

## Odd generalized exponential log logistic distribution: A new acceptance sampling plans based on percentiles

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### Article Info

#### Article history:

Received Apr 1, 2019

Revised Jul 14, 2019

Accepted Aug 2, 2019

#### Keywords:

Acceptance sampling

Consumer's risk

OC curve

Producer's risk

Truncated life tests

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### ABSTRACT

In this paper, acceptance sampling plans are developed for the odd generalized exponential log logistic distribution based on percentiles when the life test is truncated at a pre-specified (pre-determined) time. The minimum sample size necessary to ensure the specified life percentile is obtained under a given consumer's risk. The operating characteristic values of the sampling plans as well as the producer's risk are presented. One example with real data set is also given as an illustration.

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## 1. INTRODUCTION

Acceptance sampling is 'the middle of the road' approach between no inspection and 100% inspection. The objective of acceptance sampling is not to estimate the quality of the lot, but to decide whether or not the lot is likely to be acceptable. The applications stanch from real life scenarios: if every bullet was tested in advance prior to war, no bullet is at hand for the time of action and if no bullet is tested, then malfunctions may occur in the war with disastrous results. The selection of a sample from a lot or consignment and the outcome of the products totally depend on the characteristics collected from this sample which was described by [1]. This procedure is called as acceptance sampling plan (ASP) or 'lot sentencing'. In mass production, a sample is taken at random and tested on the basis of the quality characteristics, ASP is used to accept or reject a submitted lot. An ASP is a specified plan that establishes the minimum sample size to be used for testing. In most ASPs for a truncated life test, the foremost issue is to determine the minimum sample size from a lot under consideration. Traditionally, the lot of items is accepted when the life test indicates that the average life of items exceeds the specified one, otherwise it is rejected. For any industries, the objective is to reducing the cost and test time, a truncated life test may be conducted to obtain the smallest sample size to ensure a certain average life time/percentile lifetime of items, for a given acceptance number  $c$ , the number of failures observed does not exceed when the life test is terminated at a pre-assigned time. The decision is to accept the lot if a pre-determined average lifetime/percentile lifetime can be reached with a pre-determined high probability which provides protection to consumer. Therefore, the life test is ended at the time the failure is observed or at the pre-assigned time, whichever is earlier. For such a truncated life test and the associated decision rule; we are focused in obtaining the smallest sample size to arrive at a decision.

In the past few decades, much effort has gone into the investigation of acceptance sampling plans under a truncated life test. The truncated life tests in the exponential distribution were first considered by [2]. Truncated life tests are deliberated by many authors for various distributions: for example [3-7]. The ASPs could be used for the quantiles and derived the formulae for generalized Birnbaum-Saunders distribution and Marshall-Olkin extended Lomax distribution was proposed by [8, 9]. ASP based on truncated life tests for log-logistic distribution and exponentiated Fréchet distribution was proposed by [10, 11].

