19] in case of elastokinetics.

5. FREQUENCY EQUATION

In this section we shall derive the secular equation for the three dimensional vibrations cylindrical panel subjected to traction free boundary conditions at the upper and lower surfaces at

$$r=a,b$$

$$u=\left(-\overline{\phi}'-\frac{\delta\overline{\psi}'}{r}\right)\sin(m\pi z)\sin(\delta\theta)e^{i\omega t}$$

$$v=\left(-\overline{\psi}'-\frac{\delta\overline{\phi}'}{r}\right)\sin(m\pi z)\cos(\delta\theta)e^{i\omega t}$$

$$w=\overline{\chi}t_L\cos(m\pi z)\sin(\delta\theta)e^{i\omega t}$$

$$T(r,\theta,z,t)=\overline{T}(r,\theta,z,t)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t}$$

$$\overline{\sigma}_{rr}=\left[(2+\overline{\lambda})\delta\left(\frac{\overline{\psi}'}{r}-\frac{\overline{\psi}}{r^2}\right)+(2+\overline{\lambda})\left(\frac{1}{r}\overline{\varphi}'+(\alpha_i^2-\frac{\delta^2}{r^2}\overline{\varphi})\right)+\overline{\lambda}\left(\frac{\delta}{r^2}\overline{\psi}-\frac{1}{r}\overline{\phi}'-\frac{\delta^2}{r^2}\overline{\varphi}-\frac{\delta}{r}\overline{\psi}'-t_L^2\overline{\chi}\right)\right]$$

$$\sin(m\pi)z\cos(\delta\theta)e^{i\omega t}$$

$$\overline{\sigma}_{r\theta}=2\left(\frac{1}{r}\overline{\psi}+(\alpha_i^2-\frac{\delta^2}{r^2})\overline{\psi}-\frac{2\delta}{r}\overline{\varphi}^{'}+\frac{2\delta}{r^2}\overline{\varphi}+\frac{\overline{\psi}^{'}}{r}-\frac{\delta^2}{r^2}\overline{\psi}\right)\sin(m\pi)z\cos(\delta\theta)e^{i\omega t}$$

$$\overline{\sigma}_{rz}=2t_L\left(-\overline{\varphi}^{'}-\frac{\delta}{r}\overline{\psi}+\overline{\chi}^{'}\right)\cos(m\pi)z\sin(\delta\theta)e^{i\omega t} \tag{22}$$

Where prime denotes the differentiation with respect to r $\overline{u}_i=u_i/R,(i=r,\theta,z)$ are three non–dimensional displacements and $\overline{\sigma}_{rr}=\sigma_{rr}/\mu,\overline{\sigma}_{r\theta}=\sigma_{r\theta}/\mu,\overline{\sigma}_{rz}=\sigma_{rz}/\mu$ are three non-dimensional stresses

Using the result obtained in the Eqs. (1)- (3) in Eqs. (6) we can get the frequency equation of free vibration as follows

$$\left|E_{ij}\right|=0 \qquad i,j=1,2,...8 \tag{23}$$

The values of the E_{ij} are defined in Appendix.

6. NUMERICAL RESULTS AND DISCUSSION

The frequency Eq. (22) is numerically solved for Zinc material. For the purpose of numerical computation we consider the closed circular cylindrical shell with the center angle $\alpha=2\pi$ and the integer n must be even since the shell vibrates in circumferential full wave. The frequency equation for a closed cylindrical shell can be obtained by setting $\delta=l\left(l=1,2,3.....\right)$ where l is the circumferential wave number in Eq. (14). The material properties of a Zinc is taken from [10] for isotropic material

$\rho=7.14\times10^3 kgm^{-3}$ $\lambda=0.385\times10^{11}Nm^{-2}$ $\mu=0.508\times10^{11}Nm^{-2}$ $\Omega=0.3\ rps$

$\beta=5.75\times10^6 Nm^{-2}\deg^{-1}$ $T_0=296K$ $K=1.24\times10^2 Wm^{-1}\deg^{-1}$ $C_v=3.9\times10^2\ J\,kg^{-1}\deg^{-1}$

