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Discrete time reaching law based variable structure control for fast reaching with reduced chattering

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ABSTRACT

In this paper, a variable structure control law is proposed for discrete time sliding mode control so as to reduce both reaching time and quasi sliding mode band reduction. This new law is composed of two different sliding variable dynamics; one to achieve fast reaching and the other to counter its effect on widening the quasi sliding mode band. This is accomplished by introducing a boundary layer around the sliding surface about which the transformation of the sliding variable dynamics takes place. This provides the flexibility to choose the initial dynamics in such a way as to speed up the reaching phase and then at the boundary transform this dynamics to one that reduces the quasi sliding mode band. Thus, the law effectively coalesces the advantageous traits of hitherto proposed reaching laws that succeed in either the reduction of reaching phase or the elimination of quasi sliding mode band. The effectiveness of the proposed reaching law is validated through simulations.

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1. INTRODUCTION

Sliding Mode Control (SMC), since its inception in Russia in the late 1950s has acquired wide appeal in the area of control engineering [1-3], by virtue of the numerous advantages it offers, the most important of which is the invariance to external disturbances and system uncertainties. There are several approaches to the design of SMC among which the Reaching Law (RL) approach is considered in this paper. In RL based SMC, the desired evolution of the sliding variable is defined beforehand so that finite reaching of the sliding motion is ensured. This approach is advantageous in that it can prescribe the reaching dynamics i.e. the manner in which the system trajectory moves to the sliding surface. The reaching law approach was first proposed for continuous systems [2, 3], but with the widespread use of computers in control techniques, it was only natural that SMC be extended to discrete time systems [4]. The Reaching law approach for Discrete time Sliding Mode Control (DSMC) was first proposed by Gao, Wang and Homaifa [5]. In this paper, this RL is referred to as Gao's Discrete Reaching Law (GDRL). Taking advantage of the flexibility and merits offered by the RL approach, several authors have henceforth put forward modified reaching laws [7-11] that help improve the various attributes of DSMC such as reduction of chattering for reduced Quasi Sliding Mode Band (QSMB), better reaching dynamics, improved settling time etc. While the QSMB width may be decreased by using smaller controller gains, this will in turn prolong the reaching phase. In a similar manner, the use of a higher rate of change of the sliding variable to reduce the reaching time will reflect as increased width of QSMB. This dilemma as stated in [11] was attempted to be solved by incorporating an exponential term in the conventional reaching law proposed by Gao et al. [5]. While this modification [11] did result in faster reaching, the width of the QSMB remained the same as that of Gao's reaching law in [5] because the RL structure in [11] transforms into Gao's RL as the sliding variable approaches zero. In this paper, the above dilemma is solved by combining the faster reaching provided by the RL in [11] and the QSMB reduction offered by the RL proposed by Bartoszewicz et al. in [10] in such a way that there occurs a change in the structure of the system during the reaching phase. Initially, for large values of the sliding variable, the RL with the exponential term is ideal (for fast reaching) and for smaller values, the asymptotic convergence of RL in [10] is most suitable (for reduced QSMB). The proposed RL is so structured that each of the above laws comes in to play depending on the value of the sliding variable. This is accomplished by the concept of a boundary layer around the sliding surface where the RL undergoes a transformation from the fast paced reaching provided by the RL in [11] (referred as Novel Exponential Reaching Law (NERL)) to an asymptotic convergence of the sliding variable to the sliding surface as per [10] (being referred to as Bartoszewicz Latosi´nski Discrete-time Reaching Law (BLDRL) in this paper).

This paper is structured as follows. In section 2, a brief review of NERL and BLDRL is given and based on these, the proposed RL is formulated for nominal and perturbed systems. In section 3, the simulation results obtained for the proposed RL law are given together with a comparison between the various reaching laws, viz., GDRL, NERL and BLDRL. Concluding remarks are given in Section 4.

2. PROPOSED CONTROL STRATEGY

2.1. Brief review of NERL and BLDRL

Consider the following discrete linear system:

$$x(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{b}u(k) \tag{1}$$

where $\boldsymbol{A}$ is an $n \times n$ matrix, $\boldsymbol{x}(k)$ is the state vector, $\boldsymbol{b}$ is the input vector and $u(k)$ is the scalar control signal. The sliding surface is defined as:

$$s(k) = \boldsymbol{c}^T \boldsymbol{e}(k) = 0 \tag{2}$$

where $\boldsymbol{e}(k) = \boldsymbol{x}(k) - \boldsymbol{x}_d(k)$ is the error vector and $\boldsymbol{c}^T$ is the sliding coefficient vector that is so designed to yield stable dynamics when the system is in Sliding Mode Motion (SMM). $\boldsymbol{x}_d(k)$ is the desired value of the state vector.

The seminal RL proposed by Gao et al. in 1995 [5] takes the following form:

$$s(k+1) = (1 - qT)s(k) - \varepsilon T \text{sgn}[s(k)] \tag{3}$$

where q is the proportional term, ε is the constant term with $1 - qT > 0$ and $\varepsilon > 0$ and T is the sampling period. While this RL remains popular to this date, this law causes the system representative point (tip of the system trajectory and/or sliding variable) to cross the sliding surface at some finite time and then move in a perpetually zigzag manner around the surface. This motion, in contrast to the ideal SMM, was coined as the Quasi Sliding Mode (QSM) motion around the sliding surface [5]. Due to the QSM, more control effort is expended in the sliding mode. Hence, it is desirable to keep the width of the QSMB as minimum as possible. The width of QSMB is directly affected by the constant term ε in (3) and it can be reduced by the use of a smaller value of ε. However, this will in turn reduce the rate of change of the sliding variable leading to increased reaching phase. Since it is the SMM that offers robustness to system performance, increased reaching phase is undesirable. On the other hand, if the term q, that affects the reaching time, is increased, the reaching time is reduced, but causes an increase in the width of the QSMB due to the increased rate of change of the sliding variable near the sliding surface. In order to solve this dilemma, several new RLs have been proposed that use controller gains that adapt to the variations in the sliding variable. NERL, proposed by Ma et al. strives to solve this dilemma by incorporating an exponential term [11] in (3). The NERL for a nominal system is defined as:

$$s(k+1) = (1 - qT)\Phi(k)s(k) - \frac{\varepsilon}{\Phi(k)} \text{sgn}\big(s(k)\big) \tag{4}$$

where

$$\Phi(k) = \delta_0 + (1 - \delta_0)e^{-\varphi|s(k)|^\gamma} \tag{5}$$

$$\begin{cases} 0 < \delta_0 < 1, \ \varphi > 0 \\ \gamma > 0, \quad \text{and } \gamma \in N \end{cases} \tag{6}$$

From (5), it is seen that the value of $\Phi(k)$ is d_0 for large values of the sliding variable, i.e. when the system states are far away from the sliding surface and it takes the value one when systems states are closer to the sliding surface. Since d_0 is a positive value less than one, the first term becomes $(1 - qT)d_0$ which is lesser than $(1 - qT)$ and the second term becomes $\frac{\varepsilon}{d_0}$ which is higher than ε. This causes a faster rate of change of the sliding variable at the beginning as compared to (3) (T=1 sec is assumed). But, as the system representative points (RP) move closer to the sliding surface the reaching law (4) transforms in to GDRL as in (3). Thus, the width of the QSMB is bound to remain the same as that of GDRL, but the reaching time is reduced.

BLDRL in [10] offered reduced QSMB owing to a bounded rate of convergence of the sliding variable to zero. This RL acts as a compromise between the switching type RLs, where sliding variable crosses the sliding surface in each discrete instant in the QSM, and the non switching type RLs that retain the sliding variable in a specified band around the sliding surface [10]. Whether the sliding variable crosses the sliding surface or is retained in a band near the surface, the distance of the system RP from the sliding surface decreases with each step when the system is in QSM, making it the ideal reaching law near $s = 0$. The BLDRL is given by:

$$s(k+1) = f_m[s(k)](s(k) - \varepsilon \text{sgn}[s(k)]) \tag{7}$$

where

$$f_m[s(k)] = 1 - exp\{-[s(k)/\varepsilon]^{2m}\}$$

ε is a positive real number and $m \in N$. The value of f_m for large values of $s(k)$ is one, thereby causing the the sliding variable to decrease at an almost constant rate, ε. But, closer to the sliding surface the value of f_m decreases and tends to zero asymptotically. As a result, around the sliding surface, if $s(k)$ reaches exactly on the surface $s(k) = 0$, it will remain there from then on. Otherwise, the value of $s(k)$ keeps decreasing with each step. Thus, the control effort during sliding mode is greatly reduced as compared to NERL and GDRL. However, the reaching time is more than that provided by the modified constant term in (4). The proposed RL successfully utilizes the aforementioned merits of both NERL and BLDRL.

2.2. Proposed reaching law for nominal systems

Keeping in consideration both reduced reaching time and reduced QSMB width, it is desirable that NERL be used for large values of the sliding variable (initial phase of reaching), thereby providing fast rate of change of sliding variable at the beginning and BLDRL be used as the system RP inches closer to the sliding surface, resulting in smaller QSMB. This is accomplished by introducing a boundary layer around the sliding surface, say $s(k) = \pm g$, where g is a small positive value lesser than the initial value of the sliding variable. For values of $|s(k)| > g$, the controller evolving from NERL is employed and for $|s(k)| < g$, BLDRL's controller is used. Hence, it is essential to have a RL that will assume the form in (4) until $s(k) = \pm g$ and beyond that it should transform to (7). This is achieved by the proposed RL with the structure:

$$\begin{aligned} s(k+1) = {} & f_m[s(k)](s(k) - \varepsilon \text{sgn}[s(k)])G_1[s(k)] \\ & + \left((1 - qT)\Phi(k)s(k) - \frac{\varepsilon}{\Phi(k)} \text{sgn}[s(k)]\right) G_2[s(k)] \end{aligned} \tag{8}$$

where

$$G_1[s(k)] = 0.5\big(\text{sgn}[s(k)](\text{sgn}[s(k) + g] - \text{sgn}[s(k) - g])\big)$$
$$G_2[s(k)] = 0.5\big(\text{sgn}[s(k)](\text{sgn}[s(k) + g] + \text{sgn}[s(k) - g])\big)$$

From the above expressions, it can be seen that for values of $|s(k)| > g$, G_1 is zero and G_2 is one and (8) transforms into NERL given by (4). In a similar manner, for $|s(k)| < g$, G_1 becomes one and G_2 is equal to zero, thus causing (8) to metamorphose into (7). Thus, initially for large values of sliding variable, faster rate of change is obtained and near the sliding surface, a constant rate of change followed by an asymptotic convergence is achieved. This results in much faster reaching as compared to GDRL and BLDRL. The reaching time is comparable to that of NERL, but the QSMB is reduced to a great extent which is not possible with NERL. In short, it combines the benefits of NERL and BLDRL, giving a reaching law that can successfully tackle the reaching time/QSMB dilemma mentioned earlier.

