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Deriving the system equations of unbalanced two-phase induction motor

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ABSTRACT

As there is no system driven especially for the two-phase induction motor fed from unbalanced two-phase supply yet, so we start for derivation the system equations for the said motor to be generally used even for the balanced or unbalanced two-phase supply. In this paper, we will derive a system equation starting from the sequence equivalent circuit for the forward and backwards equivalent circuits, then we will re-arrange the equations with some mathematical assumptions which will lead us to the new system equations. first for the voltage equations then for the current equations and finally for both power and torque equations. Moreover, we will put an example which will cover all cases with specific values and relations charts.

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1. INTRODUCTION

When the motor works under a general operating conditions of unbalanced stator voltages or windings, and if we start to derive the system equations, it is necessary to start from the known equivalent circuits which are obtained for both forward and back word components [1]. Using a special analysis which will be required in order to completely describe the two-phase equivalent circuit or the system of the voltage equations for the two-phase motor [2]. Using the relations between the sequence and phase values, to obtain the important system of voltage equations for the case of unbalanced system [3]. After the derivation of the system equations we can learn the performance of the unbalanced two-phase electrical motor, using the symmetrical components representation for both voltages and currents [4]. These system equations will be used for all cases for the two-phase induction motor which fed from two-phase power supply under all conditions [5].

2. RESEARCH METHOD

2.1. Equivalent circuits of forward and backward components

From the usual equivalent circuits of forward and backward components, the following equations for both components are shown in Figure 1. From the figure, we can express the voltage equations as the following:

$$V_{sf} = (R_s + j\, x_{LS} + j\, x_m)\, I_{sf} + jx_m\, I_{rf}$$
$$V_{rf} = Jx_m\, I_{sf} + (j\, x_m + R_r / s)\, I_{rf}$$
$$V_{sb} = (R_s + j\, x_{LS} + j\, x_m)\, I_{sb} + j\, x_m\, I_{rb}$$
$$V_{rb} = j\, x_m\, I_{sb} + (j\, x_m + R_r / (2 - s))\, I_{rb}$$

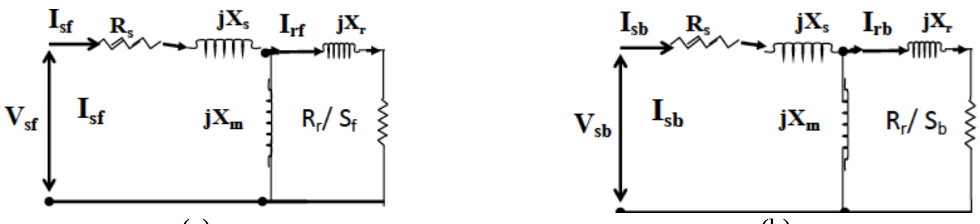

Figure 1. Single-phase equivalent circuits for a two-phase motor under unbalanced condition (a) forward field and (b) backward field

For the above equivalent circuits there are some assumptions

$$Z_m = jX_m \qquad Z_s = R_s + jXs.,$$
$$Z_{sm} = Z_m + Z_s \qquad Z_{mr} = Z_m + jX_r + (R_r / S_f)$$
$$S_f = S, \qquad S_b = (2 - S)$$

The symmetrical components relations for the voltages are

$$V_{sf} = (V_{s1} + j\, V_{s2}) / 2, \quad V_{sb} = (V_{s1} - j\, V_{s2}) / 2,$$
$$V_{rf} = (V_{r1} + j\, V_{r2}) / 2, \quad V_{rb} = (V_{r1} - j\, V_{r2}) / 2$$

and for the currents are

$$I_{sf} = (I_{s1} + j\, I_{s2}) / 2, \quad I_{sb} = (I_{s1} - j\, I_{s2}) / 2,$$
$$I_{rf} = (I_{r1} + j\, I_{r2}) / 2, \quad I_{rb} = (I_{r1} - j\, I_{r2}) / 2.$$

2.2. Solution of the forward and backward currents

Mathematical analysis for derivation of the system equations is to obtain the system equations of the voltages and currents.