The roots of the algebraic Eq. (12) were calculated using a combination of Birge-Vita method and Newton-Raphson method. In the present case simple Birge-Vita method does not work for finding the root of the algebraic equation. After obtaining the roots of the algebraic equation using Birge-Vita method, the roots are corrected for the desired accuracy using the Newton-Raphson method. This combination has overcome the difficulties in finding the roots of the algebraic equations of the governing equations. To validate the present analysis a comparative study is presented in Table.1 for different values of thickness to inner radius ratio (h/b=0.1, 0.2, 0.3) and center angle $\alpha=30^0,60^0,90^0$ of a cylindrical panel in the absence of thermal and rotational effect. A comparison is made between the non dimensional frequencies of thermally insulated and isothermal modes of vibration of a rotating and non rotating cylindrical shell with respect to different rotational speed in Tabl.2 and Table.3, respectively. From Table.2 and Table.3 it is clear that as the rotational speed increases, the non dimensional frequencies are also increases in both rotating and non rotating cases. As the rotation of the cylindrical shell increases, the coupling effect of various interacting fields also increases resulting in higher frequency.

Table 1.The lowest natural frequency of Zinc cylindrical panel with respect to thickness to inner radius ratio.

h/b	(α)	Ref[23]	Ref[24]	Present
0.1	30	0.7207	0.7207	0.7190
	60	0.8262	0.8257	0.8192
	90	0.9697	0.9680	0.9533
0.2	30	1.3448	1.3429	1.3325
	60	1.3118	1.3055	1.1990
	90	1.3015	1.2901	1.2877
0.3	30	1.9803	1.9706	1.9690
	60	1.8362	1.8099	1.8135
	90	1.6937	1.6552	1.6743

Table 1. Comparison between the non dimensional frequencies of Rotating and Non-Rotating thermo-elastic cylindrical shell for thermally insulated boundary in the first three modes of vibration.

Ω	Rotating			Non-Rotating		
	n=1	n =2	n =3	n =1	n =2	n=3
0.1	0.1033	0.1159	0.1462	0.0899	0.1059	0.1259
0.3	0.3721	0.4821	0.5250	0.2897	0.2707	0.3779
0.5	0.5285	0.6221	0.6614	0.5406	0.5241	0.6327
0.7	0.9898	0.9053	0.7999	0.7840	0.9005	0.8945
1.0	1.3144	1.3728	1.4663	1.1353	1.2064	1.3977

Table 2. Comparison between the non dimensional frequencies of Rotating and Non-Rotating thermo-elastic cylindrical shell for isothermal boundary in the first three modes of vibration.

Ω	Rotating			Non-Rotating		
	n=1	n=2	n=3	n=1	n=2	n=3
0.1	0.1026	0.1215	0.1413	0.0741	0.1078	0.1214
0.3	0.4443	0.4549	0.5245	0.2243	0.3550	0.4247
0.5	0.6077	0.7075	0.7378	0.5922	0.7071	0.7077
0.7	0.9196	0.8200	0.9044	0.9094	0.9909	0.9909
1.0	1.4149	1.4256	1.4644	1.4142	1.4156	1.4142

A dispersion curve is drawn between the non-dimensional wave number versus dimensionless phase velocity in case of rotating and non-rotating thermally insulated cylindrical shell with respect to different thickness parameters $t^* = b - a/R = 0.1, 0.25, 0.5$ for thermally insulated and isothermal boundaries is shown in Fig.1 and Fig.2 respectively. The solid line curves correspond to rotating thermo elastic cylindrical shell and the dotted line curves to that of non-rotating shell. From the Figs.1 and 2, it is observed that the non-dimensional phase velocity decreases rapidly to become linear at higher values of wave number for both thermally insulated and isothermal cases.The phase velocity of lower value of t^* in case of non rotating shell is observed to increase from zero wave number and become stable at higher values of wave number for both the thermal boundaries. The phase velocity at higher value of t^* attain quite large values at the vanishing wave number and are non-dispersive due to rotation. When the thickness parameter of the cylindrical panel is increased, the dimensionless phase velocity is decreases for both rotating and non-rotating cylindrical shell.

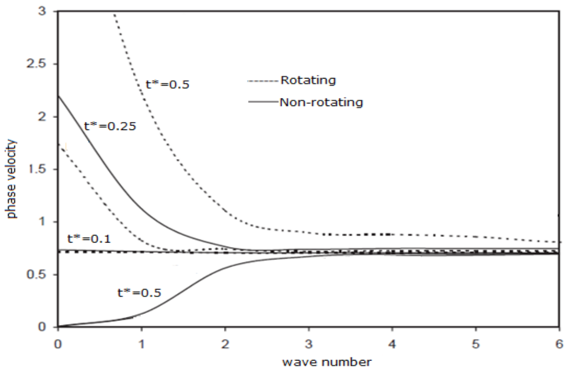

Fig.1.Variation of wave number verses phase velocity with different t* for thermally insulated Zinc shell.

