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Nonlinear systems identification with discontinuous nonlinearity

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ABSTRACT

In this paper, nonparametric nonlinear systems identification is proposed. The considered system nonlinearity is nonparametric and is of hard type. This latter can be discontinuous and noninvertible. The entire nonlinear system is structured by Hammerstein model. Furthermore, the linear dynamic block is of any order and can be nonparametric. The problem identification method is done within two stages. In the first stage, the system nonlinearity is identified using simple input signals. In the first stage, the linear dynamic block parameters are estimated using periodic signals. The proposed algorithm can be used of large class of nonlinear systems.

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1. INTRODUCTION

Nonlinear systems identification is an active research area in the last decades [1-4]. The actual systems are generally nonlinear in nature [5, 6]. Then, the nonlinear effect can not be neglected. Furthermore, the nonlinear systems parameters vary according to the time, temperature, etc. It is therefore necessary to develop a nonlinear model to take into account the real behavior of process.

Nonlinear systems identification is a necessary step for system control [7-10]. The nonlinear systems structured in blocks are increasingly used. It consists of series connexion of linear and nonlinear blocks [5, 6] and [11, 12]. Nonlinear systems structured in blocks have been widely studied lately [13-17].

One the most used models of these systems are Hammerstein models. The Hammerstein models consist in a nonlinearity element followed by a linear dynamic block (Figure 1). These nonlinear models are widely studied in the last decades [18-22]. Several systems can be practically modelled by Hammerstein models [23, 24].

The identification problem of these nonlinear systems is an important stage for control and stability [18]. The diversity of nonlinear systems and the considerable assumptions on the system has led to a large variety of identification solutions and approaches [9, 10] and [23, 24]. Generally, there exist frequency solutions [23, 24], recursive algorithms [8, 9], hierarchical and filtering approaches [25], subspace identification methods [26].

In this paper, an identification method is proposed to deal in the kind of nonlinear systems structured by Hammerstein models. Compared to most of previous methods, the present work is more general. Indeed, the assumptions imposed on the linear subsystem and the nonlinear blocks are reduced. Presently, the considered Hammerstein model is characterized by nonparametric linear subsystem block. This latter can be of unknown structure and of infinite order.

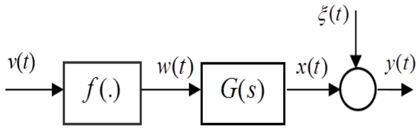
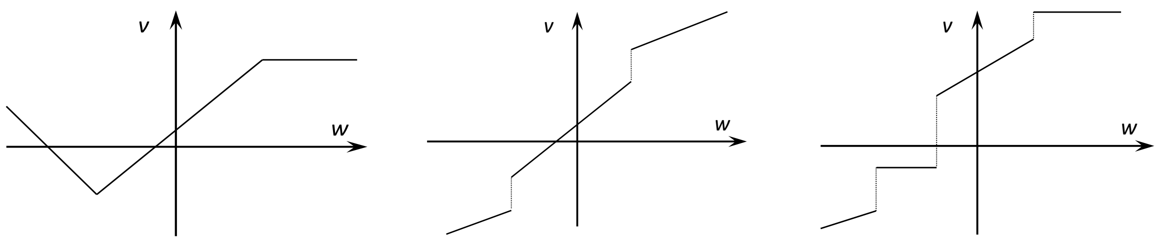


Figure 1. Hammerstein nonlinear system

On the other hand, the considered system nonlinearity *f*(.) is a function of hard type (Figure 2a-c). These examples of nonlinearities are a very attractive research area in the last decades [21]. In fact, it is very difficult to approximate these nonlinearities type by polynomial functions [21].

Furthermore, it is assumed here that, the system nonlinearity can be discontinuous and noninvertible (Figure 2a-c). Finally, in order to identifiability considerations, the nonlinearity *f*(.) contains at least one segment of nonzero slope (Figure 2a-c). Presently, the system nonlinearity *f*(.) can contains saturation (Figure 2a), or discontinuity (Figure 2b), as it can contains, simultaneously, saturation and discontinuity (Figure 2b). Then, the present study is more general using nonparametric linear dynamic block which can be of infinite order. Unlike several previous works, the linear element is parametric or of known structure.

The paper is organized as follows: the identification problem is formulated in Section 2; the nonlinear operator identification is coped with in Section 3; the linear subsystem frequency response determination is investigated in Section 4. An example of simulation is proposed with in Section 5.