2.2.1. For voltage system equations using the symmetrical components

For voltage system equations using the symmetrical components, we use:

$$V_{s1} = (V_{sf} + V_{sb}), \qquad V_{s2} = -j\, (V_{sf} - V_{sb}),$$
$$V_{r1} = (V_{rf} + V_{rb}), \qquad V_{r2} = -j\, (V_{rf} - V_{rb}),$$

The similar relations for the currents are:

$$I_{s1} = (I_{sf} + I_{sb}), \qquad I_{s2} = -j\, (I_{sf} - I_{sb}),$$
$$I_{r1} = (I_{rf} + I_{rb}), \qquad I_{r2} = -j\, (I_{rf} - I_{rb}),$$

From the previous analysis, the above system of equations, can be used to obtain the following general system of equations for the phase voltages of the two-phase induction machine in the general forms.

$$\begin{aligned} V_{s1} &= (V_{sf} + V_{sb}) \\ &= (R_s + j\, x_{LS} + j\, x_m)\, I_{sf} + jx_m\, I_{rf} + (R_s + j\, x_{LS} + j\, x_m)\, I_{sb} + j\, x_m\, I_{rb} \\ &= (R_s + j\, x_{LS} + j\, x_m) \,.\, (I_{sf} + I_{sb}) + jx_m\, (I_{rf} + I_{rb}) \end{aligned}$$

using the previous current equations we get

$$V_{s1} = (R_s + j\, x_{LS} + j\, x_m) \,.\, I_{s1} + jx_m \,.\, I_{r1}$$

By the same manner,

$$\begin{aligned} V_{s2} &= -j\, (V_{sf} - V_{sb}) \\ &= -j\, ((R_s + j\, x_{LS} + j\, x_m)\, I_{sf} + jx_m\, I_{rf} - (R_s + j\, x_{LS} + j\, x_m)\, I_{sb} - j\, x_m\, I_{rb} \\ &= (((R_s + j\, x_{LS} + j\, x_m) \,.\, -j\, (I_{sf} - I_{sb}) + j\, x_m\, (-j\, (I_{rf} - I_{rb})) \\ V_{s2} &= (R_s + j\, x_{LS} + j\, x_m) \,.\, I_{s2} + j\, x_m\, I_{r2} \end{aligned}$$

For the other voltage equations

$$V_{r1} = (V_{rf} + V_{rb})$$
$$V_{rf} = Jx_m I_{sf} + (j x_m + R_r / s) I_{rf}$$
$$V_{rb} = j x_m I_{sb} + (j x_m + (R_r / (2 - s))) I_{rb}$$
$$V_{r1} = jx_m I_{sf} + (j x_m + R_r / s) I_{rf} + j x_m I_{sb} + (j x_m + (R_r / (2 - s))) I_{rb}$$

We will divide the previous equation into two parts to solve them separately then we will collect them again:

$$V_{r1} = [jx_m I_{sf} + j x_m I_{sb.}] + [(j x_m + R_r / s) I_{rf} + (j x_m + (R_r / (2 - s))) I_{rb}]$$

