

Adaptive Fuzzy and Robust H_∞ Compensation Control for Uncertain Robot

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ABSTRACT

In this paper, two types of robust adaptive compensation control schemes for the trajectory tracking control of robot manipulator with uncertain dynamics are proposed. The proposed controllers incorporate the computed-torque control scheme as a nominal portion of the controller; an adaptive fuzzy control algorithm to approximate the structured uncertainties; and a nonlinear H_∞ tracking control model as a feedback portion to eliminate the effects of the unstructured uncertainties and approximation errors. The validity of the robust adaptive compensation control schemes is investigated by numerical simulations of a two-link rotary robot manipulator.

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1. INTRODUCTION

In robot manipulator, similar to many engineering applications, it is impossible or very difficult to obtain an exact dynamic model, due to the presence of model uncertainties. These model uncertainties can be divided into the structured uncertainties stemming from the unknown kinematic parameters or nonlinear coupling dynamic, and the unstructured uncertainties including changing payload, nonlinear friction and unknown external disturbance. Therefore, there is a need to design a control strategy with robustness, adaptive capability, fast convergence and simple structure.

In the last few decades, many well-known nonlinear control methods [1-5] have been devoted to the design or improvement of the controller for robot manipulator with these model uncertainties. Adaptive and robust control schemes are the most effective and popular means to handle these model uncertainties [6]. Due to having the capability of learning uncertain parameters of structured uncertainties, adaptive controller can achieve a good control performance and compensate for the structured uncertainties and bounded disturbances. On the other hand, adaptive control scheme is restricted to the parameterization of known functional dependency but of unstructured uncertainties. Consequently, these unstructured uncertainties affect the control performances of the nonlinear adaptive controllers in cases with a poorly known dynamic model or when the fast real-time control is required [7]. Robust control schemes are naturally attractive because of their capability to deal with unstructured uncertainties, good transient performance and fast response. But, it is not properly suitable for the slow time-varying structured uncertainties with unknown upper bound [8].

Some robust adaptive control strategies [9-14] have been investigated to overcome some shortcomings of adaptive control scheme and robust controller. Their idea is to apply the adaptive control scheme to estimate the unknown parameters of the dynamical system and to use the robust controller to

compensate for the external disturbances. As a matter of fact, there exists a great differences between the structured and unstructured uncertainties, for example, structured uncertainties are characterized by the existence of the upper bound, but some unstructured uncertainties may be of finite energy only, but not bounded. However, most robust adaptive control schemes just provide an overall compensation control for these structured and unstructured uncertainties, and no attention is paid to separately compensation control for these structured and unstructured uncertainties. Furthermore, undesirable chattering control signals in these robust adaptive controllers are inevitably induced from some upper bound of the uncertainties determined inaccurately by virtue of designers' experiences [15]. These discontinuous chattering control signals might excite high order vibrations and cause lots of wear and tear in actuators. Especially, a large number of parameters and an adaptation gain (i.e., design parameter) corresponding to each unknown parameter introduce more complexity, and even the problem of adaptation time and computation burden can lead to severe stability and robustness problem.

In this paper, dynamical model of robot manipulator are separated as the following three subsystems: nominal system with precise dynamical knowledge, structured uncertain system and unstructured uncertain system. Two types of novel robust adaptive compensation control schemes combined with an adaptive fuzzy control algorithm and a nonlinear H_∞ tracking control model are proposed for the trajectory tracking control of robot manipulator with structured and unstructured uncertainties. The adaptive fuzzy control algorithm is employed to approximate the structured uncertainties, and the nonlinear robust H_∞ control model is designed to eliminate the effects of the unstructured uncertainties and approximation errors. Another main feature for the robust adaptive fuzzy control scheme is that no matter how many rules in the fuzzy system are utilized; only one tuning parameter will be adjusted on-line, so the computation burden of the proposed fuzzy control algorithm can be significantly reduced.

This paper is organized into five sections. Following the introduction, Section 2 addresses some comprehensive theoretical basis, which consist of dynamical models of robot manipulators with uncertainties and detailed explanations of computed-torque controller. Section 3 is devoted to design of robust adaptive compensation controller, and stability proof based on Lyapunov stability theorem. In Section 4, some simulation results are illustrated feasibility of the proposed control schemes by using simulations of a two-link rotary robot manipulator. Some conclusion remarks are finally included in Section 5.

2. THE COMPREHENSIVE THEORETICAL BASIS

In joint space, the dynamic equation of a robot manipulator with n degrees-of-freedom can be expressed as follows.

$$D(q)\ddot{q} + H(q, \dot{q}) + F(q, \dot{q}) = \tau \quad (1)$$

where $q, \dot{q}, \ddot{q} \in R^n$ are the vectors of the position, velocity and acceleration, respectively; $D(q) \in R^{n \times n}$ is a symmetric positive definite inertia matrix; $H(q, \dot{q}) = C(q, \dot{q})\dot{q} + G(q)$, $C(q, \dot{q})\dot{q} \in R^n$ expresses the Coriolis and centrifugal forces; $G(q) \in R^n$ denotes the gravity force; $F(q, \dot{q}) \in R^n$ includes the friction terms and external disturbances; $\tau \in R^n$ represents the torque exerted on joints.