The design of ASPs based on the population mean under a truncated life test is considered by all the authors who are in the above. For a skewed distribution, the median represents a better quality parameter than the mean was suggested by [12]. On the other hand, for a symmetric distribution, mean is preferable to use as a quality parameter. ASPs based on the truncated life tests to Birnbaum-Saunders distribution and Burr type XII for percentiles was considered by [13, 14] and they proposed that the ASPs based on mean may not satisfy the requirement of engineering on the specific percentile of strength or breaking stress. When the quality of a specified low percentile is concerned, the ASPs based on the population mean could pass a lot which has the low percentile below the required standard of consumers. Furthermore, a small diminution in the mean with a simultaneous small intensification in the variance can result in a significant downward shift in small percentiles of interest. This means that a lot of products could be accepted due to a small decrease in the mean life after inspection. But the material strengths of products are deteriorated significantly and may not meet the consumer's expectation. Therefore, engineers should pay more attention to the percentiles lifetimes than the mean life in life testing applications. Moreover, most of the employed life distributions are not symmetric. Actually, percentiles provide more information regarding a life distribution than the mean life does. When the life distribution is symmetric, the 50<sup>th</sup> percentile or the median is equivalent to the mean life. Several authors developed the acceptance sampling plans based on percentile. ASPs from truncated life tests based on the log-logistic and inverse Rayleigh distributions, Marshall – Olkin extended Lomax distribution, Linear Failure Rate distribution, Half Normal distribution, Gompertz distribution for percentiles were developed by [15-20]. ASPs based on median life for Fréchet distribution was discussed by [21]. An ASPs from truncated life tests based on the weighted exponential distribution was considered by [22]. New acceptance sampling plans based on percentiles for exponentiated Fréchet distribution was constructed by [23]. An ASPs based on percentiles for Odds exponential log logistic distribution (OELLD) was discussed by [24]. New ASPs based on life tests for Birnbaum–Saunders distributions was considered by [25]. Acceptance sampling for attributes via hypothesis testing and the hyper-geometric distribution was developed by [26]. Acceptance sampling based on life tests from some specific distributions was constructed [27]. These reasons we are motivate to develop ASPs based on the percentiles, since odd generalized exponential log logistic distribution, we prefer to use the percentile point as the quality parameter, and it will be denoted by  $t_q$ . The rest of the paper is organized as follows: In Section 2, we describe concisely the odd generalized exponential log logistic distribution. In Section 3, the design of proposed acceptance sampling plan for lifetime percentiles under a truncated life test is presented. In Section 4, we present the description of the proposed plan and obtain the necessary results. An example with real data set and comparison of the proposed sampling scheme with the OELLD is also given as an illustration. Finally, conclusions are made in Section 5.

## 2. THE ODD GENERALIZED EXPONENTIAL LOG LOGISTIC DISTRIBUTION

In this section, we provide a brief summary about the odd generalized exponential log logistic distribution (OGELLD). The OGELLD was introduced and studied quite extensively by [28]. The probability density function (pdf) and cumulative distribution function (cdf) of OGELLD respectively are given as follows

$$f(t; \sigma, \lambda, \theta, \gamma) = \frac{\gamma \theta}{\lambda \sigma} \left(\frac{t}{\sigma}\right)^{\theta-1} \left[1 - e^{-\frac{t}{\lambda} \left(\frac{t}{\sigma}\right)^\theta}\right]^{\gamma-1} \quad \text{for } t > 0, \sigma, \lambda > 0, \theta, \gamma > 1 \quad (1)$$

$$F(t; \sigma, \lambda, \theta, \gamma) = \left[1 - e^{-\frac{t}{\lambda} \left(\frac{t}{\sigma}\right)^\theta}\right]^\gamma, \quad t > 0, \sigma, \lambda, \theta \text{ and } \gamma > 1 \quad (2)$$

where  $\sigma, \lambda$  are the scale parameters and  $\theta, \gamma$  are shape parameters respectively. The  $100q$ -th quantile of the OGELLD is given as

$$t_q = \sigma \eta_q, \quad \text{where } \eta_q = \left[-\lambda \ln \left(1 - q^{\frac{1}{\gamma}}\right)\right]^{\frac{1}{\theta}} \quad (3)$$

Hence, for the fixed values of  $\lambda = \lambda_0$ ,  $\theta = \theta_0$  and  $\gamma = \gamma_0$ , the quantile  $t_q$  given in (3) is the function of scale parameter  $\sigma = \sigma_0$ , that is  $t_q \geq t_q^0 \Leftrightarrow \sigma \geq \sigma_0$ , where

$$\sigma_0 = \frac{t_q^0}{\left[-\lambda_0 \ln\left(1 - q^{\frac{1}{\gamma_0}}\right)\right]^{\frac{1}{\theta_0}}} = t_q^0 \left[-\lambda_0 \ln\left(1 - q^{\frac{1}{\gamma_0}}\right)\right]^{\frac{1}{\theta_0}} = t_q^0 \eta_q \quad (4)$$

Note that  $\sigma_0$  also depends on  $\lambda_0, \theta_0$  and  $\gamma_0$ , to construct the acceptance sampling plans for the OGELLD ascertain  $t_q \geq t_q^0$ , equivalently that  $\sigma$  exceeds  $\sigma_0$ .