(a) (b) (c)

Figure 2. Hard nonlinearity with (a) saturation, (b) discontinuity, and (c) discontinuity and saturation

2. PROBLEM STATEMENT

The aim in this work is to deal the nonlinear systems identification problem. The considered nonlinear system is structured by Hammerstein models (Figure 1). Presently, the nonlinear function *f*(.) is of hard type (Figure 2a-c) and the nonparametric linear block is of unknown structure. Furthermore, the linear element is characterized by the transfer function *G*(*s*). A standard Hammerstein model is analytically described as follows (Figure 1):

$$w(t) = f\big(v(t)\big) \quad (1)$$

$$x(t) = g(t) * w(t) \quad (2)$$

$$y(t) = x(t) + \xi(t) = g(t) * w(t) + \xi(t) \quad (3)$$

where $g(t) = L^{-1}\big(G(s)\big)$ and L^{-1} denotes the inverse Laplace operator. Except the input and output system ($u(t)$ and $y(t)$), all internal signals $w(t)$, $x(t)$ and $\xi(t)$ are not accessible to measurement. On the other hand, the extra-input signal $\xi(t)$ accounts to disturbances and measurement errors.

On the other hand, note that Hammerstein systems problem identification doesn't have a unique solution [11-12]. Indeed, if the couple $\big(f(u), G(s)\big)$ (Figure 1) is solution of the above problem identification. Then, any nonlinear system of the form $\left(f(u)K, \frac{G(s)}{K}\right)$ (Figure 3) is also solution of this problem identification for any nonzero real *K*. Currently, the question that arises is how to choose the constant *K*? The problem of the judicious choice of *K* can be resolved in the following section. The aim in this paper is to develop a frequency identification method allowing the estimate the system nonlinearity parameters, as well as, the frequency linear subsystem estimation.

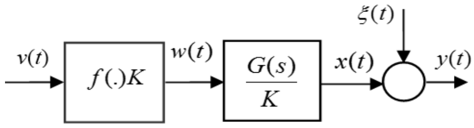


Figure 3. Solution multiplicity of Hammerstein models

3. NONLINEARITY FUNCTION IDENTIFICATION

In this section, the objective is to provide an identification solution letting the estimate of nonlinearity function *f*(.). To this end, the nonlinear system structured by Hammerstein model (Figure 1) is excited by a set of constant input:

$$v(t) = V_j \text{ for } t \in [(j-1)T, jT] \tag{4}$$

where T is much greater than the system rise time ($T > t_r$) and $j = 1 \dots N$. Note that the integer number N is arbitrarily chosen by the designer. Then, the internal signal $w(t)$ is also a piecewise constant signal. Specifically, it follows from (1) and (4) one has:

$$w(t) = W_j = f(V_j) \text{ for } t \in [(j-1)T, jT] \tag{5}$$

for $j = 1 \dots N$. In section 2, we have shown that the Hammerstein system identification possesses a multiple solutions (Figure 3). Several studies were made for the choice of K. It is preferable to take the following factor K:

$$K = G(0) \tag{6}$$

Accordingly, it readily follows from (Figure 3) and (6) that the linear element to be identified is characterized by a unit static gain. Specifically, one has without loss of generality:

$$G(0) = 1 \tag{7}$$

Accordingly, it readily follows from (2), (5) and (7) that the steady state of undisturbed output $x(t)$ is constant for any input value $V_j (j = 1 \dots N)$. Then, it can be expressed as:

$$x(t) = X_j = f(V_j) \text{ for } t > t_r \tag{8}$$

for any $j \in \{1 \dots N\}$. Therefore, using (3) and (8) the steady state system output $y(t)$ is close to a set of constant values up to noise, for any input $V_j (j = 1 \dots N)$. Specifically, the system output $y(t)$ can rewritten as follows:

$$y(t) = X_j + \xi(t) = f(V_j) + \xi(t) \text{ for } t > t_r \tag{9}$$

where $j = 1 \dots N$. Presently, the difficulty is how to determine $f(V_j)$, for any input $V_j (j = 1 \dots N)$, using uniquely the system input $u(t)$ and output $y(t)$?