2.2.1. Reachability of the proposed reaching law

Assume that $s(k) > 0$ with $s(k) > g$. From (8), we can write:

$$s(k+1) = (1-qT)\Phi(k)s(k) - \frac{\varepsilon}{\Phi(k)} \tag{9}$$

which can be rewritten as:

$$s(k+1) - s(k) = -qT\Phi(k)s(k) - \frac{\varepsilon}{\Phi(k)} \tag{10}$$

We have $q > 0$, $\varepsilon > 0$ and $\Phi(k) > 0$. Thus, the RHS of (10) is negative whereby with $s(k) > 0$, it follows that $s(k+1) < s(k)$. Similarly, it can be seen that for $s(k) < 0$ and $s(k) < -g$, the sliding variable value decreases in each discrete step. It can thus be concluded that outside the boundary layer, $s(k+1) < s(k)$. Once the sliding variable enters the boundary layer, $s(k) > 0$ and $s(k) < g$ or $s(k) < 0$ and $s(k) > -g$. Let us first consider the case when $s(k) > 0$ and $s(k) < g$. From (8),

$$s(k+1) = f_m[s(k)](s(k) - \varepsilon) \tag{11}$$

The sliding surface is crossed when $s(k+1) < 0$. Thus,

$$f_m[s(k)](s(k) - \varepsilon) < 0 \tag{12}$$

Since $f_m[s(k)]$ is always positive,

$$s(k) - \varepsilon < 0 \tag{13}$$

From (13), it can be concluded that the sliding surface will be crossed when $s(k)$ becomes less than ε. In a similar manner, this can be proved for the case when $s(k) < 0$. In general, we can write if $|s(k)| < \varepsilon$, then $sgn[s(k+1)] = -sgn[s(k+1)]$. On the other hand, if $|s(k)| > \varepsilon$, $s(k+1) < s(k)$. Thus, we can conclude that the value of the sliding variable keeps decreasing outside the boundary as well as inside the boundary. Once the value of $s(k)$ becomes less than ε, it crosses the sliding surface. This is ensured due to the steady decrease in the value of sliding variable because of which at some point $s(k) < \varepsilon$. Thus, it is ensured that the sliding surface is reached in finite time.

2.2.2. Quasi-sliding mode of the proposed reaching law

Once the sliding variable reaches the vicinity of the sliding surface $s(k) = 0$, the function f_m begins to drop. As a result, in the quasi sliding mode band, instead of maintaining the same value as is seen with NERL and GDRL, the sliding variable value drops in each subsequent discrete instant.

$$s(k+1) = f_m[s(k)](s(k) - \varepsilon) \tag{14}$$

where $f_m < 1$ in the near vicinity of $s(k) = 0$. Whether the sliding variable switches in every instant or not, it is assured that the value of $s(k)$ reduces in each sampling time in QSM. Thus, it reduces the width of the QSMB to a negligibly small value.

2.3. Proposed reaching law for perturbed systems

Consider the following perturbed system:

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{b}u(k) + \boldsymbol{f}(k) \tag{15}$$

where $\boldsymbol{x}$ is the state vector, $\boldsymbol{A}$ is an $n \times n$ matrix and $\boldsymbol{b}$ is the input vector. $f(k)$ is the perturbation term, whose upper and lower bounds are assumed to be known. In order to satisfy the matching conditions, we can write $\boldsymbol{f} = \boldsymbol{b}\tilde{f}$ [5]. Equation (9) becomes

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{b}\big(u(k) + \tilde{f}(k)\big) \tag{16}$$

Now, assuming the control law to be generated from the same RL (8) for the system (16), we obtain $u(k)$ as:

$$u(k) = (\boldsymbol{c}^T\boldsymbol{b})^{-1}\Big\{f_m[s(k)](s(k) - \varepsilon\text{sgn}[s(k)])G_1\big(s(k)\big) + \left((1-qT)\Phi(k)s(k) - \frac{\varepsilon}{\Phi(k)}\text{sgn}[s(k)]\right)G_2\big(s(k)\big) - \boldsymbol{c}^T\boldsymbol{A}\boldsymbol{x}(k) - \tilde{f}(k)\Big\} \tag{17}$$

Since $\tilde{f}(k)$ is not usually known, this term has to be adequately dealt with. This term is hence replaced by another term $f_c(k)$ in such a way so that the reaching conditions are still satisfied [5].

$$u(k) = (\boldsymbol{c}^T\boldsymbol{b})^{-1}\Big\{f_m[s(k)](s(k) - \varepsilon\text{sgn}[s(k)])G_1\big(s(k)\big) + \left((1-qT)\Phi(k)s(k) - \frac{\varepsilon}{\Phi(k)}\text{sgn}[s(k)]\right)G_2\big(s(k)\big) - \boldsymbol{c}^T\boldsymbol{A}\boldsymbol{x}(k) - f_c(k)\Big\} \tag{18}$$

With the control law as in (18), RL in (8) becomes:

$$s(k+1) = f_m[s(k)](s(k) - \varepsilon\text{sgn}[s(k)])G_1\big(s(k)\big) + \left((1-qT)\Phi(k)s(k) - \frac{\varepsilon}{\Phi(k)}\text{sgn}[s(k)]\right)G_2\big(s(k)\big) - \boldsymbol{c}^T\boldsymbol{b}\big(\tilde{f}(k) - f_c\big) \tag{19}$$