The task of trajectory tracking control for robot manipulator can be described as follows: given desired trajectories $q_d \in R^n$, the control input torque τ should be found such that the actual trajectories q in Eq. (1) can tend to q_d , as time goes to infinity. The actual parameters $D(q)$ and $H(q, \dot{q})$ in dynamic Eq. (1) are assumed to be separated as the nominal parts denoted as $\hat{D}(q)$ and $\hat{H}(q, \dot{q})$, and the uncertain parts defined by $\Delta D(q)$ and $\Delta H(q, \dot{q})$. These parameters satisfy the following relations.

$$\begin{cases} \hat{D}(q) = D(q) - \Delta D(q) \\ \hat{H}(q, \dot{q}) = H(q, \dot{q}) - \Delta H(q, \dot{q}) \end{cases} \quad (2)$$

And, the control law of computed-torque controller can be written as follows.

$$\tau = \hat{D}(q)(\ddot{q}_d + K_v \dot{e} + K_p e) + \hat{H}(q, \dot{q}) \quad (3)$$

where K_v and K_p are symmetric positive definite gain matrices; and $e = q_d - q$ denotes the trajectory tracking error.

Computed-torque controller is an effective means for trajectory tracking control on strong assumption that exact knowledge of robot dynamics is precisely known. However, it is very difficult for computed-torque controller to achieve the ideal control performances in practice, due to the presence of the inevitable uncertainties in robot manipulator. Substituting Eq. (3) into Eq. (1) yields the following closed loop tracking error dynamic equation.

$$\ddot{e} + K_v \dot{e} + K_p e = \psi + \delta \quad (4)$$

where $\psi = \hat{D}^{-1}(\Delta D(q)\ddot{q} + \Delta H(q, \dot{q}))$, which is defined as the structured uncertainties; $\delta = \hat{D}^{-1}F(q, \dot{q})$ is denoted as the unstructured uncertainties. Hence, $\psi + \delta$ is termed as lumped uncertainties in robot dynamics.

To improve robustness of computed-torque controller in the presence of uncertainties, a composite control scheme composed of adaptive fuzzy control algorithm and robust control model will be attached to the computed-torque controller as a compensator. In this way, the overall control law can be written as follows.

$$\tau = \tau_0 + \tau_c \quad (5)$$

where τ_0 expresses the control input torque of computed-torque controller defined by Eq. (3), and τ_c denotes the control input torque of the composite compensation control scheme.

Substituting the overall control law (5) into Eq. (1) yields the following closed loop tracking error dynamic equation.

$$\dot{x} = Ax + B(\psi + \delta - \hat{D}(q)^{-1} \tau_c) \quad (6)$$

where $A = \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ I \end{bmatrix}$, and $x = [e, \dot{e}]^T$.

In this paper, a multi-input and multi-output fuzzy logic system will be introduced to eliminate the structured uncertainties, and its final output results are defined as follows.

Definition 1: A multi-input and multi-output fuzzy logic system performs a mapping from fuzzy sets in $U \in R^n$ to another fuzzy sets in $V \in R^m$. The j th output of fuzzy logic system with center-average defuzzifier, product inference, and singleton fuzzifier can be expressed as the following form:

$$y_j = \frac{\sum_{l=1}^M \left(\prod_{i=1}^n \mu_{A_i^l}(x_i) \right) \bar{y}_j^l}{\sum_{l=1}^M \left(\prod_{i=1}^n \mu_{A_i^l}(x_i) \right)} = \sum_{i=1}^M \bar{y}_j^l \xi_i^l(x) = \Theta_j^T \xi(x), \quad j = 1, 2, \dots, m \quad (7)$$

where \bar{y}_j^l is a specific value in fuzzy sets V_j at which fuzzy membership function $\mu_{B_j^l}(y_j)$ achieves its maximum value; A_i^l and B_j^l are the linguistic variables in the subspace U_i and V_j , described by their membership functions $\mu_{A_i^l}(x_i)$ and $\mu_{B_j^l}(y_j)$; M is the rule number of fuzzy IF-THEN rules; $\xi(x) = (\xi_1(x), \dots, \xi_M(x))^T$ is the fuzzy basis function vector, and $\Theta_j = (\bar{y}_j^1, \dots, \bar{y}_j^M)^T$ denotes the parameter vector. And, the l th fuzzy basis function vector is defined as follows.

$$\xi_l(x) = \frac{\prod_{i=1}^n \mu_{A_i^l}(x_i)}{\sum_{l=1}^M \left(\prod_{i=1}^n \mu_{A_i^l}(x_i) \right)}, \quad l = 1, 2, \dots, M \quad (8)$$

Given that the input universe of discourse U is a compact set in R , then, for any given real structured uncertain term ψ in Eq. (6) and an arbitrary small positive constant ε , there exists a fuzzy logic system in the form Eq. (7) such that $\sup_{x \in U} |\psi - \Theta^T \xi(x)| \leq \varepsilon$ [16]. Hence, a boundary for ψ can be expressed as follows.

$$\|\psi\| \leq \Theta^T \xi(x) + \varepsilon \tag{9}$$

where $\xi(x)$ is an unknown fuzzy base function vector; Θ denotes the tune weight parameters of fuzzy logic system; and ε is an arbitrary small positive constant.

3. RESEARCH METHOD

In this section, two types of robust adaptive compensation control schemes serving as a compensator for computed-torque controller is designed to separately compensate the structured and unstructured uncertainties. In this way, the compensation control input torque τ_c and the state space tracking error dynamic equation can be written as follows.

$$\tau_c = \tau_h + \tau_f \tag{10}$$

$$\dot{x} = Ax + B(\psi + \delta - \tau_h - \tau_f) \tag{11}$$

where $\psi = \hat{D}^{-1}(\Delta D(q)\ddot{q} + \Delta H(q, \dot{q}))$, $\delta = \hat{D}^{-1}F(q, \dot{q})$, $x = [e, \dot{e}]^T$; τ_h and τ_f are the control input torques of robust H_∞ controller and adaptive fuzzy control scheme, respectively.