The comparison of Fig.1 and Fig.2 shows that the non-dimensional phase velocity decreases exponentially for smaller wave number in case of thermally insulated and isothermal boundaries for all value of t*,but the case of higher wave number the non-dimensional phase velocity is steady and slow for all values of t*.

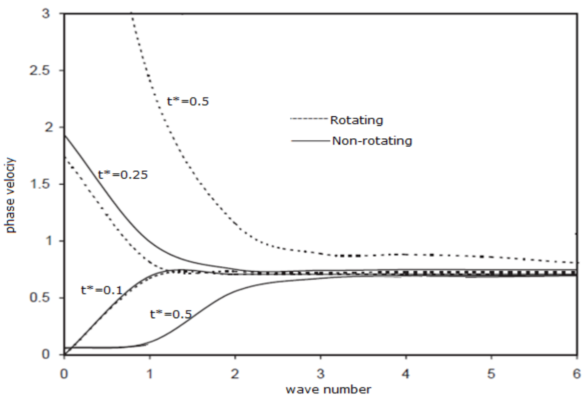

Fig.2.Variation of wave number verses phase velocity with different t^* for isothermal Zinc shell.

7. CONCLUSION

The three dimensional dispersion analysis of a homogeneous isotropic rotating cylindrical panel subjected to the traction free boundary conditions has been considered for this paper. For this problem, the governing equations of three dimensional linear thermo elasticity have been employed and solved by the Bessel function solution with complex argument. The effect of the wave number on the phase velocity of a closed Zinc cylindrical shell is investigated and the results are presented as dispersion curves. The rotational speed and different thermal boundaries influence the wave propagation characteristics. In addition, a comparative study is made between the rotating and non rotating cylindrical shell and the frequency change is observed to be highest for the rotating case. Also, a comparison of the non dimensional frequencies for the different thickness to inner radius ratio of cylindrical panel with out thermal and rotational effects shows well agreement with those of existing literature.

APPENDIX

The parameters E_{ij} in Eq. (22) are defined as

$$E_{11} = (2+\bar{\lambda})\left((\delta J_{\delta}(\alpha_1 t_1)/t_1^{2} - \frac{\alpha_1}{t_1} J_{\delta+1}(\alpha_1 t_1)) - ((\alpha_1 t_1)^2 R^2 - \delta^2) J_{\delta}(\alpha_1 t_1)/t_1^{2} \right)$$
$$+\bar{\lambda}\left(\delta(\delta-1) J_{\delta}(\alpha_1 t_1)/t_1^{2} - \frac{\alpha_1}{t_1} J_{\delta+1}(\alpha_1 t_1) \right) + \bar{\lambda} d_1 t_L{}^2 J_{\delta}(\alpha_1 t_1) - \beta T_0 R^2 e_i \bar{\lambda} \quad \text{(A1)}$$

$$E_{13} = (2+\bar{\lambda})\left((\delta J_{\delta}(\alpha_2 t_1)/t_1^{2} - \frac{\alpha_2}{t_2} J_{\delta+1}(\alpha_2 t_1)) - ((\alpha_2 t_1)^2 R^2 - \delta^2) J_{\delta}(\alpha_2 t_1)/t_1^{2} \right)$$
$$+\bar{\lambda}\left(\delta(\delta-1) J_{\delta}(\alpha_2 t_1)/t_1^{2} - \frac{\alpha_2}{t_1} J_{\delta+1}(\alpha_2 t_1) \right) + \bar{\lambda} d_2 t_L{}^2 J_{\delta}(\alpha_2 t_1) - \beta T_0 R^2 e_i \bar{\lambda} \quad \text{(A2)}$$

$$E_{15} = (2+\bar{\lambda})\left((\delta J_{\delta}(\alpha_3 t_1)/t_1^{2} - \frac{\alpha_2}{t_2} J_{\delta+1}(\alpha_3 t_1)) - ((\alpha_3 t_1)^2 R^2 - \delta^2) J_{\delta}(\alpha_3 t_1)/t_1^{2} \right)$$
$$+\bar{\lambda}\left(\delta(\delta-1) J_{\delta}(\alpha_3 t_1)/t_1^{2} - \frac{\alpha_2}{t_1} J_{\delta+1}(\alpha_3 t_1) \right) + \bar{\lambda} d_2 t_L{}^2 J_{\delta}(\alpha_3 t_1) - \beta T_0 R^2 e_i \bar{\lambda} \quad \text{(A3)}$$