Let $\boldsymbol{c}^T\boldsymbol{b}\big(\tilde{f}(k) - f_c\big) = \tilde{F} - F_c$. Rewriting (19),

$$s(k+1) = f_m[s(k)](s(k) - \varepsilon\text{sgn}[s(k)])G_1\big(s(k)\big) + \left((1-qT)\Phi(k)s(k) - \frac{\varepsilon}{\Phi(k)}\text{sgn}[s(k)]\right)G_2\big(s(k)\big) - \boldsymbol{c}^T\boldsymbol{b}\big(\tilde{F} - F_c\big) \tag{20}$$

Since, it is assumed the upper and lower bounds of $\boldsymbol{f}(k)$ are known, it follows that $\tilde{F}$ is also bounded by known values, say F_U and F_L. In (29), F_c should be so chosen that the reaching conditions are satisfied [5]. Therefore, F_c is taken as:

$$F_c = F_1 + F_2\text{sgn}[s(k)]$$

where $F_1 = \frac{F_U+F_L}{2}$ and $F_2 = \frac{F_U-F_L}{2}$. The proposed reaching law for perturbed systems can hence be formulated as:

$$s(k+1) = f_m[s(k)](s(k) - \varepsilon\text{sgn}[s(k)])G_1\big(s(k)\big) + \left((1-qT)\Phi(k)s(k) - \frac{\varepsilon}{\Phi(k)}\text{sgn}[s(k)]\right)G_2\big(s(k)\big) + \tilde{F}(k) - F_1 - F_2\text{sgn}[s(k)] \tag{21}$$

In the next section, the proposed RLs as in (8) and (19) are simulated on the same system on which Bartosewicz et al. verified BLDRL [10].

3. RESULTS AND ANALYSIS

3.1. Nominal systems

The system as in [10] is being considered for simulating the performance when the system is controlled by the proposed RLs. The matrices/vectors for the system (1) and the initial states as in [10] are given by:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 0.5 \\ 0 & 1 & 1 \\ 0 & -0.5 & 1 \end{bmatrix}$$

$\boldsymbol{b}^T = [0 \quad 0 \quad 1]$ and $\boldsymbol{c}^T = [1 \quad 1.5 \quad 1]$. The initial value of states are also taken as in [10], with $\boldsymbol{x_0}^T = [10 \quad 10 \quad 10]$ and $\boldsymbol{x_d}^T = [0 \quad 0 \quad 0]$.

The control input, $u(k)$ for system (1) with the sliding surface (2) using GDRL (3) can be derived as [5]:

$$u(k) = (\boldsymbol{c}^T\boldsymbol{b})^{-1}\{(1 - qT)\Phi(k)s(k) - \varepsilon T\mathrm{sgn}(s(k)) - \boldsymbol{c}^T\boldsymbol{A}\boldsymbol{x}(k)\} \tag{22}$$

With NERL (4) $u(k)$ turns out to be [11]:

$$u(k) = (\boldsymbol{c}^T\boldsymbol{b})^{-1}\left\{(1 - qT)\Phi(k)s(k) - \frac{\varepsilon}{\Phi(k)}\mathrm{sgn}(s(k)) - \boldsymbol{c}^T\boldsymbol{A}\boldsymbol{x}(k)\right\} \tag{23}$$

and with BLDRL (7), $u(k)$ is [10]:

$$u(k) = (\boldsymbol{c}^T\boldsymbol{b})^{-1}\{f_m[s(k)](s(k) - \varepsilon\mathrm{sgn}[s(k)]) - \boldsymbol{c}^T\boldsymbol{A}\boldsymbol{x}(k)\} \tag{24}$$

With the proposed RL in (8), the control input $u(k)$ is derived as:

$$\begin{aligned} u(k) = (\boldsymbol{c}^T\boldsymbol{b})^{-1}\Bigg\{ & f_m[s(k)](s(k) - \varepsilon\mathrm{sgn}[s(k)])G_1(s(k)) \\ & + \left((1 - qT)\Phi(k)s(k) - \frac{\varepsilon}{\Phi(k)}\mathrm{sgn}[s(k)]\right)G_2(s(k)) \\ & - \boldsymbol{c}^T\boldsymbol{A}\boldsymbol{x}(k)\Bigg\} \end{aligned} \tag{25}$$

The values of the various parameters are taken as q = 0.2, ε = 0.6 , δ_0 = 0.6, φ = 5, γ = 10, m = 1 and g = 2 . The simulation results (using same values for the common RL parameters) are shown in Figures 1-8. Figure 1 depicts the sliding variable of the proposed RL and Figure 2 gives the sliding variables for NERL, BLDRL and GDRL. Figure 4(a) and Figure 6(a) highlight the QSMB in GDRL and NERL respectively and Figure 4(b) and Figure 6(b) reveal the negligible QSMB of BLDRL and the proposed RL. From these figures, it is evident that the proposed RL achieves the best reaching while it embraces the negligible QSMB of BLDRL. The zoomed views are incorporated in Figures 4 and 6 to highlight the QSMB width of the four RLs. Comparing the control inputs shown in Figure 5(a) and 5(b), it is noted that the proposed RL achieves faster reaching with an input comparable to that of NERL. Any attempt to increase the reaching speed of GDRL by providing more control effort (through its RL parameters) will lead to wider QSMB and making this effort on BLDRL will not yield positive results in terms of reaching time because its inherent structure focuses on QSMB reduction preventing fast reaching. Table 1 has the figure of merits in terms of width of QSMB and performance speed. With NERL, the sliding variable enters QSM at 7 sec (see Figure 6(a)), where the QSM is defined as [5]:

$$\{\boldsymbol{x}| -\Delta < s(\boldsymbol{x}) < +\Delta\}$$

In the proposed reaching law, the system RP reaches the sliding surface ($s(k) = 0$) at 5 sec, which is then retained as in BLDRL. However, in BLDRL, the reaching phase is longer. The faster reaching of the proposed RL and NERL demands greater control effort in the reaching phase. However, in NERL, due to the larger QSMB, excessive control effort is dissipated in the sliding phase. The proposed RL emulates BLDRL's trait of retaining the QSMB, thus ensuring that unnecessary dissipation of control energy in sliding phase is avoided. Figures 7 and 8 depict the dynamics of the states of the system. It can be observed that the proposed reaching law allows fast settling of the states as compared to other laws. The proposed RL returns a settling time of 6 sec as compared to 8 sec with NERL.