3.1. Design of robust adaptive compensation control schemes

Acted as a nonlinear compensator for computed-torque controller, two types of novel robust adaptive compensation control schemes incorporated with adaptive fuzzy control algorithm and a nonlinear robust H_∞ control model are designed to eliminate the structured and unstructured uncertainties. The adaptive fuzzy control algorithm is employed to approximate the structured uncertainties ψ in Eq. (11), and the nonlinear robust H_∞ control model is designed to eliminate the effects of the unstructured uncertainties and approximation errors. As shown in Figure 1 (a), the robust adaptive compensation control scheme based on feed-back compensator (FBC) takes the actual output commands as the input variables of the fuzzy logic system, and utilizes the trajectory tracking errors as tune parameters of adaptive fuzzy controller. Another control scheme based on feed-forward compensator (FFC) is illustrated in Figure 1 (b).

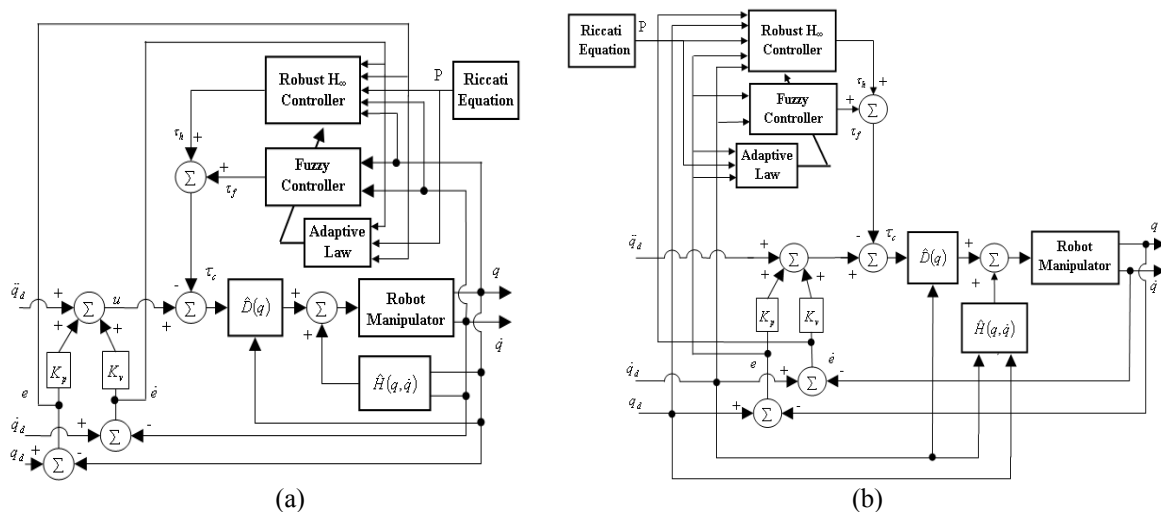


Figure 1. Configurations of control schemes: (a) FBC-based robust adaptive compensation control scheme; (b) FFC-based robust adaptive compensation control scheme.

Both the FBC-based and the FFC-based control schemes have a common adaptive learning concept, which is the tune parameters of adaptive fuzzy controller. Furthermore, the output control torques τ_c in the two control schemes are employed as the compensator. However, a closer investigation reveals many differences in the two control schemes, such as type of training signals and process of taming uncertainties. The main difference is that the input vectors in the FBC-based control scheme are calculated as a function of the actual positions $q(t)$ and velocities $\dot{q}(t)$, while the input vectors in the FFC-based control scheme are expressed as a function of the desired positions $q_d(t)$ and velocities $\dot{q}_d(t)$. The control law in the two control schemes and their closed loop tracking error dynamic equation are calculated as follows.

$$\tau = \hat{D}(\ddot{q} + K_v \dot{e} + K_p e + \tau_c) + \hat{H} \quad (12)$$

$$\ddot{e} + K_v \dot{e} + K_p e = \hat{D}^{-1}(\Delta D(q)\ddot{q} + \Delta H(q, \dot{q}) + F(q, \dot{q})) - \tau_c \quad (13)$$

where $\Delta D(q) = D(q) - \hat{D}(q)$, $\Delta H(q, \dot{q}) = H(q, \dot{q}) - \hat{H}(q, \dot{q})$, and $F(q, \dot{q})$ are uncertainties including friction terms and external disturbances, and so on.

One can see from the tracking error dynamic Eq. (13) that the robust adaptive compensation control scheme can generate a control compensation vector τ_c to reduce the tracking error to zero, so that clearly minimizing the error allows us to achieve the ideal control performances.

3.2. Derivation of robust adaptive compensation control schemes

In this section, the tuning weight parameters in the adaptive fuzzy control algorithm will be derived and the convergence and stability of the closed-loop control system will also be proved.

Lemma 1. The following inequality holds for any $\varepsilon > 0$ and for any $\eta \in R$

$$0 \leq |\eta| - \eta \tanh\left(\frac{\eta}{\varepsilon}\right) \leq \kappa \varepsilon \quad (14)$$

where κ is a constant that satisfies $\kappa = e^{-(\kappa+1)}$, i.e. $\kappa = 0.2785$. The proof of Lemma 1 is given in [17].

Lemma 2. The following inequality holds for any two matrices $X \in R^{n \times m}$ and $Y \in R^{n \times m}$

$$2X^T Y \leq \varepsilon X^T X + \varepsilon^{-1} Y^T Y \quad (15)$$

where ε denotes an arbitrary small positive constant.