This problem can be avoided using the fact that the noise signal $\{\xi(t)\}$ is of zero mean. Let $\hat{f}(v)$ denotes the estimate of $f(v)$. Then, an accurate estimate $\hat{f}(V_j)$ of $f(V_j)$, for any input $V_j (j = 1 \dots N)$, can be obtained using the following estimator:

$$\hat{f}(V_j) = \frac{1}{M} \sum_{t=1}^{M} y(t) \tag{10}$$

where M is any large integer and $j = 1 \dots N$. Then, one immediately gets from (9) and (10) that:

$$\hat{f}(V_j) = f(V_j) + \frac{1}{M}\sum_{t=1}^{M} \xi(t) \quad (11)$$

Furthermore, noticing that the term on the right of (11) boils down to zero with probability equals to 1:

$$\frac{1}{M}\sum_{t=1}^{M} \xi(t) = 0 \qquad \text{(w.p.1)} \quad (12)$$

Finally, it follows from (11) and (12) that:

$$\hat{f}(V_j)_{M\to\infty}^{\approx} f(V_j) \qquad \text{(w.p.1)} \quad (13)$$

for any input V_j ($j = 1 \ldots N$). To conclude, an accurate estimate of $f(V_j)$, for any input V_j $j = 1 \ldots N$, can be obtained by averaging the system output $y(t)$ over any interval $[(j-1)T, jT]$.

4. IDENTIFICATION OF THE LINEAR ELEMENT

The aim in this section is to develop an identification approach to determine the linear subsystem parameters. Bear in mind that the system nonlinearity is of hard type function having at least one segment of nonzero slope. Presently, we suggest a frequency approach allowing an accurate estimate of complex gain (i.e. the modulus gain and the phase) of linear dynamic element.

Then, using the nonlinearity estimator performed in section 3 and excite the Hammerstein system by the sine input signal:

$$w(t) = v_{Offset} + V\sin(\omega t) \quad (14)$$

where V is amplitude of sinusoidal part of $v(t)$ and v_{Offset} is the offset value. Recall that the system nonlinearity has at least one segment of nonzero slope (for identifiability reasons). Accordingly, the couple (V, v_{Offset}) is chosen such that the input signal $v(t)$ remains belonging to one segment having a nonzero slope. The problem that currently arises is how to make sure that $v(t)$ remains belonging to one segment (of nonzero slope)?

This question will be dealt using the system nonlinearity estimator (section 3). Accordingly, by making a linear connection of all estimated points in section 3. Some of candidate segments are deduced and let take one. On the other hand, starting from any amplitude V and choosing an offset value v_{Offset} near of the segment center. Then, the amplitude V can be adjusting until the system output $y(t)$ becomes sine signal up to noise. If this latter no longer becomes sinusoidal (up to noise), then another candidate segment can be considered.

Therefore, let $a \neq 0$ and b designates the parameters of this segment having a nonzero slope. The parameters couple (a, b) is already determined in section 3. Under these conditions, it readily follows that the internal signal $w(t)$ and the input signal $v(t)$ are related by the following relationship:

$$w(t) = av(t) + b \quad (15)$$

Then, it readily follows from (1), (14) and (15) that, the internal signal $w(t)$ is also sine signal. This latter can be expressed as:

$$w(t) = av\sin(\omega t) + av_{Offset} + b \quad (16)$$

On the other hand, use the fact that the stability assumption of the linear block, then the undisturbed system output $x(t)$ is also sine signal. Specifically, one immediately gets from (1), (7) and (16) that the signal $x(t)$ can expressed as follows:

$$x(t) = aV|G(j\omega)|\sin(\omega t + \varphi(\omega)) + av_{Offset} + b \quad (17)$$

Finally, from (3) and (17), one has the resulting system output $y(t)$:

$$y(t) = aV|G(j\omega)|\sin(\omega t + \varphi(\omega)) + av_{offset} + b + \xi(t) \quad (18)$$

Accordingly, this latter equation shows that the steady state of system output $y(t)$ is a sine signal up to noise. Further, the unique unknowns in the amplitude and phase of sinusoidal term are the modulus gain $|G(j\omega)|$ and phase $\varphi(\omega)$ of linear block.

Then, by filtering the sinusoidal term in (18), the frequency parameters of linear subsystem can be easily obtained. To this end, the noise part in (18) is filtered as follows. Let $\hat{x}(t)$ denotes the undisturbed output estimate. The estimator of $x(t)$ can be obtained using the following T-periodic average:

$$\hat{x}(t) = \frac{1}{M}\sum_{k=1}^{M} y(t + kT) \quad (19)$$

where $T = 2\pi/\omega$ is the period of the input $u(t)$ and output $y(t)$ signals. Indeed, using the fact that the undisturbed system output $x(t)$ is periodic signal of same period of the input $u(t)$. Then, one immediately gets from (3) and (19) that the estimate $\hat{x}(t)$ can be expressed as:

$$\begin{aligned}\hat{x}(t) &= \frac{1}{M}\sum_{k=1}^{M}\left(x(t + kT) + \xi(t + kT)\right)\\ &= x(t) + \frac{1}{M}\sum_{k=1}^{M}\xi(t + kT)\end{aligned} \quad (20)$$