### 3. THE ACCEPTANCE SAMPLING PLAN

The problem considered is that of finding the minimum sample size necessary to ensure a percentile lifetime of the product, when the life test is terminated at a pre-assigned time  $t_q^0$  and when the observed number of failures does not exceed a given acceptance number  $c$ . The decision procedure is to accept a lot only if the specified percentile lifetime can be established with a pre-assigned high probability  $\alpha$ , which provides protection to the consumer. The life test experiment gets terminated at the time at which  $(c + 1)^{st}$  failure is observed or at quantile time  $t_q$ , whichever is earlier. The probability of accepting lot based on the number of failures from a sample under a truncated life test at the test time schedule  $t_0$  is given by

$$P_a(p) = \sum_{i=0}^c \binom{n}{i} p^i (1-p)^{n-i} \quad (5)$$

where  $n$  is the sample size,  $c$  is the acceptance number and  $p$  is the probability of getting a failure within the life test schedule,  $t_0$ . If the product lifetime follows an OGELLD, then  $p = F(t_0; \sigma, \lambda_0, \theta_0, \gamma_0)$ . Usually, it would be convenient to express the experiment termination time  $t_0$  as  $t_0 = \delta_q^0 t_q^0$  for a constant  $\delta_q^0$  and the targeted 100q-th lifetime percentile,  $t_q^0$ . Suppose  $t_q$  is the true 100q-th lifetime percentile. Then,  $p$  can be rewritten as

$$p = \left[1 - \exp\left\{-\frac{1}{\lambda} \left(\frac{t_0}{\sigma}\right)^\theta\right\}\right]^\gamma = \left[1 - \exp\left\{-\frac{1}{\lambda} \left(\frac{\eta_q \delta_q^0}{t_q^0}\right)^\theta\right\}\right]^\gamma \quad (6)$$

In order to obtain the proposed design parameters of the proposed plan, we prefer the approach based on two points on the Operating Characteristic (O.C) curve by considering the Type I and Type II errors (i.e., producer's and consumer's risk). In our methodology, the quality level is intended through the ratio of its percentile lifetime to the true lifetime,  $t_q \div t_q^0$ . These ratios are very useful for the producer to give the better quality of products. Meanwhile the producer's perspective, the probability of lot acceptance should be at least  $1 - \alpha$  at acceptable reliability level (ARL),  $p_1$ . Therefore, the producer demands that a lot should be accepted at various levels, say  $t_q \div t_q^0 = 2, 4, 6, 8$  in (5). Whereas the consumer's viewpoint, the lot is rejection should be at most  $\beta$  at the lot tolerance reliability level (LTRL),  $p_2$ . However, the consumer considers that a lot should be rejected when  $t_q \div t_q^0 = 1$ . From (5), we have

$$P_a(p_1) = \sum_{i=0}^c \binom{n}{i} p_1^i (1-p_1)^{n-i} \geq 1 - \alpha \quad (7)$$

$$P_a(p_2) = \sum_{i=0}^c \binom{n}{i} p_2^i (1-p_2)^{n-i} \leq \beta \quad (8)$$

where  $p_1$  and  $p_2$  are given by

$$p_1 = \left[1 - \exp\left\{\left(\frac{-1}{\lambda}\right) \left(\frac{\eta_q \delta_q^0}{t_q^0}\right)^\theta\right\}\right]^\gamma \quad \text{and} \quad p_2 = \left[1 - \exp\left\{\left(\frac{-1}{\lambda}\right) (\eta_q \delta_q^0)^\theta\right\}\right]^\gamma \quad (9)$$