Theorem 1. If there exists a continuous function $V(\square): R^n \rightarrow R^+$ for the continuous system described by Eq. (11), and there are scalars \bar{V} and \underline{V} with $0 \leq \underline{V} \leq \bar{V} < \infty$, such that whenever $\underline{V} \leq V(x) \leq \bar{V}$, V is continuously differentiable and $\dot{V} = \frac{\partial V}{\partial t} f(x, t) \leq -q\alpha [V(x) - \underline{V}]$ for all $t \in R^+$, then, the system (11) is uniformly exponentially convergent to $S = \Phi(r)$ with rate α . The proof was given by Corless [18].

Theorem 2. If the control law for a nonlinear robotic system represented by Eq. (1) is designed as follows:

$$\tau = \tau_0 + \tau_c \quad (16)$$

$$\tau_0 = \hat{D}(q)(\ddot{q}_d + K_v \dot{e} + K_p e) + \hat{H}(q, \dot{q}) \quad (17)$$

$$\tau_c = \tau_h + \tau_f \quad (18)$$

where the control law of the adaptive fuzzy controller is given as $\tau_f = -\rho(x, \hat{\Theta}) \tanh\left(\frac{\rho(x, \hat{\Theta}) B^T P x}{\varepsilon}\right)$, and

the control law of robust H_∞ controller τ_h is defined as $\tau_h = -R^{-1} B^T P x$, with

$$\rho(x, \hat{\Theta}) = \Theta^T \xi(x) \quad (19)$$

$$\dot{\hat{\Theta}} = -\lambda \hat{\Theta} + L \xi(x) \|B^T P x\| \quad (20)$$

The symmetric positive definite matrix P satisfies the following Riccati-like equation:

$$A^T P + PA + Q + PB \left(\frac{1}{\varepsilon} I - R^{-1} \right) B^T P = 0 \quad (21)$$

where ε is an arbitrary small positive constant.

Then, the proposed robust adaptive compensation control law can guarantee the closed loop system to convergent towards a residual set $\Phi(r)$ with rate $\mu/2$, where $\mu = \frac{1}{2} \min \left\{ \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \lambda \right\}$, $r = \sqrt{\frac{\varepsilon}{\mu}}$,

$$\bar{\varepsilon} = \gamma \left[\frac{\lambda \|\Theta\|}{2l_{\min}} + \varepsilon \right] + \frac{1}{2} \varepsilon \|F\|^2, \quad l_{\min} = \min \{l_1, l_2, \dots, l_p\}.$$

Proof: Define a Lyapunov function candidate as follows:

$$V = \frac{1}{2} x^T P x + \frac{1}{2} \gamma \tilde{\Theta}^T L^{-1} \tilde{\Theta} = z^T \bar{P} z := \bar{V}(z, t) \quad (22)$$

where $\tilde{\Theta}^T = \hat{\Theta}^T - \Theta^T$.

The derivative of the Lyapunov function V with respect to time along the tracking error dynamic Eq. (11) is given by

$$\dot{V} = \frac{1}{2} x^T (A^T P + PA) x - x^T P B \tau_f + x^T P B \psi - x^T P B \tau_h + x^T P B \delta + \gamma \tilde{\Theta}^T L^{-1} \dot{\tilde{\Theta}} \quad (23)$$

By using of the inequality (9), the adaptive fuzzy control law τ_f with Eqs. (19) and (20), and the Lemma 1 for $\eta = \hat{\Theta}^T \xi(x) B^T P x$, we can get the following inequality of the term of $-x^T P B \tau_f + x^T P B \psi$ in Eq. (23).

$$\begin{aligned} -x^T P B \tau_f + x^T P B \psi &\leq -x^T P B \rho(x, \hat{\Theta}) \tanh \left(\frac{\rho(x, \hat{\Theta}) B^T P x}{\varepsilon} \right) + \|\psi\| \|B^T P x\| \\ &\leq \hat{\Theta}^T \xi(x) \|B^T P x\| - x^T P B \hat{\Theta}^T \xi(x) \tanh \left(\frac{\hat{\Theta}^T \xi(x) B^T P x}{\varepsilon} \right) - \tilde{\Theta}^T \xi(x) \|B^T P x\| \\ &\leq \gamma \varepsilon - \gamma \tilde{\Theta}^T \xi(x) \|B^T P x\| \end{aligned} \quad (24)$$

Since $\frac{1}{2} (\tilde{\Theta} + \Theta)^T L^{-1} (\tilde{\Theta} + \Theta) \geq 0$, such that $\tilde{\Theta}^T L^{-1} \tilde{\Theta} + \tilde{\Theta}^T L^{-1} \Theta \geq \frac{1}{2} (\tilde{\Theta}^T L^{-1} \tilde{\Theta} - \Theta^T L^{-1} \Theta)$. In addition, the term $\dot{\tilde{\Theta}} = -\lambda (\tilde{\Theta} + \Theta) + L \xi(x) \|B^T P x\|$ can also be inferred from the adaptive control law Eq. (20). Thus, the term of $\gamma \tilde{\Theta}^T L^{-1} \dot{\tilde{\Theta}}$ in Eq. (23) can be rewritten as follows.

$$\begin{aligned} \gamma \tilde{\Theta}^T L^{-1} \dot{\tilde{\Theta}} &= -\gamma \lambda \tilde{\Theta}^T L^{-1} (\tilde{\Theta} + \Theta) + \gamma \tilde{\Theta}^T \xi(x) \|B^T P x\| \\ &\leq -\frac{1}{2} \gamma \lambda \tilde{\Theta}^T L^{-1} \tilde{\Theta} + \frac{1}{2} \gamma \lambda \Theta^T L^{-1} \Theta + \gamma \tilde{\Theta}^T \xi(x) \|B^T P x\| \end{aligned} \quad (25)$$