For the first part

$$[jx_m I_{sf} + j x_m I_{sb.}] = jx_m (I_{sf} + I_{sb}) = jx_m I_{s1}$$

For the seconsd part

$$\begin{aligned} &[(j x_m + R_r / s) I_{rf} + (j x_m + (R_r / (2 - s))) I_{rb}] = \\ &= j x_m (I_{rf} + \underline{I}_{rb}) + (R_r / s) I_{rf} + (R_r / (2 - s)) I_{rb} \\ &= j x_m I_{r1} + ((2 - s) R_r / (s (2 - s))) I_{rf} + (sR_r / (s (2 - s))) I_{rb} \\ &= j x_m I_{r1} + [(R_r / (s (2 - s))) . ((2 - s) I_{rf} + s I_{rb})] \\ &= j x_m I_{r1} + (R_r / (s (2 - s))) . [(1 + 1 - s) I_{rf} - (- 1 + 1 - s) I_{rb}] \\ &= j x_m I_{r1} + (R_r / (s (2 - s))) . [I_{rf} + I_{rb} + (1 - s) . (I_{rf} - I_{rb})] \\ &= j x_m I_{r1} + (R_r / (s (2 - s))) . [I_{r1} + j (1 - s) I_{r2}] \\ &= [j x_m I_{r1} + (R_r / (s (2 - s)))] I_{r1} + j (1 - s) [(R_r / (s (2 - s))) . I_{r2} \end{aligned}$$

Then for both parts

$$V_{r1} = jx_m I_{s1} + [j x_m + (R_r / (s (2 - s)))] I_{r1} + j (1 - s) [(R_r / (s (2 - s))) . I_{r2}$$

for the following equation

$$V_{r2} = - j (V_{rf} - V_{rb}),$$
$$V_{rf} = jx_m I_{sf} + (j x_m + R_r / s) I_{rf}, \quad V_{rb} = j x_m I_{sb} + (j x_m + (R_r / (2 - s))) I_{rb}$$

as the same for the previous equation of the V_{r1}

$$\begin{aligned} V_{r2} &= - j [jx_m I_{sf} + (j x_m + R_r / s) I_{rf} - j x_m I_{sb} - (j x_m + (R_r / (2 - s))) I_{rb} \\ &= - j [jx_{m\,.} (I_{sf} - I_{sb}) + j x_{m\,.} (I_{rf} - I_{rb}) + R_r / s . I_{rf} - (R_r / (2 - s)) I_{rb} \\ &= - j [(- x_{m\,.} I_{s2}) - (x_{m\,.} I_{r2}) + [R_r / s . I_{rf} - (R_r / (2 - s)) I_{rb}]] \end{aligned}$$

for the second part of the equation

$$\begin{aligned} &R_r / s . I_{rf} - (R_r / (2 - s)) I_{rb}] \\ &= ((2 - s) R_r / (s (2 - s))) I_{rf} - (sR_r / (s (2 - s))) I_{rb} \\ &= (R_r / (s (2 - s))) . ((2 - s) I_{rf} - s I_{rb}) \\ &= (R_r / (s (2 - s))) . [(1 + 1 - s) I_{rf} + (- 1 + 1 - s) I_{rb}] \\ &= (R_r / (s (2 - s))) . [j (- j) (I_{rf} - I_{rb}) + (1 - s) . (I_{rf} + I_{rb})] \\ &= (R_r / (s (2 - s))) [j I_{r2} + j (1 - s) . I_{r1}] \end{aligned}$$

Then for both parts

$$\begin{aligned} j V_{r2} = &[(1 - s) . R_r / (s (2 - s))) . I_{r1} + (R_r / (s (2 - s)) + j x_m) . I_{r1} \\ &+ j (R_{r\,.} (1 - s) / (s (2 - s))) . I_{r2} \end{aligned}$$

Then we drive the following system equations for voltages

$$\begin{aligned} V_{s1} &= (R_s + j x_{LS} + j x_m) . I_{s1} + jx_{m\,.} I_{r1} \\ j V_{s2} &= (R_s + j x_{LS} + j x_m) . j I_{s2} + j x_m j I_{r2} \\ V_{r1} &= jx_m I_{s1} + [j x_m + (R_r / (s (2 - s)))] I_{r1} + j (1 - s) [(R_r / (s (2 - s))) . I_{r2} \\ j V_{r2} &= [(1 - s) . R_r / (s (2 - s))) . I_{r1} + (R_r / (s (2 - s)) + j x_m) . I_{r1} \\ &\quad + j (R_{r\,.} (1 - s) / (s (2 - s))) . I_{r2} \end{aligned}$$