$$E_{17} = (2+\bar{\lambda})\left((\frac{k_1\delta}{t_1} J_{\delta+1}(k_1 t_1) - \delta(\delta-1) J_{\delta}(k_1 t_1)/t_1^{2} \right) + \bar{\lambda}\left(\delta(\delta-1) J_{\delta}(k_1 t_1)/t_1^{2} - \frac{k_1\delta}{t_1} J_{\delta+1}(k_1 t_1) \right) \quad \text{(A4)}$$

$$E_{21} = 2\delta\left((\alpha_1/t_1) J_{\delta+1}(\alpha_1 t_1) - \delta(\delta-1) J_{\delta}(\alpha_1 t_1)\right) \quad \text{(A6)}$$

$$E_{23} = 2\delta\left((\alpha_2/t_1) J_{\delta+1}(\alpha_2 t_1) - \delta(\delta-1) J_{\delta}(\alpha_2 t_1)\right) \quad \text{(A7)}$$

$$E_{25} = 2\delta\left((\alpha_3/t_1) J_{\delta+1}(\alpha_3 t_1) - \delta(\delta-1) J_{\delta}(\alpha_3 t_1)\right) \quad \text{(A8)}$$

$$E_{27} = (k_1 t_1)^2 R^2 J_{\delta}(k_1 t_1) - 2\delta(\delta-1) J_{\delta}(k_1 t_1)/t_1^{2} + k_1/t_1 J_{\delta+1}(k_1 t_1) \quad \text{(A9)}$$

$$E_{31} = -t_L(1+d_1)\left(\delta/t_1 J_{\delta}(\alpha_1 t_1) - \alpha_1 J_{\delta+1}(\alpha_1 t_1)\right) \quad \text{(A10)}$$

$$E_{33} = -t_L(1+d_2)\left(\delta/t_1 J_{\delta}(\alpha_2 t_1) - \alpha_2 J_{\delta+1}(\alpha_2 t_1)\right) \quad \text{(A11)}$$

$$E_{35} = -t_L(1+d_3)\left(\delta/t_1 J_{\delta}(\alpha_3 t_1) - \alpha_2 J_{\delta+1}(\alpha_3 t_1)\right) \quad \text{(A12)}$$

$$E_{37} = -t_L(\delta/t_1) J_{\delta}(k_1 t_1) \quad \text{(A13)}$$

$$E_{41} = e_1[(\delta/t_1) J_{\delta}(\alpha_1 t_1) - (\alpha_1) J_{\delta+1}(\alpha_1 t_1) + h J_{\delta}(\alpha_1 t_1)] \quad \text{(A14)}$$

$$E_{43} = e_2[(\delta/t_1) J_{\delta}(\alpha_2 t_1) - (\alpha_2) J_{\delta+1}(\alpha_2 t_1) + h J_{\delta}(\alpha_2 t_1)] \quad \text{(A15)}$$

$$E_{45} = e_3[(\delta/t_1) J_{\delta}(\alpha_3 t_1) - (\alpha_3) J_{\delta+1}(\alpha_3 t_1) + h J_{\delta}(\alpha_3 t_1)] \quad \text{(A16)}$$

$$E_{47} = 0 \quad \text{(A17)}$$

In which $t_1 = a/R = 1 - t^*/2$, $t_2 = b/R = 1 + t^*/2$ and $t^* = b - a/R$ is the thickness -to-mean radius ratio of the panel. Obviously $E_{ij}\left(j = 2,4,6,8\right)$ can obtained by just replacing modified Bessel function of the first kind in $E_{ij}\left(i = 1,3,5,7\right)$ with the ones of the second kind, respectively, while $E_{ij}\left(i = 5,6,7,8\right)$ can be obtained by just replacing t_1 in $E_{ij}\left(i = 1,2,3,4\right)$ with t_2 .