Substituting Eqs. (24) and (25) into Eq. (23) yields the following inequality

$$\dot{V} \leq \frac{1}{2} x^T (A^T P + PA) x + \gamma \varepsilon - \frac{1}{2} \gamma \lambda \tilde{\Theta}^T L^{-1} \tilde{\Theta} + \frac{1}{2} \gamma \lambda \Theta^T L^{-1} \Theta - x^T P B \tau_h + x^T P B \delta \quad (26)$$

Taking into account the Lemma 2, the term of $x^T P B \delta$ in Eq. (23) can be defined as

$$x^T PB\delta \leq \frac{1}{2}(\varepsilon\delta^T\delta + \varepsilon^{-1}x^T PBB^T Px) \quad (27)$$

In the end, substituting Eq. (27), the robust control law $\tau_h = -R^{-1}B^T Px$, and the Riccati-like equation (21) into Eq. (26), then, the derivative of the Lyapunov function \dot{V} can be bounded as

$$\begin{aligned} \dot{V} &\leq \frac{1}{2}x^T(A^T P + PA)x + \gamma\varepsilon - \frac{1}{2}\gamma\lambda\tilde{\Theta}^T L^{-1}\tilde{\Theta} + \frac{1}{2}\gamma\lambda\Theta^T L^{-1}\Theta - x^T PB\tau_h + \frac{1}{2}(\varepsilon\delta^T\delta + \varepsilon^{-1}x^T PBB^T Px) \\ &\leq \frac{1}{2}x^T(A^T P + PA + \varepsilon^{-1}PBB^T P - PBR^{-1}B^T P)x + \gamma\varepsilon - \frac{1}{2}\gamma\lambda\tilde{\Theta}^T L^{-1}\tilde{\Theta} + \frac{1}{2}\gamma\lambda\Theta^T L^{-1}\Theta + \frac{1}{2}\varepsilon F^T F \\ &\leq -\frac{1}{2}x^T Qx + \gamma\varepsilon - \frac{1}{2}\gamma\lambda\tilde{\Theta}^T L^{-1}\tilde{\Theta} + \frac{1}{2}\gamma\lambda\Theta^T L^{-1}\Theta + \frac{1}{2}\varepsilon F^T F \\ &\leq \frac{1}{2}(-x^T Qx - \gamma\lambda\tilde{\Theta}^T L^{-1}\tilde{\Theta}) + \gamma\lambda\Theta^T L^{-1}\Theta + \gamma\varepsilon + \frac{1}{2}\varepsilon\|F\|^2 \end{aligned} \quad (28)$$

Given that $\bar{Q} = \begin{bmatrix} Q & 0 \\ 0 & \gamma\lambda L^{-1} \end{bmatrix}$, the above expression can be given as follows

$$\dot{\bar{V}} \leq -z^T \bar{Q}z + \bar{\varepsilon} \quad (29)$$

Substituting the parameters given in Theorem 1 into the above expression, we get

$$\dot{\bar{V}} \leq -2\mu\bar{V} + \bar{\varepsilon} \quad (30)$$

Then, using the Theorem 1, one can see that the tracking error converges towards a residual set $\Phi(r)$ with the convergence rate $\frac{\mu}{2}$.

4. RESULTS AND ANALYSIS

In this section, the proposed control schemes are applied to control simulation of a two-link planar rotary robot manipulator (shown in Figure 2) gripping an unknown load. The actual parameters of the robot manipulator are selected as follows: $r_1 = 1(m)$, $m_1 = 1(kg)$, $r_2 = 0.8(m)$, $m_2 = 1.5(kg)$. And, their corresponding nominal values are assumed as follows: $\hat{r}_1 = 1(m)$, $\hat{m}_1 = 0.5(kg)$, $\hat{r}_2 = 0.8(m)$, $\hat{m}_2 = 0.8(kg)$. Furthermore, a payload uncertainty $\delta(t) = [10\sin(t), 10\cos(t)]^T$ is attached to the second link, and a coulomb and viscous friction force $F(q, \dot{q}) = 80\text{sgn}(\dot{q}) + 10\dot{q}$ is added to each joint. $q_{d1} = 0.5\cos(t) + 0.2\sin(3t)$ and $q_{d2} = -0.2\sin(2t) - 0.5\cos(t)$ are chosen as the desired joint trajectories. The dynamic equation of the two-link planar rotary robot manipulator is derived as follows.

$$\begin{bmatrix} D_{11}(q_2) & D_{12}(q_2) \\ D_{21}(q_2) & D_{22}(q_2) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -C_{12}(q_2)\dot{q}_2 & -C_{12}(q_2)(\dot{q}_1 + \dot{q}_2) \\ C_{12}(q_2)\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} G_1(q_1, q_2)g \\ G_2(q_1, q_2)g \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (31)$$

where $D_{11}(q_2) = (m_1 + m_2)r_1^2 + m_2r_2^2 + 2m_2r_1r_2\cos q_2$, $D_{12}(q_2) = D_{21}(q_2) = m_2r_2^2 + m_2r_1r_2\cos q_2$, $D_{22}(q_2) = m_2r_2^2$, $C_{12}(q_2) = m_2r_1r_2\sin q_2$, $G_1(q_1, q_2) = (m_1 + m_2)r_1\cos q_2 + m_2r_2\cos(q_1 + q_2)$, $G_2(q_1, q_2) = m_2r_2\cos(q_1 + q_2)$ with q_i ($i = 1, 2$) denote the link angular position (rad); m_i ($i = 1, 2$) are the link masses (kg), and r_i ($i = 1, 2$) represent the link lengths (m).