2.2.2. For current system equations

Using the previous equations for the single-phase equivalent circuits will give as the following:

$$V_{sf} = Z_{sm} . I_{sf} - Z_m . I_{rf}$$
$$V_{rf} = Z_{mr} . I_{rf} - Z_m . I_{sf}$$
$$I_{rf} = (Z_{sm} . I_{sf}) / Z_m - V_{sf} / Z_m$$

Also, according to the relation between current, impedance and voltage

$$I_{rf} = (Z_m / Z_{mr}) . I_{sf}$$

Then,

$$I_{rf} = (Z_m / Z_{mr}) . I_{sf} = (Z_{sm} . I_{sf}) / Z_m - V_{sf} / Z_m$$
$$V_{sf} / Z_m = [(Z_{sm} / Z_m) - (Z_m / Z_{mr})] . I_{sf}$$
$$I_{sf} = [Z_{mr} / ((Z_{sm} . Z_{mr}) - Z_m^{2})] . V_{sf}$$

Now we get the first current equation as following

$$I_{sf} = [Z_{mr} / ((Z_{sm} . Z_{mr}) - Z_m^{2})] . V_{sf}$$
$$I_{sf} = [(Z_m + jX_r + (R_r / S_f)) / (((Z_m + Z_s) . (Z_m + jX_r + (R_r / S_f))) - Z_m^{2})] . V_{sf}$$

Using the following equation

$$V_{rf} = Z_{mr} . I_{rf} - Z_m . I_{sf} = 0$$

Then we get

$$I_{rf} = [Z_m / ((Z_{sm} . Z_{mr}) - Z_m^{2})] . V_{sf}$$
$$I_{rf} = [Z_m / (((Z_m + Z_s) . (Z_m + jX_r + (R_r / S_f))) - Z_m^{2})] . V_{sf}$$

Then we can express for the forward as the following

$$I_{sf} = [(Z_m + jX_r + (R_r / S_f)) / (((Z_m + Z_s) . (Z_m + jX_r + (R_r / S_f))) - Z_m^{2})] . V_{sf}$$
$$I_{rf} = [Z_m / (((Z_m + Z_s) . (Z_m + jX_r + (R_r / S_f))) - Z_m^{2})] . V_{sf}$$

For the backwards,

$$I_{sb} = [(Z_m + jX_r + (R_r / S_b)) / (((Z_m + Z_s) . (Z_m + jX_r + (R_r / S_b))) - Z_m^{2})] . V_{sb}$$
$$I_{rb} = [Z_m / (((Z_m + Z_s) . (Z_m + jX_r + (R_r / S_b))) - Z_m^{2})] . V_{sb}$$

To obtain the air gab power equation

$$I_{rf} = (I_{r1} + j I_{r2}) / 2, \qquad I_{rb} = (I_{r1} - j I_{r2}) / 2$$

For the P_{gf} we use the expression

$$P_{gf} = m_s I_{rf} I_{rf}^{*} R_r / s_f,$$
$$2 I_{rf} = I_{r1r} + j I_{r1i} + j I_{r2r} - I_{r2i}$$
$$= (I_{r1r} - I_{r2i}) + j (I_{r1i} + I_{r2r})$$

Then

$$2 I_{rf} = (I_{r1r} - I_{r2i}) + j (I_{r1i} + I_{r2r}), \quad 2 I_{rf}^{*} = (I_{r1r} - I_{r2i}) - j (I_{r1i} + I_{r2r})$$
$$P_{gf} = m_s I_{rf} I_{rf}^{*} R_r / s_f$$
$$= [((m_s . R_r) / 4 s) . (((I_{r1r} - I_{r2i}) + j (I_{r1i} + I_{r2r})) . ((I_{r1r} - I_{r2i}) - j (I_{r1i} + I_{r2r}))$$
$$= [((m_s . R_r) / 4 s) . ((I_{r1r} - I_{r2i})^{2} + (I_{r1i} + I_{r2r})^{2})$$
$$= [((m_s . R_r) / 4 s) . (I_{r1r}^{2} - 2 I_{r1r} . I_{r2i} + I_{r2i}^{2} + I_{r1i}^{2} + 2 I_{r1i} . I_{r2r} + I_{r2r}^{2}))]$$