REFERENCES

[1] Nowacki W. *Dynamical problems of thermo elasticity*, Noordhoff, Leyden, The Netherlands, 1975.
[2] Lord Shulman. "A generalized dynamical theory of thermo elasticity", *Journal of Mechanics of Physics of Solids*, Vol. 15. Pp. 299–309, 1967.
[3] A.E Green and K.A Lindsay. "Thermo elasticity", *Journal of Elasticity*, Vol. 2. Pp.1–7, 1972.
[4] X.Wang. "Three dimensional analysis of multi layered functionally graded anisotropic cylindrical panel under thermo mechanical load", *Mechanics of materials*, Vol. 40. Pp. 235-254, 2008.
[5] C.B Hallam and E. Ollerton. "Thermal stresses in axially connected circular cylinders", *Journal of Strain Analysis,* Vol/Issue: 8(3). Pp. 160-167, 1973.
[6] C.F Gao and N. Noda. "Thermal-induced interfacial cracking of magneto electro elastic materials.", *International Journal of Engineering Sciences*, Vol. 42, Pp.1347-1360, 2000.
[7] W.Q Chen et.al. "Point temperature solution for a penny-shaped crack in an infinite transversely isotropic thermo-piezo-elastic medium", *Engineering Analysis with Boundary elements*, Vol. 29. Pp. 524-532, 2005.
[8] M.Abouhamze and M.Shakeri. "Multi objective stacking sequence optimization of laminated cylindrical panels using the genetic algorithm and nueral network", *Composite structures*, Vol. 81 Pp. 253-263, 2007.
[9] P.Chadwick. "Basic prosperities of plane harmonic waves in a pre stressed heat conducting elastic material", *Journal of Thermal Stresses*, Vol. 2. Pp. 193-214, 2002.
[10] J.N Sharma. "Three dimensional vibration analysis of homogenous transversely isotropic thermo elastic cylindrical panel", *Journal of Acoustical Society of America*, Vol. 110. Pp. 648-653, 2001.
[11] J.H Prevost and D.Tao "Finite element analysis of dynamic coupled thermo elasticity problems with relaxation times", *J. Appl. Mech. Trans. ASME*, Vol. 50. Pp. 817–822, 1983.
[12] M.R Eslami and H Vahedi. "Coupled thermo elasticity beam problems", *AIAA Journal*, Vol/Issue: 27(5). Pp. 662–665, 1989.
[13] N.N Huang and T.R Tauchert. "Thermo elastic solution for cross-ply cylindrical panels", *Journal of Thermal Stresses,* Vol. 14. Pp. 227–237, 1991.
[14] P.Ponnusamy and R.Selvamani. "Wave propagation in generalized thermo elastic plate embedded in an elastic medium", *Interaction and multiscale mechanics,* Vol. 5. Pp.13-26, 2012.
[15] P.Ponnusamy and R.Selvamani. "Dispersion analysis of generalized magneto thermo elastic waves in a transeversely isotropic cylindrical panel", *Journal of thermal stresses,* Vol. 35. Pp. 1119-1142, 2012.
[16] R.Selvamani and P.Ponnusamy. "Damping of generalized thermoelastic plate in homogeneous isotropic plate", *Materials Physics and mechanics*, Vol. 14. Pp. 64-73, 2012.
[17] C.T Loy and K.Y Lam. "Vibration of Rotating Thin Cylindrical Panel", *Applied Acoustic*, Vol.46. Pp. 327-343, 1995.
[18] A.A.Bhimraddi. "A higher order theory for free vibration analysis of circular cylindrical shell", *International Journal of Solid and Structure,* Vol. 20. Pp. 623-630, 1984.
[19] X.M Zhang. "The parametric analysis of frequency of rotating laminated composite cylindrical shell using wave propagation approach", *Computer methods in applied mechanics and engineering,* Vol. 191. Pp. 2027-2043, 2002.
[20] J.N Sharma and D. Grover. "Body wave propagation in rotating thermo elastic media", *Mechanical Research Communications,* Vol. 36. Pp. 715-721, 2009.
[21] A.M Abd-Alla and F.S Bayones. "Effect of rotation in a generalized magneto thermo visco elastic media", *Advances in Theoretical and Applied Mechanics*, Vol. 4. Pp. 15-42, 2011.
[22] J.Wauer, "Waves in rotating and conducting piezoelectric media", *Journal of Acoustical Society of America*, Vol/Issue: 106(2). Pp. 626-636, 1999.
[23] S.K Roychoudhuri and S. Mukhopadhyay. "Effect of rotation and relaxation times on plane waves in generalized thermo visco elasticity", *IJMMS*, Vol/Issue: 23(7). Pp. 497-505, 2000.
[24] Chen et.al. "Vibration analysis of orthotropic cylindrical shells with free ends by the Rayleigh–Ritz method", *Journal of Sound and Vibration*, Vol. 195. Pp. 117–135, 1996.
[25] L.I Hua and K.Y Lam. "Frequency characteristics of a thin rotating cylindrical shell using general differential quadrature method", *International Journal of Mechanical Sciences*, Vol. 40. Pp. 443-459, 1998.