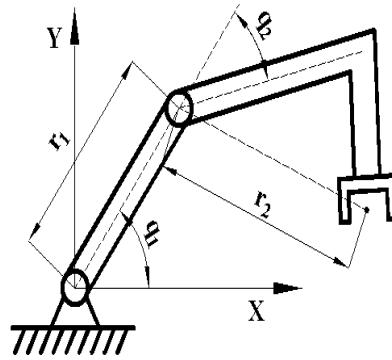


Figure 2. A two-link planar rotary robot manipulator.

4.1 Simulation results for the proposed control schemes

For the purpose of comparison studies, four simulation cases are conducted and their tracking performances are demonstrated in Figs.3, 4 and 5.

Case 1: Computed-torque controller is applied to robot manipulator with precise dynamical models. The parameters of the robot manipulator are selected as the nominal values. Its control parameters are picked as $K_v = \text{diag}(50, 50)$, $K_p = \text{diag}(100, 100)$. As shown in Figure 3, the tracking errors occur at the initial stages, and after a few seconds the actual trajectories almost overlap with the desired values. Moreover, it can be seen from the Figure 3 (a) and (b) that this case has smooth control torques.

Computed-torque controller utilizes the existing knowledge of the robotic model to linearize or decouple the robot dynamics, and cancels out nonlinear dynamics of robot manipulators. It exhibits excellent tracking performances for robot manipulator with nominal dynamic parameters, but it is not robust in the presence of modeling uncertainties. In practice, it is not possible to obtain a perfect or even reasonably accurate dynamic model of a manipulator. So, it will be very difficult for computed-torque controller to achieve the ideal control performances in practical applications.

Case 2: Computed-torque controller having the same control parameters as the first case is applied to robot manipulator with uncertain dynamics. In other words, the control scheme is designed according to the nominal parameters instead of actual values. Furthermore, a payload uncertainty $\delta(t) = [10\sin(t), 10\cos(t)]^T$ is attached to the second link, and a coulomb and viscous friction force $F(q, \dot{q}) = 80\text{sgn}(\dot{q}) + 10\dot{q}$ is added to each joint. A set of simulation results of the tracking positions and control torques for the two joints are illustrated in Figure 4.

Compared with the first case, the tracking performances of the positions and velocities in the second case have a tendency to deteriorate, due to imprecision dynamic parameters of robot manipulator. Especially, a large gaps between the simulation results and the desired trajectories in the second joint, resulting from the uncertain payload, may destroy the stability of the closed loop system. In addition, it is worth noting that the evaluations of control torques for two joints display relatively large oscillations, which might go beyond the limits of actuators. Therefore, a conclusion can be drawn that some additional compensation control schemes should be appropriately designed to eliminate the effects of imprecision dynamic parameters of robot manipulator and external disturbances.

Case 3: Computed-torque controller plus adaptive fuzzy compensator proposed by Chen [19] is used to control robot manipulator with uncertain dynamics. But, the adaptive fuzzy compensator just provides an overall compensation control for the structured and unstructured uncertainties. The input vector of the fuzzy compensator is defined as $X = \{x_i | i = 1, 2, \dots, 4\} = \{q_1, \dot{q}_1, q_2, \dot{q}_2\}$, the universe of discourse of each fuzzy input vector is divided into five fuzzy labels, i.e., *NB*, *NS*, *ZO*, *PS*, *PB*, and their corresponding membership functions are defined as $\mu_{A_i^l}(x_i) = \exp\left[-\frac{(x_i - C_i)^2}{2\sigma_i^2}\right]$, where C_i are -1, -0.5, 0, 0.5, and 1, σ_i is equal to 0.2124. The external disturbances and the friction forces are also the same as the second case. The tracking performances are illustrated in Figure 5.