Accordingly, the last term in the right side of (20) vanishes with probability equal to 1. Then, it follows from (20) that:

$$\hat{x}(t) \approx x(t) \quad (21)$$

5. SIMULATION

The nonlinear system of Hammerstein models considered in simulation is characterized by the system nonlinearity shown in Figure 4. Then, the transfer function of linear element is given as follows:

$$G(s) = \frac{0.1}{(s + 0.2)(s + 0.5)} \quad (22)$$

In the first stage and using the identification method presented in section 3, the system nonlinearity parameters can be obtained. Then, the nonlinear system is excited by the constant input of Figure 5. The corresponding system output $y(t)$ is displayed by Figure 6. Furthermore, the nonlinear system of Hammerstein model is excited by others constant inputs. The system output $y(t)$ for $v(t) = V_2 = -1$ is shown in Figure 7.

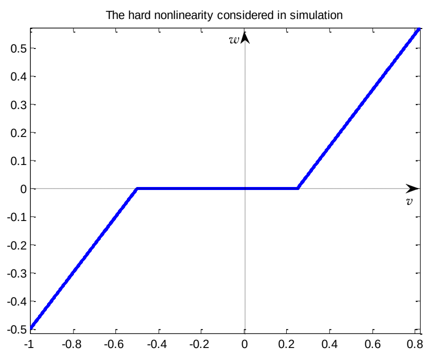


Figure 4. The system nonlinearity *f*(.)

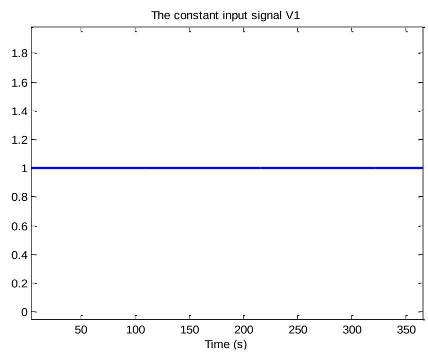


Figure 5. The input *v*(*t*)

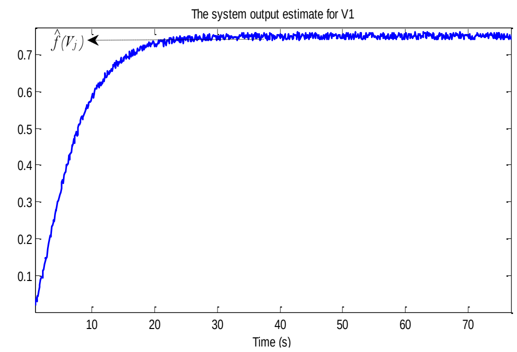


Figure 6. The resulting output signal $y(t)$

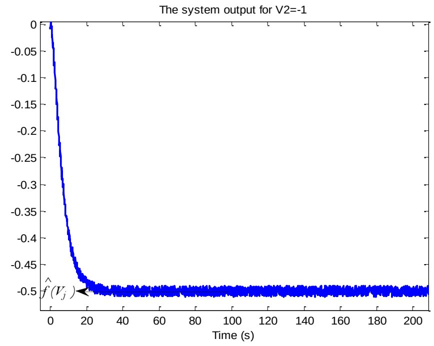


Figure 7. The resulting output signal $y(t)$

In the second stage and using the frequency algorithm presented in section 4, the linear block parameters can be identified. To this end, the nonlinear system is excited by the sine input (14). Then, for an arbitrarily amplitude V and offset v_{offset}, the system output $y(t)$ is shown in Figure 8. It is clear that the system output $y(t)$ is not a sine signal. Then, by adjusting the parameters couple (V, v_{offset}) of the input (14) and observing the system output $y(t)$ until that this latter becomes sine signal. This result is shown in Figure 9.

This result (Figure 9) shows that the output signal $y(t)$ is close to a sine signal up to noise. Then, the estimate $\hat{x}(t)$ of inner signal $x(t)$ using the estimator (19) is represented by Figure 10. Then, by representing the output $y(t)$ according to the signal $z(t)$. The corresponding locus for any phase δ is shown in Figure 11.