The proposed plan parametric quantities for different values of parameters  $\lambda, \theta$  and  $\gamma$  are constructed. Given the producer's risk  $\alpha = 0.05$  and termination time schedule  $t_0 = \delta_q^0 t_q^0$  with  $\delta_q^0 = 1.0, 1.5, 2.0$  and  $2.5$  the four parameters of the proposed plan under the truncated life test at the pre-specified time,  $t_0$  with  $\theta = 2, \gamma = 2$  and  $\lambda = 0.5, 1.0, 1.5, 2.0$  are obtained according to the consumer's confidence levels  $\beta = 0.25, 0.10, 0.05, 0.01$  for 50<sup>th</sup> percentile and the O.C. values are also obtained and the results are framed in Table 1 to Table 4. The proposed plan parameters are presented in Table 1 to Table 4 for  $\theta=2, \gamma = 2$  and  $\lambda = 0.5, 1.0, 1.5, 2.0$  with 50<sup>th</sup> percentiles, whereas Table 5 shows the plan parameters for  $\hat{\lambda} = 10.7592, \hat{\theta} = 2.4083$  and  $\hat{\gamma}=1.3177$  at 50<sup>th</sup> percentile. On clear observation, we noticed from Table 1 to Table 4 that the percentile ratio increases, the sample size 'n' decreases.

Table 1. Minimum sample size necessary to assert the 50th percentile life and the corresponding O.C values of OGELLD for  $\lambda = 0.5, \theta = 2.0, \gamma = 2.0$

$\beta$	$\frac{t_q}{t_q^0}$	$\delta_q=1.0$				$\delta_q=1.5$				$\delta_q=2$				$\delta_q=2.5$			
		c	n	$P_a(p_1)$	c	n	$P_a(p_1)$	c	n	$P_a(p_1)$	c	n	$P_a(p_1)$	c	n	$P_a(p_1)$	
0.25	2	10	25	0.9574	8	14	0.9531	9	13	0.9539	8	10	0.9526				
	4	3	10	0.9658	3	7	0.9672	3	5	0.9810	2	3	0.9653				
	6	2	7	0.9811	1	3	0.9513	2	4	0.9806	1	2	0.9513				
	8	1	5	0.9609	↑	↑	0.9718	1	3	0.9513	↑	↑	0.9723				
0.10	2	14	37	0.9578	13	24	0.9600	12	18	0.9519	13	17	0.9522				
	4	4	14	0.9701	4	10	0.9678	3	6	0.9549	3	5	0.9582				
	6	2	9	0.9604	2	6	0.9652	2	5	0.9580	2	4	0.9641				
	8	↑	↑	0.9814	↑	↑	0.9840	1	3	0.9513	↑	↑	0.9839				
0.05	2	17	47	0.9512	16	31	0.9505	15	23	0.9534	16	21	0.9618				
	4	5	18	0.9749	4	11	0.9504	4	8	0.9660	4	7	0.9588				
	6	3	13	0.9765	3	9	0.9771	2	5	0.9580	2	4	0.9641				
	8	2	11	0.9670	2	7	0.9741	↑	↑	0.9807	↑	↑	0.9839				
0.01	2	-	-	-	-	-	-	19	30	0.9506	-	-	-				
	4	6	25	0.9593	6	16	0.9739	5	11	0.9563	5	9	0.9619				
	6	4	19	0.9773	3	11	0.9518	3	8	0.9622	3	7	0.9534				
	8	3	17	0.9763	↑	↑	0.9813	↑	↑	0.9858	2	5	0.9646				

The upward arrow (↑) indicates the same values as the cell above.

Table 2. Minimum sample size necessary to assert the 50th percentile life and the corresponding O.C values of OGELLD for  $\lambda = 1.0, \theta = 2.0, \gamma = 2.0$