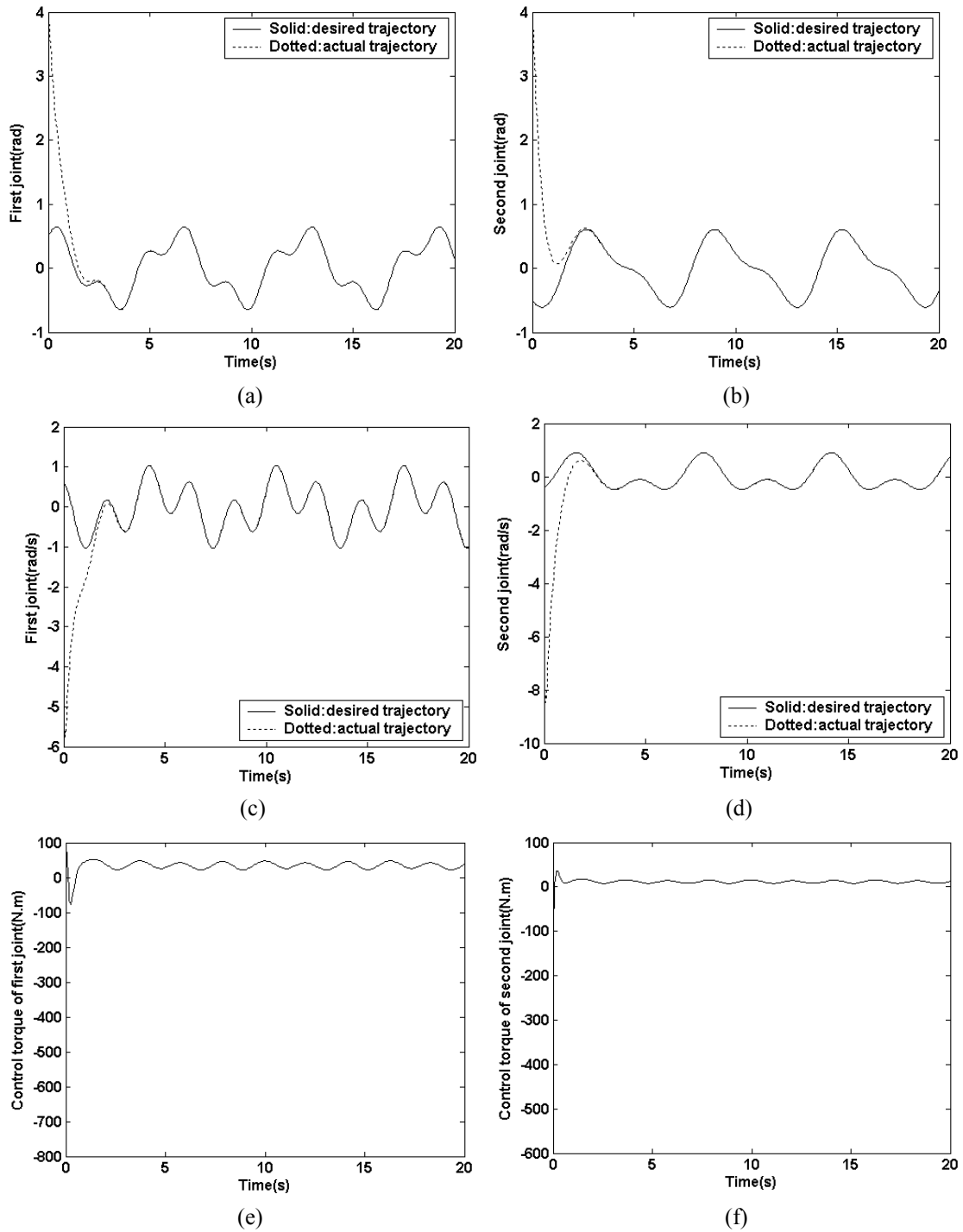


Figure 3. Tracking performances of two joints when computed-torque controller is applied to robot manipulator without uncertainties and disturbances.

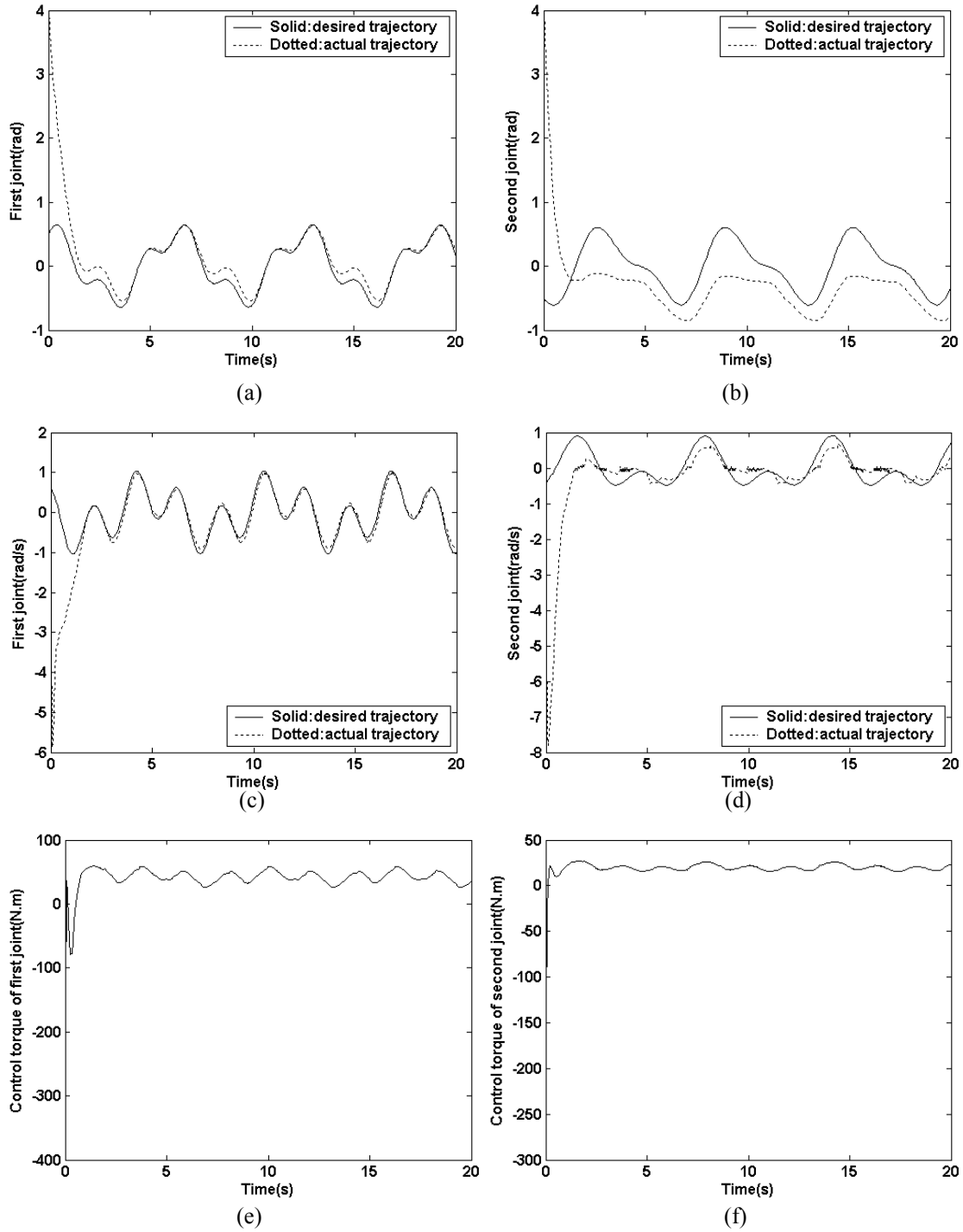


Figure 4. Tracking performances when computed-torque controller is applied to robot manipulators with uncertainties and external disturbances.