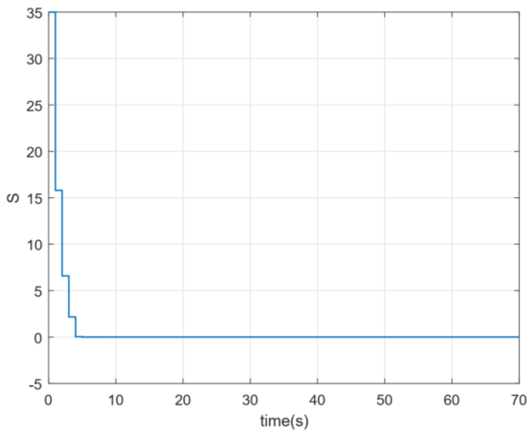


Figure 1. Sliding variable with the Proposed RL

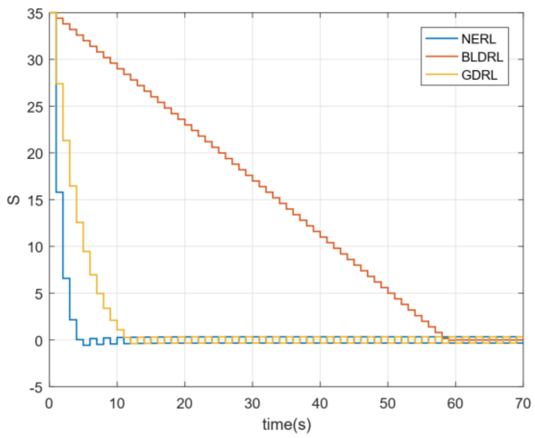
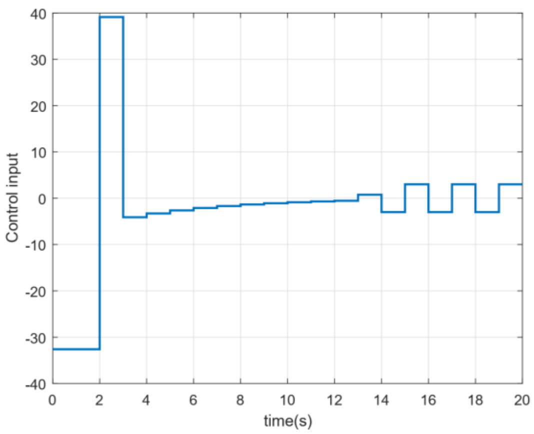
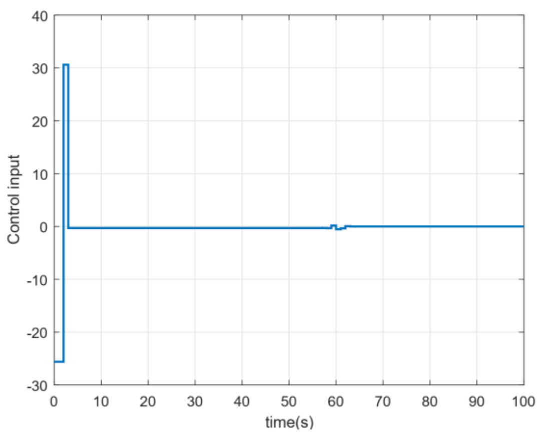