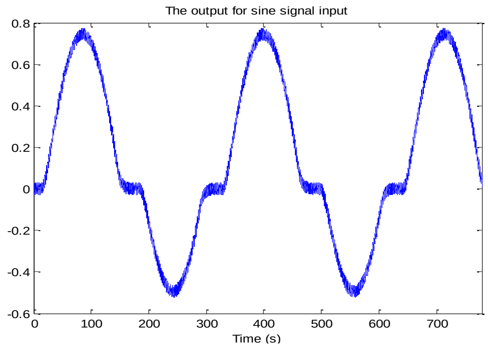


Figure 8. The output $y(t)$ for an arbitrarily V and v_{offset}

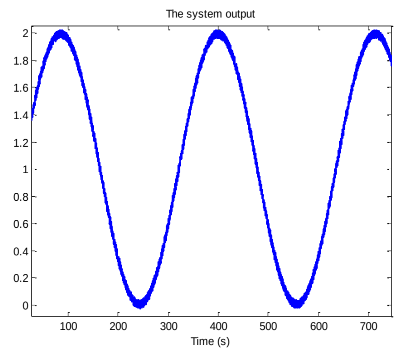


Figure 9. The system output signal $y(t)$

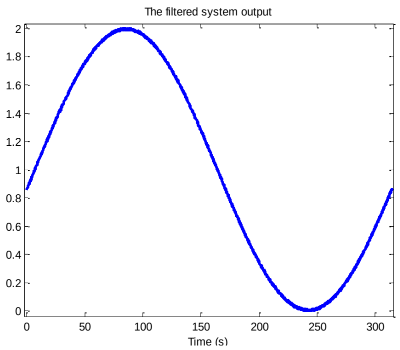


Figure 10. The estimate of inner signal $x(t)$

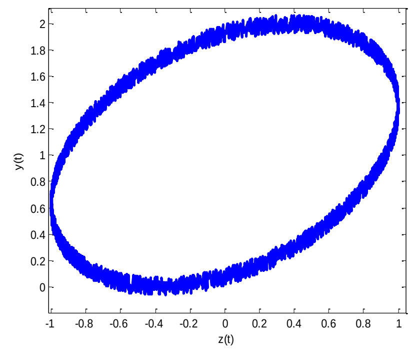


Figure 11. The locus $(z(t), y(t))$

It is clear that the undisturbed system output $x(t)$ is very close to a sine signal. Accordingly, to determine the linear block parameters, let consider the following reference signal having the same frequency of input $v(t)$:

$$z(t) = V\sin(\omega t + \delta) \tag{23}$$

This result points out that this locus is different from a straight line. Then, adjusting the parameter δ in (23) until the locus $(z(t), y(t))$ becomes straight line up to noise. One immediately concludes that this value of δ is the phase estimate of linear element. This case is shown in Figure 12. Then, the modulus frequency gain $|G(j\omega)|$ can be estimated by representing the filtered locus $(z(t), \hat{x}(t))$ using the estimator (19). This result is shown in Figure 13. Finally, the linear block parameters (i.e. the phase $\varphi(\omega)$ and the modulus frequency gain $|G(j\omega)|$) can be determined for any frequency ω.

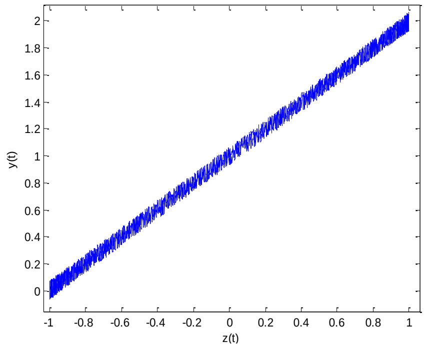


Figure 12. The locus $(z(t), y(t))$

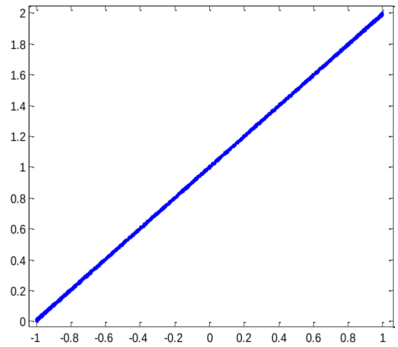


Figure 13. The filtered locus $(z(t), \hat{x}(t))$

6. CONCLUSION

In this work, the identification problem of nonlinear systems is dealt. The nonlinear system is structured by Hammerstein models. The considered nonlinear system is more general. The system nonlinearity can be discontinuous and noninvertible function. Presently, this latter is a function of hard type. Furthermore, the dynamic linear block can be nonparametric and of unknown structure. Then, in this work a new two stages solution is proposed. In the first stage, an identification method is proposed to estimate the system nonlinearity parameters. In the second stage, the frequency gain of the linear element can be identified.

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