$\beta$	$\frac{t_q}{t_q^0}$	$\delta_q=1.0$				$\delta_q=1.5$				$\delta_q=2$				$\delta_q=2.5$			
		c	n	$P_a(p_1)$	c	n	$P_a(p_1)$	c	n	$P_a(p_1)$	c	n	$P_a(p_1)$	c	n	$P_a(p_1)$	
0.25	2	4	12	0.9697	4	7	0.9605	6	8	0.9648	7	8	0.9634				
	4	1	5	0.9835	1	3	0.9757	1	2	0.9747	2	3	0.9867				
	6	0	2	0.9622	0	1	0.9576	↑	↑	0.9945	1	2	0.9871				
	8	↑	↑	0.9786	↑	↑	0.9759	0	1	0.9576	↑	↑	0.9957				
0.10	2	5	17	0.9587	5	9	0.9637	6	8	0.9648	11	13	0.9645				
	4	1	7	0.9673	1	4	0.9544	2	4	0.9858	2	3	0.9867				
	6	↑	↑	0.9928	↑	↑	0.9898	1	3	0.9843	1	2	0.9871				
	8	0	4	0.9576	0	2	0.9524	0	1	0.9576	↑	↑	0.9957				
0.05	2	6	21	0.9616	7	13	0.9714	8	11	0.9673	11	13	0.9645				
	4	1	8	0.9576	1	4	0.9544	2	4	0.9858	2	3	0.9867				
	6	↑	↑	0.9906	↑	↑	0.9898	1	3	0.9843	1	2	0.9871				
	8	↑	↑	0.9969	0	2	0.9524	↑	↑	0.9948	↑	↑	0.9957				
0.01	2	8	30	0.9610	8	16	0.9590	9	13	0.9539	14	17	0.9598				
	4	2	14	0.9805	2	7	0.9789	2	5	0.9687	2	4	0.9561				
	6	1	11	0.9822	1	5	0.9835	1	4	0.9702	1	3	0.9644				
	8	↑	↑	0.9940	↑	↑	0.9945	↑	↑	0.9898	↑	↑	0.9877				

The upward arrow (↑) indicates the same values as the cell above.

Table 3. Minimum sample size necessary to assert the 50<sup>th</sup> percentile life and the corresponding O.C values of OGELLD for  $\lambda = 1.5, \theta = 2.0, \gamma = 2.0$

$\beta$	$\frac{t_q}{t_0^q}$	$\delta_q=1.0$			$\delta_q=1.5$			$\delta_q=2$			$\delta_q=2.5$		
		c	n	$P_a(p_1)$	c	n	$P_a(p_1)$	c	n	$P_a(p_1)$	c	n	$P_a(p_1)$
0.25	2	2	7	0.9720	3	5	0.9760	4	5	0.9688	8	9	0.9596
	4	0	2	0.9706	0	1	0.9529	1	2	0.9893	1	2	0.9667
	6	↑	↑	0.9910	↑	↑	0.9852	0	1	0.9662	↑	↑	0.9960
	8	↑	↑	0.9962	↑	↑	0.9936	↑	↑	0.9852	0	1	0.9719
0.10	2	3	12	0.9715	4	7	0.9786	6	8	0.9648	8	9	0.9596
	4	1	7	0.9956	1	3	0.9935	1	2	0.9843	1	2	0.9667
	6	0	4	0.9821	0	2	0.9706	0	1	0.9662	↑	↑	0.9960
	8	↑	↑	0.9924	↑	↑	0.9873	↑	↑	0.9852	0	1	0.9719
0.05	2	3	13	0.9621	4	8	0.9560	6	8	0.9648	8	9	0.9596
	4	1	8	0.9942	1	4	0.9875	1	3	0.9702	1	2	0.9667
	6	0	5	0.9777	0	2	0.9706	0	1	0.9662	↑	↑	0.9960
	8	↑	↑	0.9905	↑	↑	0.9873	↑	↑	0.9852	0	1	0.9719
0.01	2	4	19	0.9603	6	12	0.9731	8	11	0.9673	12	14	0.9524
	4	1	11	0.9890	1	5	0.9798	1	3	0.9702	1	2	0.9667
	6	0	7	0.9690	0	3	0.9563	↑	↑	0.9966	↑	↑	0.9960
	8	↑	↑	0.9867	↑	↑	0.9810	0	2	0.9706	0	1	0.9719

The upward arrow (↑) indicates the same values as the cell above.