Figure 2. Sliding variables with different RLs

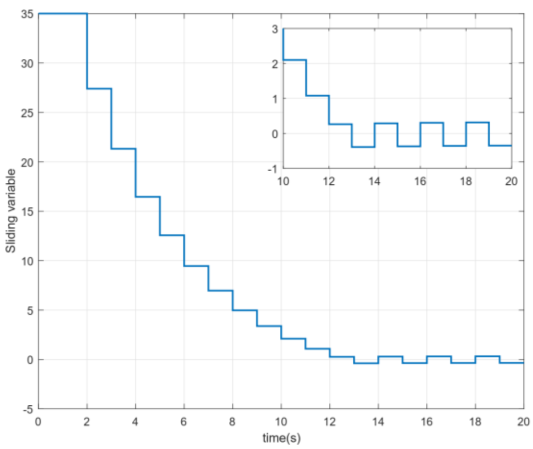


(a) GDRL

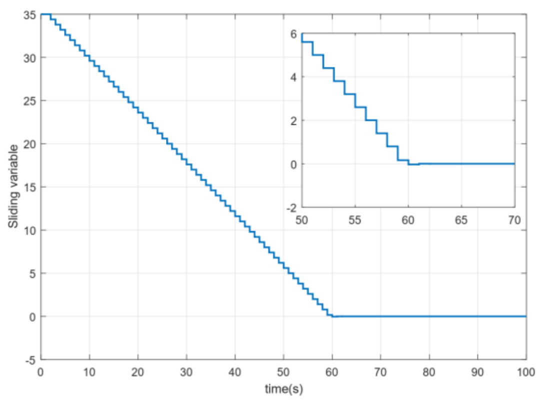


(b) BLDRL

Figure 3. Control input for GDRL and BLDRL

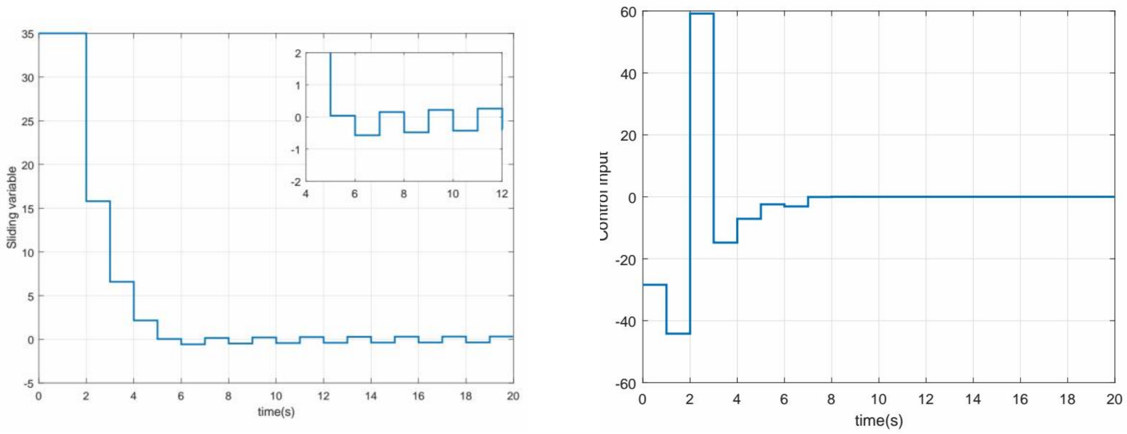


(a) GDRL



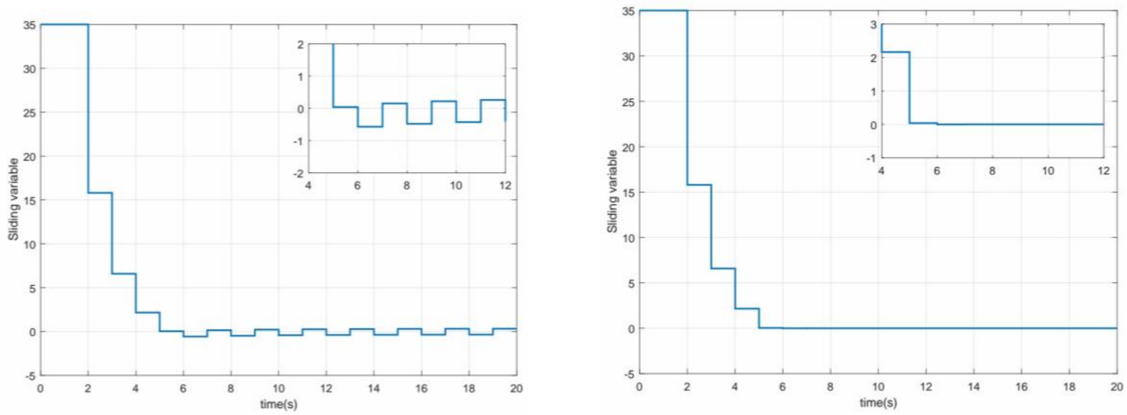
(b) BLDRL

Figure 4. Sliding variable for GDRL and BLDRL



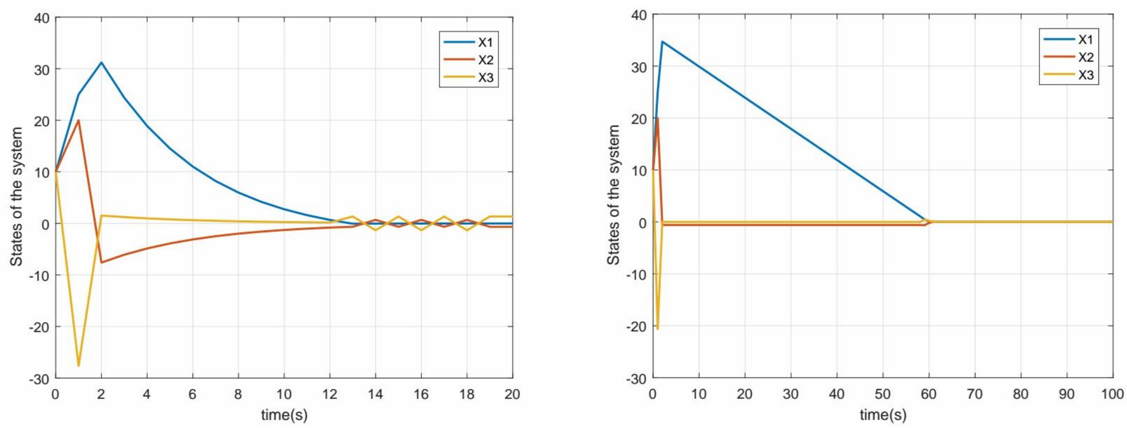
(a) NERL (b) Proposed RL

Figure 5. Control input for NERL and Proposed RL



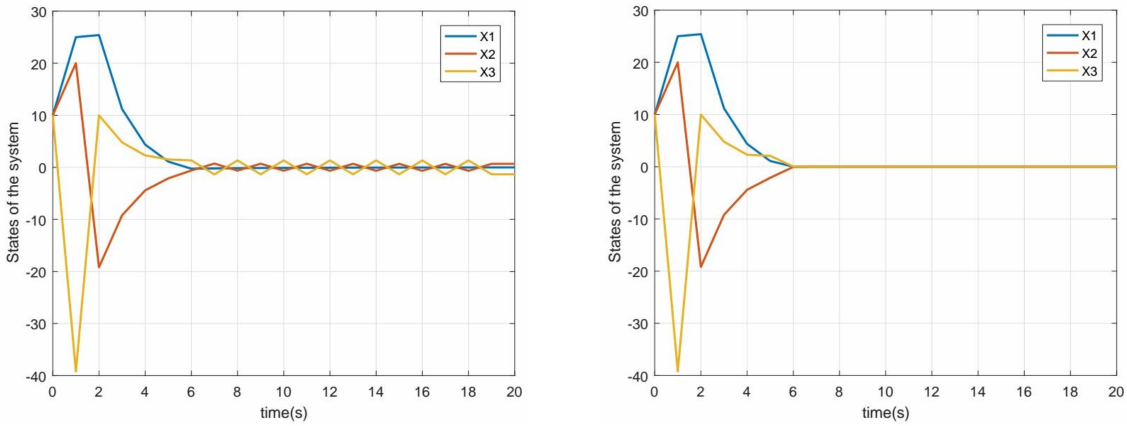
(a) NERL (b) Proposed RL

Figure 6. Sliding variable for NERL and Proposed RL



(a) GDRL (b) BLDRL

Figure 7. States of the system for GDRL and BLDRL



(a) GDRL (b) BLDRL

Figure 8. States of the system for NERL and Proposed RL

Table 1. Figures of merit with nominal system

Comparison of RLs	Proposed	NERL	BLDRL	GDRL
Reaching time	5	7	60	14
Width of QSMB	na	0.6	na	0.6
Settling time	6	8	61	15

3.2. Perturbed systems

From (15), (2) and (21), the control law for perturbed systems is derived as:

$$u(k) = (\boldsymbol{c}^T\boldsymbol{b})^{-1}\Big\{f_m[s(k)](s(k) - \varepsilon\text{sgn}[s(k)])G_1(s(k)) + \left((1 - qT)\Phi(k)s(k) - \frac{\varepsilon}{\Phi(k)}\text{sgn}[s(k)]\right)G_2(s(k)) - \boldsymbol{c}^T\boldsymbol{A}\boldsymbol{x}(k) - F_1 - F_2\text{sgn}[s(k)]\Big\} \quad (26)$$