And for, I_{rb}

$$P_{gb} = m_s I_{rb} I_{rb}^{*} R_r / s_b,$$
$$I_{rb} = (I_{r1} - j I_{r2}) / 2.$$

With the same manner

$$2\,I_{rb} = I_{r1r} + j\,I_{r1i} - j\,I_{r2r} + I_{r2i}$$
$$= (I_{r1r} + I_{r2i}) + j\,(I_{r1i} - I_{r2r})$$

then:

$$2\,I_{rb} = (I_{r1r} + I_{r2i}) + j\,(I_{r1i} - I_{r2r}), \quad 2\,I_{rb}^{*} = (I_{r1r} + I_{r2i}) - j\,(I_{r1i} - I_{r2r})$$
$$P_{gb} = m_s\, I_{rb}\, I_{rb}^{*} R_r / s_b,$$
$$= [((m_s\,.\,R_r)/4\,s_b)\,.\,((I_{r1r} + I_{r2i}) + j\,(I_{r1i} - I_{r2r}))\,.\,((I_{r1r} + I_{r2i}) - j\,(I_{r1i} - I_{r2r}))$$
$$= [((m_s\,.\,R_r)/4\,s_b)\,.\,((I_{r1r} + I_{r2i})^2 + ((I_{r1i} - I_{r2r})^2)$$
$$= [((m_s\,.\,R_r)/4\,s_b)\,.\,(I_{r1r}^2 + 2\,I_{r1r}\,.\,I_{r2i} + I_{r2i}^2 + I_{r1i}^2 - 2\,I_{r1i}\,.\,I_{r2r} + I_{r2r}^2))]$$

where

$$P_g = P_{gf} - P_{gb}$$
$$= [((m_s\,.\,R_r)/4\,s)\,.\,(I_{r1r}^2 - 2\,I_{r1r}\,.\,I_{r2i} + I_{r2i}^2 + I_{r1i}^2 + 2\,I_{r1i}\,.\,I_{r2r} + I_{r2r}^2))] -$$
$$[((m_s\,.\,R_r)/4\,s_b)\,.\,(I_{r1r}^2 + 2\,I_{r1r}\,.\,I_{r2i} + I_{r2i}^2 + I_{r1i}^2 - 2\,I_{r1i}\,.\,I_{r2r} + I_{r2r}^2))]$$
$$= [((m_s\,.\,R_r)/(2\,s\,.\,(2 - s)))\,.\,(I_{r1r}^2 + I_{r2i}^2 + I_{r2r}^2 + (1 - s)\,.\,(2\,I_{r1r}\,.\,I_{r2i} + 2\,I_{r1i}\,.\,I_{r2r} + I_{r1i}^2))$$

For the power equations

$$P_{gf} = m_s\, I_{rf}\, I_{rf}^{*} R_r / s_f,$$
$$P_{gb} = m_s\, I_{rb}\, I_{rb}^{*} R_r / s_b,$$

where, s_f = s and s_b = 2-s

$$P_g = P_{gf} - P_{gb}$$
$$Pm = Pg\,.\,(1 - s)$$
$$T = Pg / \omega_s$$

where ωs = ω / p, with p = number of pole pair.

2.3. Performance of two-phase motor under unbalanced supply

Case study for different values of the phase angle of the second voltage source uses Matlab software for simulation the results according to different values of the voltage angle values (90° 60° 30° 15° 0°) and other assumptions as the following

```
V=220 v ac, f=50 Hz, p=2, w=2.*pi.*f,ws=w./p,ms=2,Vs1=v,
xm=40 Ω,Zm=j.*xm Ω,rs=2 Ω,Xs=2 Ω,Xr=2 Ω,rr=2 Ω,
no=1.0,Bv2=(pi./2.).*1./no,vs2=v.*((cos(Bv2))-(j.*sin(Bv2))),
s=0.00:0.001:1,
sf=s,sb=2.-s
```

From the previous values and using Matlab we can get the result as in Figures 2–5.