Table 4. Minimum sample size necessary to assert the 50<sup>th</sup> percentile life and the corresponding O.C values of OGELLD for  $\lambda = 2.0, \theta = 2.0, \gamma = 2.0$

$\beta$	$\frac{t_q}{t_0^q}$	$\delta_q=1.0$			$\delta_q=1.5$			$\delta_q=2$			$\delta_q=2.5$		
		c	n	$P_a(p_1)$	c	n	$P_a(p_1)$	c	n	$P_a(p_1)$	c	n	$P_a(p_1)$
0.25	2	1	5	0.9576	3	5	0.9847	4	5	0.9688	9	10	0.9582
	4	0	3	0.9837	0	1	0.9748	1	2	0.9951	1	2	0.9789
	6	↑	↑	0.9966	↑	↑	0.9945	0	1	0.9837	0	1	0.9631
	8	↑	↑	0.9989	↑	↑	0.9982	↑	↑	0.9945	↑	↑	0.9872
0.10	2	2	9	0.9792	3	6	0.9630	4	5	0.9688	9	10	0.9582
	4	0	4	0.9783	0	2	0.9503	1	2	0.9951	1	2	0.9789
	6	↑	↑	0.9955	↑	↑	0.9891	0	1	0.9837	0	1	0.9631
	8	↑	↑	0.9986	↑	↑	0.9964	↑	↑	0.9945	↑	↑	0.9872
0.05	2	2	11	0.9632	3	6	0.9630	6	8	0.9648	9	10	0.9582
	4	0	5	0.9730	0	2	0.9503	1	2	0.9951	1	2	0.9789
	6	↑	↑	0.9944	↑	↑	0.9891	0	1	0.9837	0	1	0.9631
	8	↑	↑	0.9982	↑	↑	0.9964	↑	↑	0.9945	↑	↑	0.9872
0.01	2	3	17	0.9728	4	9	0.9520	6	8	0.9648	9	10	0.9582
	4	0	7	0.9624	1	4	0.9963	1	3	0.9860	1	2	0.9789
	6	↑	↑	0.9922	0	3	0.9837	0	2	0.9677	0	1	0.9631
	8	↑	↑	0.9975	↑	↑	0.9947	↑	↑	0.9891	↑	↑	0.9872

The upward arrow (↑) indicates the same values as the cell above.

Table 5. Minimum sample size necessary to assert the 50<sup>th</sup> percentile life and the corresponding O.C values of OGELLD for  $\hat{\lambda} = 10.7592, \hat{\theta} = 2.4083, \hat{\gamma} = 1.3177$

$\beta$	$\frac{t_q}{t_0^q}$	$\delta_q=1.0$			$\delta_q=1.5$			$\delta_q=2$			$\delta_q=2.5$		
		c	n	$P_a(p_1)$	c	n	$P_a(p_1)$	c	n	$P_a(p_1)$	c	n	$P_a(p_1)$
0.25	2	2	7	0.9831	3	5	0.9817	4	5	0.9688	9	10	0.9599
	4	0	3	0.9692	0	1	0.9637	1	2	0.9927	1	2	0.9740
	6	↑	↑	0.9913	↑	↑	0.9896	0	1	0.9747	0	1	0.9501
	8	↑	↑	0.9965	↑	↑	0.9958	↑	↑	0.9896	↑	↑	0.9792
0.10	2	2	9	0.9643	3	6	0.9565	4	5	0.9688	9	10	0.9599
	4	0	4	0.9591	1	3	0.9961	1	2	0.9927	1	2	0.9740
	6	↑	↑	0.9884	0	2	0.9794	0	1	0.9747	0	1	0.9501
	8	↑	↑	0.9953	↑	↑	0.9916	↑	↑	0.9896	↑	↑	0.9792
0.05	2	3	13	0.9795	3	6	0.9565	6	8	0.9648	9	10	0.9599
	4	1	8	0.9971	1	3	0.9961	1	2	0.9927	1	2	0.9740
	6	0	5	0.9856	0	2	0.9794	0	1	0.9747	0	1	0.9501
	8	↑	↑	0.9942	↑	↑	0.9916	↑	↑	0.9896	↑	↑	0.9792
0.01	2	4	19	0.9806	4	8	0.9674	6	8	0.9648	9	10	0.9599
	4	1	11	0.9944	1	4	0.9925	1	3	0.9792	1	2	0.9740
	6	0	7	0.9799	0	3	0.9692	↑	↑	0.9981	0	1	0.9501
	8	↑	↑	0.9918	↑	↑	0.9874	0	2	0.9794	↑	↑	0.9792