For the system (15), $\boldsymbol{A}$, $\boldsymbol{b}$, $\boldsymbol{C}^T$, $\boldsymbol{x}(0)$, and $\boldsymbol{x}_d$ are taken to be the same as in the above subsection. $T = 1$ sec. The perturbation term is taken as $f(k) = 0.2(-1)^{[k/10]}$ (as in [10]). Thus, $F_1 = 0$ and $F_2 = 0.2$. The control law proposed by Gao et al. is defined as [5]:

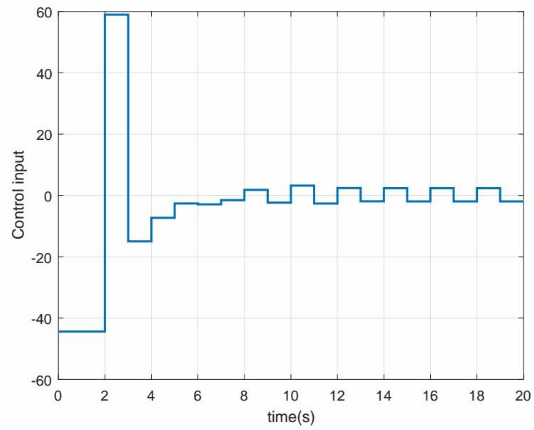
$$u(k) = (\boldsymbol{c}^T\boldsymbol{b})^{-1}\left[(1 - qT)\Phi(k)s(k) - \varepsilon T\text{sgn}(s(k)) - \boldsymbol{c}^T\boldsymbol{A}\boldsymbol{x}(k) - F_1 - F_2\text{sgn}(s(k))\right] \quad (27)$$

The simulation results of the law (26) and GDRL for perturbed systems (27) are shown in Figures 9-12.

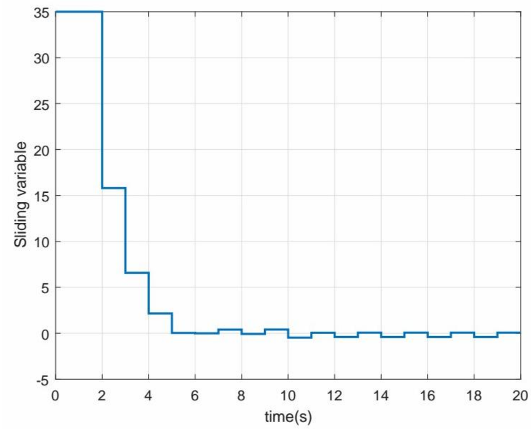
It is observed, as in the case of nominal systems that the proposed reaching law has much faster reaching and less width of QSMB. The reaching time with the proposed law is 7 sec and that with GDRL is 13 sec. It is also observed that the sliding variable is contained in a much smaller band with the proposed RL as compared to GDRL. A zoomed in view of the QMSB with the two reaching laws is shown in Figure 12. Table 2 gives the performance traits of the two RLs.