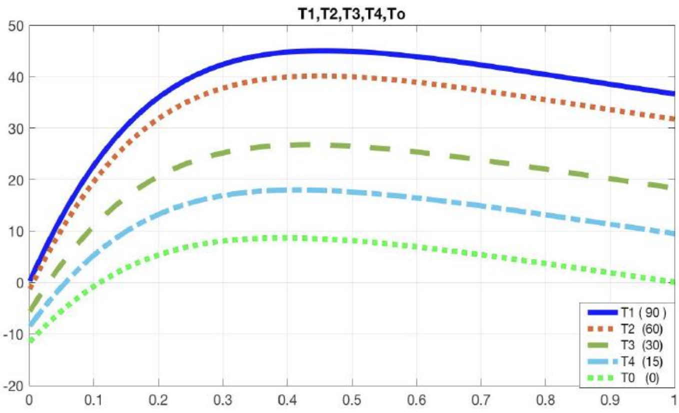

Figure 2. The control of the torque

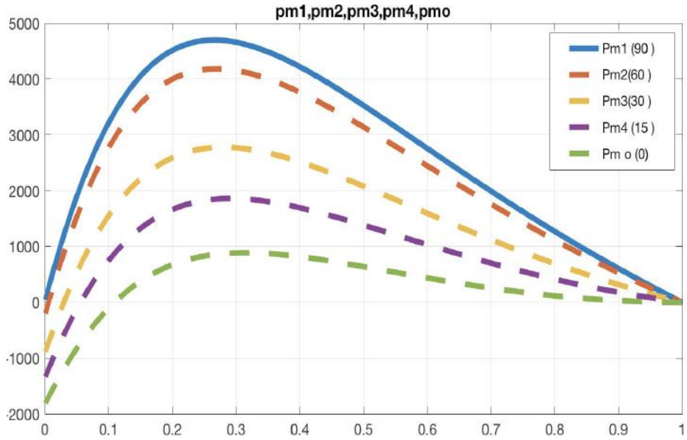

Figure 3. The control of Pm

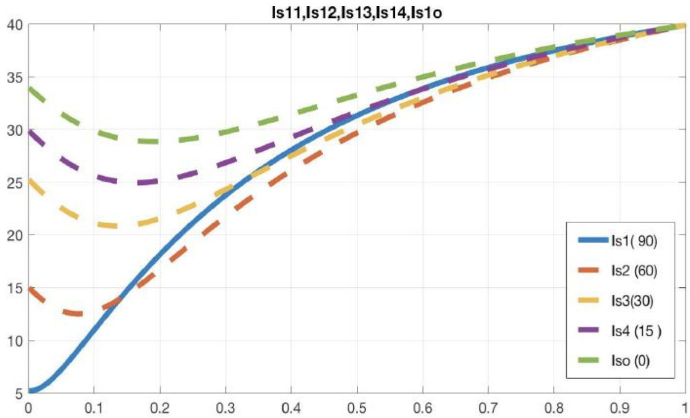

Figure 4. The control of ls1

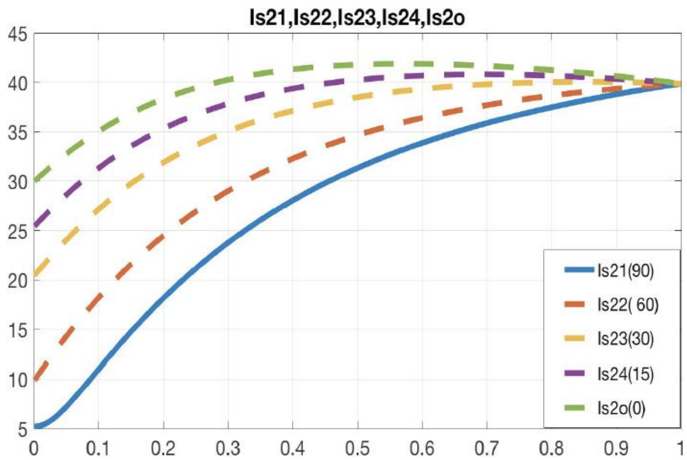

Figure 5. The control of ls2

3. RESULTS AND DISCUSSION

The target achieved, as we derive a system equations of unbalanced two-phase induction motor could be applied for any running conditions of the motor, even it runs in balanced or un-balanced conditions according to the following equations:

$$V_{s1} = (R_s + j\, x_{LS} + j\, x_m) \,.\, I_{s1} + jx_m \,.\, I_{r1}$$
$$V_{s2} = (R_s + j\, x_{LS} + j\, x_m) \,.\, I_{s2} + j\, x_m\, I_{r2}$$
$$V_{r1} = jx_m\, I_{s1} + [\, j\, x_m + (R_r / (s (2 - s)))]\, I_{r1} + j (1 - s) [(R_r / (s (2 - s))) \,.\, I_{r2}$$
$$j\, V_{r2} = [(1 - s) \,.\, R_r / (s (2 - s))) \,.\, I_{r1} + (R_r / (s (2 - s)) + j\, x_m) \,.\, I_{r1} + j (R_r \,.\, (1 - s) / (s (2 - s))) \,.\, I_{r2}$$
$$I_{sf} = [(Z_m + jX_r + (R_r / S_f)) / (((Z_m + Z_s) \,.\, (Z_m + jX_r + (R_r / S_f))) - Z_m^2)] \,.\, V_{sf}$$
$$I_{rf} = [\, Z_m / (((Z_m + Z_s) \,.\, (Z_m + jX_r + (R_r / S_f))) - Z_m^{\,2})] \,.\, V_{sf}$$
$$I_{sb} = [(Z_m + jX_r + (R_r / S_b)) / (((Z_m + Z_s) \,.\, (Z_m + jX_r + (R_r / S_b))) - Z_m^2)] \,.\, V_{sb}$$