The upward arrow (↑) indicates the same values as the cell above.

**4. DESCRIPTION OF METHODOLOGY FOR PROPOSED PLAN WITH REAL DATA EXAMPLE**

**4.1. Description of the proposed plan**

Let us assume that the producer desires to implement a proposed plan for assuring that the 50<sup>th</sup> percentile life of the products under inspection is at least 1000 hours when  $\beta = 0.10$  at the percentile ratio  $t_q \div t_q^0 = 2$ . He desires to run this experiment 1000 hrs. From the past data, if it is observed that the lifetime of the item follows OGELLD with  $\lambda = \theta = \gamma = 2$ . The optimal plan from Table 4 or specified requirements such as,  $\beta=0.10, \lambda = \theta = \gamma=2, t_q \div t_q^0 = 2$  and  $\delta_q = 1.0$  is obtained as  $n = 9$  and  $c=2$  with the acceptance probability is 0.9792. Most of the life testing with ASPs for various life time distributions available in the literature is based on one point on the OC curve approach for assuring mean or percentile lifetime. But in this study, we have designed sampling plans based on two-points on the OC curve approach for assuring percentile lifetime of the products under OGELLD.

**4.2. Real data example**

The following real data set corresponds to an uncensored data set from [29-32] on breaking stress of carbon fibres (in Gba). We describe the proposed plan for this data set:

0.39, 0.81, 0.85, 0.98, 1.08, 1.12, 1.17, 1.18, 1.22, 1.25, 1.36, 1.41, 1.47, 1.57, 1.57, 1.59, 1.59, 1.61, 1.61, 1.69, 1.69, 1.71, 1.73, 1.80, 1.84, 1.84, 1.87, 1.89, 1.92, 2.00, 2.03, 2.03, 2.05, 2.12, 2.17, 2.17, 2.17, 2.35, 2.38, 2.41, 2.43, 2.48, 2.48, 2.50, 2.53, 2.55, 2.55, 2.56, 2.59, 2.67, 2.73, 2.74, 2.76, 2.77, 2.79, 2.81, 2.81, 2.82, 2.83, 2.85, 2.87, 2.88, 2.93, 2.95, 2.96, 2.97, 2.97, 3.09, 3.11, 3.11, 3.15, 3.15, 3.19, 3.19, 3.22, 3.22, 3.27, 3.28, 3.31, 3.31, 3.33, 3.39, 3.39, 3.51, 3.56, 3.60, 3.65, 3.68, 3.68, 3.68, 3.70, 3.75, 4.20, 4.38, 4.42, 4.70, 4.90, 4.91, 5.08, 5.56

We show a rough indication of the goodness of fit for our model by plotting the density (together with the data histogram) for the data shows that the OGELLD is a good fit in Figure 1 and also goodness of fit is emphasized with Q-Q plot, displayed in Figure 1. The maximum likelihood estimates of the parameters of OGELLD for the breaking stress of carbon fibres are  $\hat{\lambda} = 10.7592, \hat{\theta} = 2.4083$  and  $\hat{\gamma} = 1.3177$  and the K-S test and found that the maximum distance between the data and the fitted of the OGELLD is 0.0644 with p-value is 0.8006. Therefore, the four-parameter OGELLD provides good fit for the breaking stress of carbon fibres.