Table 2. Figures of merit with perturbed system

Comparison of RLs	Proposed	GDRL
Reaching time	7	13
Width of QSMB	0.8	2
Settling time	8	14

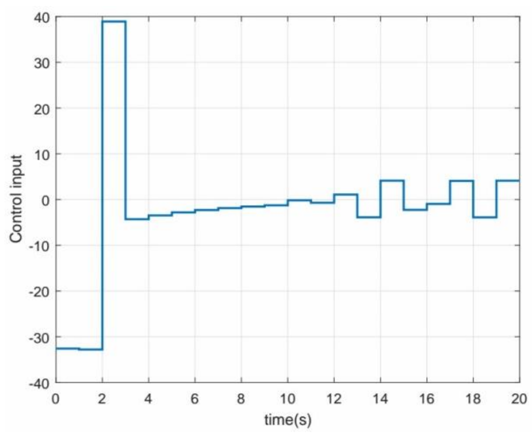


(a) Control input

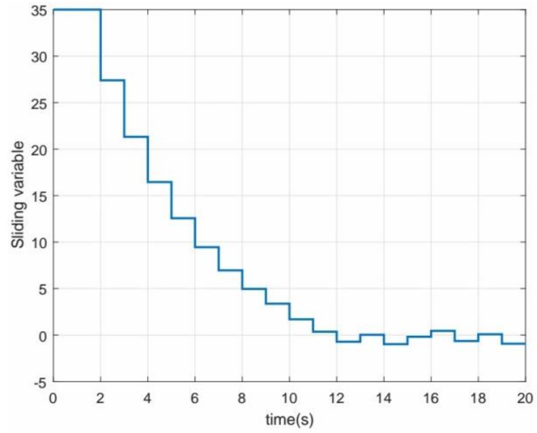


(b) Sliding variable

Figure 9. Control and sliding variable of the proposed RL

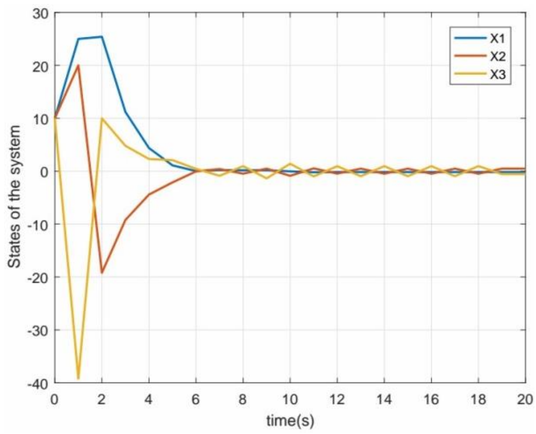


(a) Control input

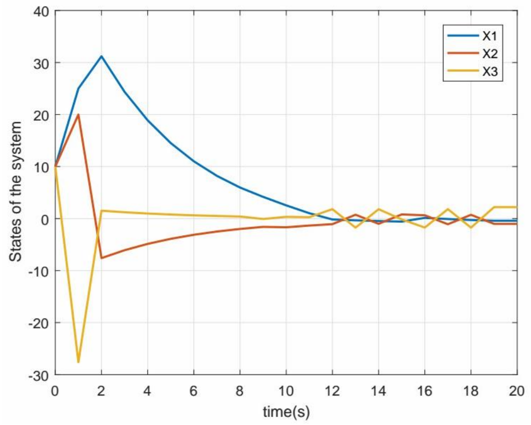


(b) Sliding variable

Figure 10. Control and sliding variable for GDRL

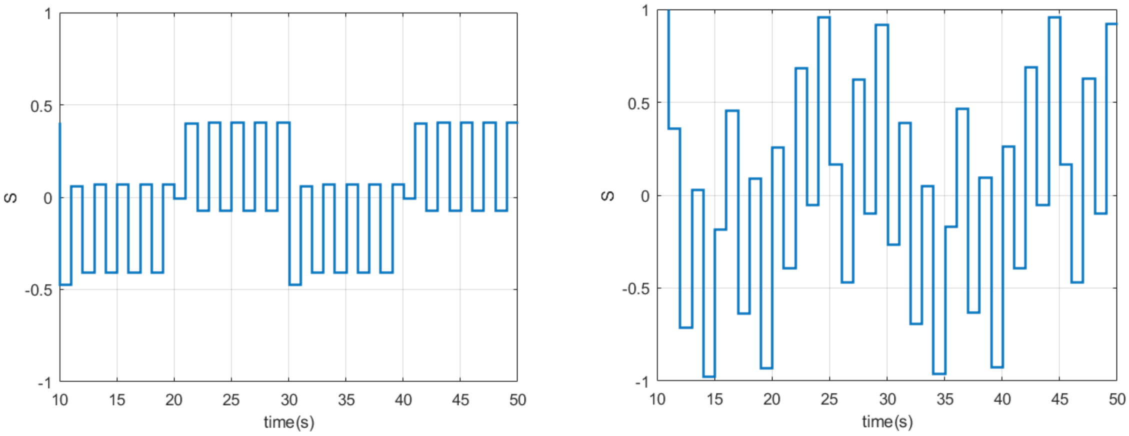


(a) Proposed RL



(b) GDRL

Figure 11. States of the system for the Proposed RL and GDRL



(a) Proposed RL (b) GDRL

Figure 12.

4. CONCLUSION

A new structure for Reaching Law (RL) based sliding mode control has been presented in this paper. This RL structure operates in the true essence of variable structure control system in that it switches the system structure by transforming one RL into another, encompassing the desirable traits of each structure in the form of reduced reaching time and smaller width of Quasi Sliding Mode Band (QSMB). The merits of the proposed RL are substantiated through simulation and are compared to other existing reaching laws. The proposed RL is then modified for application to perturbed systems. Its performance traits are also verified through simulation and are compared with Gao's reaching law for perturbed systems. The proposed RL structure is seen to fetch reduced reaching time and minimal QSMB.

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