$$I_{rb} = [\, Z_m / (((Z_m + Z_s) \,.\, (Z_m + jX_r + (R_r / S_b))) - Z_m^{\,2})] \,.\, V_{sb}$$
$$I_{rf} = (I_{r1} + j\, I_{r2}) / 2, \qquad I_{rb} = (I_{r1} - j\, I_{r2}) / 2.$$
$$P_{gf} = m_s\, I_{rf}\, I_{rf}{}^{*} R_r / s_f,$$
$$P_{gb} = m_s\, I_{rb}\, I_{rb}{}^{*} R_r / s_b,$$
$$P_g = [((m_s \,.\, R_r) / (2\, s \,.\, (2 - s))) \,.\, (I_{r1r}^2 + I_{r2i}^2 + I_{r2r}^2 + (1 - s) \,.\, (2\, I_{r1r} \,.\, I_{r2i} + 2\, I_{r1i} \,.\, I_{r2r} + I_{r1i}^2))$$
$$Pm = Pg \,.\, (1 - s)$$
$$T = Pg / \omega_s$$

4. CONCLUSION

As there is no system driven especially for the two-phase induction motor fed from unbalanced two-phase supply, so we start for derivation the system equations for the said motor to be generally used even for the balanced or unbalanced two-phase supply. In this paper, we derive a system equation starting from the sequence equivalent circuit for the forward and backwards equivalent circuits, then we rearrange the equations with some mathematical assumptions which lead us to the new system equations. first for the voltage equations then for the current equations and finally for both power and torque equations. Moreover, we put an example which cover all cases with specific values and relations charts.

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