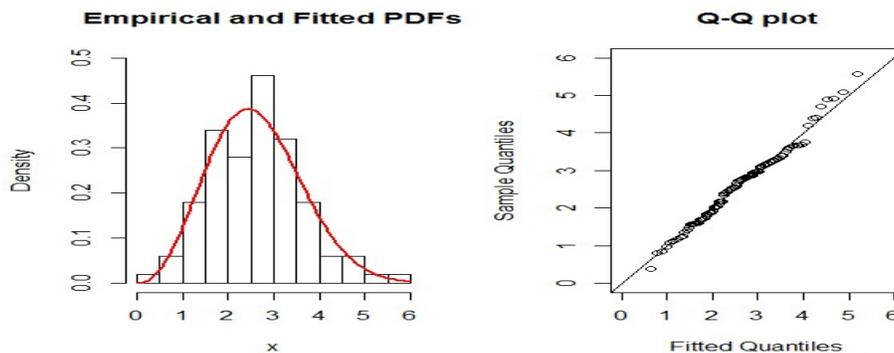


Figure 1. The density plot and Q-Q plot of the fitted OGELLD for the strength data

Let us suppose that it is desired to develop the single ASP to satisfy that the 50<sup>th</sup> percentile lifetime is greater than breaking stress of carbon fibres 0.35 through the experiment to be completed by breaking stress of carbon fibres 0.35. Let us fix that the consumer's risk is at 25% when the true 50<sup>th</sup> percentile is breaking stress of carbon fibres 0.35 and the producer's risk is 5% when the true 50<sup>th</sup> percentile is breaking stress of carbon fibres 0.70. Since  $\hat{\lambda} = 10.7592, \hat{\theta} = 2.4083$  and  $\hat{\gamma} = 1.3177$ , the consumer's risk is 25%,  $\delta_q^0 = 1.0$  and  $t_q/t_q^0 = 2$ , the minimum sample size and acceptance number given by  $n =9$  and  $c=2$  from Table 5. Thus, the design can be implemented as follows. Select a sample of 9 breaking stress of carbon fibres, we will accept the lot when no two or more failure occurs before breaking stress of carbon fibres 0.70. According to this proposed plan, the breaking stress of carbon fibres could have been accepted because there is only one failure before the termination time of breaking stress of carbon fibres 0.70.

### 4.3. Comparison of distributions

In Table 6, we compare the plan parameters of the proposed sampling plan with the odds exponential log –logistic distribution (OELLD) which was studied by [24] when  $\beta = 0.05$  and for various levels of  $\delta_q = 1.0, 1.5, 2.0, 2.5$ . The sample size for the OGELLD is smaller as compared to OELLD for 50<sup>th</sup> percentiles.

Table 6. Comparison between OGELLD and OELLD for the combination of  $\lambda = \theta = \gamma = 2 \beta = 0.05$

$t_q/t_q^0$	OGELLD				OELLD			
	$\delta_q = 1.0$	$\delta_q = 1.5$	$\delta_q = 2.0$	$\delta_q = 2.5$	$\delta_q = 1.0$	$\delta_q = 1.5$	$\delta_q = 2.0$	$\delta_q = 2.5$
	n	n	n	n	n	n	n	n
2	11	6	8	10	21	13	11	13
4	5	2	2	2	8	4	4	3
6	5	2	1	1	8	4	3	2
8	5	2	1	1	8	2	3	2

## 5. CONCLUSION

In this manuscript, we established the single ASPs based on the OGELLD percentiles when the life test is truncated at a pre-fixed time. To ensure that the life quality of products exceeds a specified one in terms of the percentile life, the ASPs based on percentiles can be used. We have designed sampling plans based on two-points on the OC curve approach for assuring percentile lifetime of the products. Some tables are provided for practical use in industry and also proposed plan illustrated with real data set. We fitted the proposed OGELLD curve for the above data which is shown in the following graphs. The proposed sampling scheme is illustrated with a real data set and results shows that our methodology performs well as compared with existing sampling plans.

## ACKNOWLEDGEMENTS

The authors are deeply thankful to the editor and reviewers for their valuable suggestions to improve the manuscript.

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