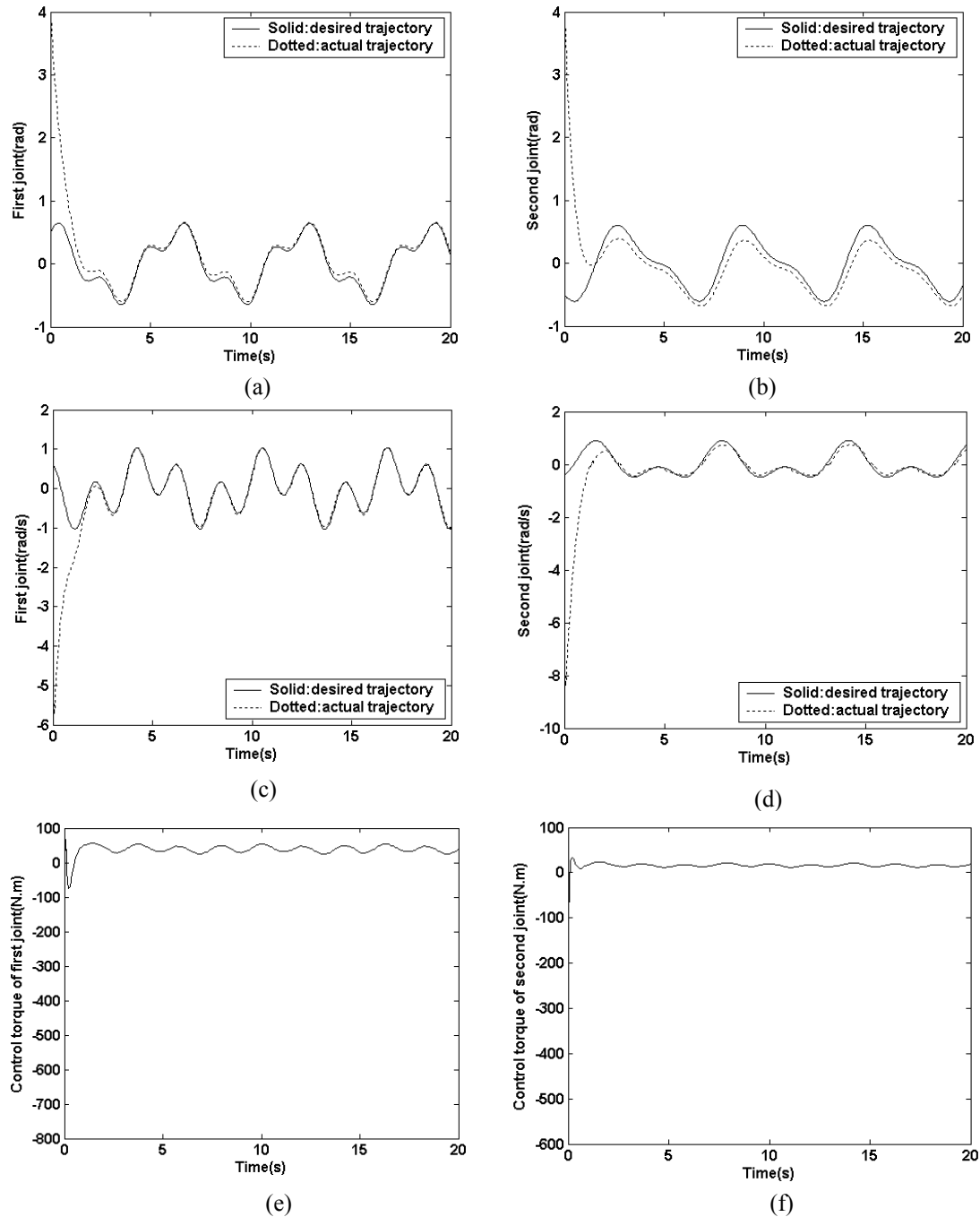


Figure 5. Tracking performances when computed-torque controller plus adaptive fuzzy controller is applied to robot manipulators with uncertainties and disturbances.

A significant difference between the second case and this case is that the controller in this case consists of an adaptive fuzzy compensator. As can be seen from Figs.4 and 5, the oscillations of the tracking errors in this case are remarkably smaller than the second case, and the evaluations of control torques for two joints also exhibit a smooth control action. Thus, the adaptive fuzzy compensator in this case can compensate some uncertainties effectually. However, from Figure 5, it is obviously to see that there still is a big errors between the desired and actual trajectories, which shows that only adaptive fuzzy compensator cannot completely compensate the structured and unstructured uncertainties with different features. Therefore, a composite compensation controller should be developed to separately compensate the structured and unstructured uncertainties.

Case 4: In this case, the simulations of the proposed control scheme incorporated with adaptive fuzzy control algorithm and robust H_∞ controller are carried out to verify the theoretical results. Here, the adaptive fuzzy controller is designed to compensate the structured uncertainties, and the robust H_∞ controller is employed to approximate the unstructured uncertainties. The external disturbances and the friction forces are also the same as the previous two cases. The tracking performances in the case are shown in Figure 6.

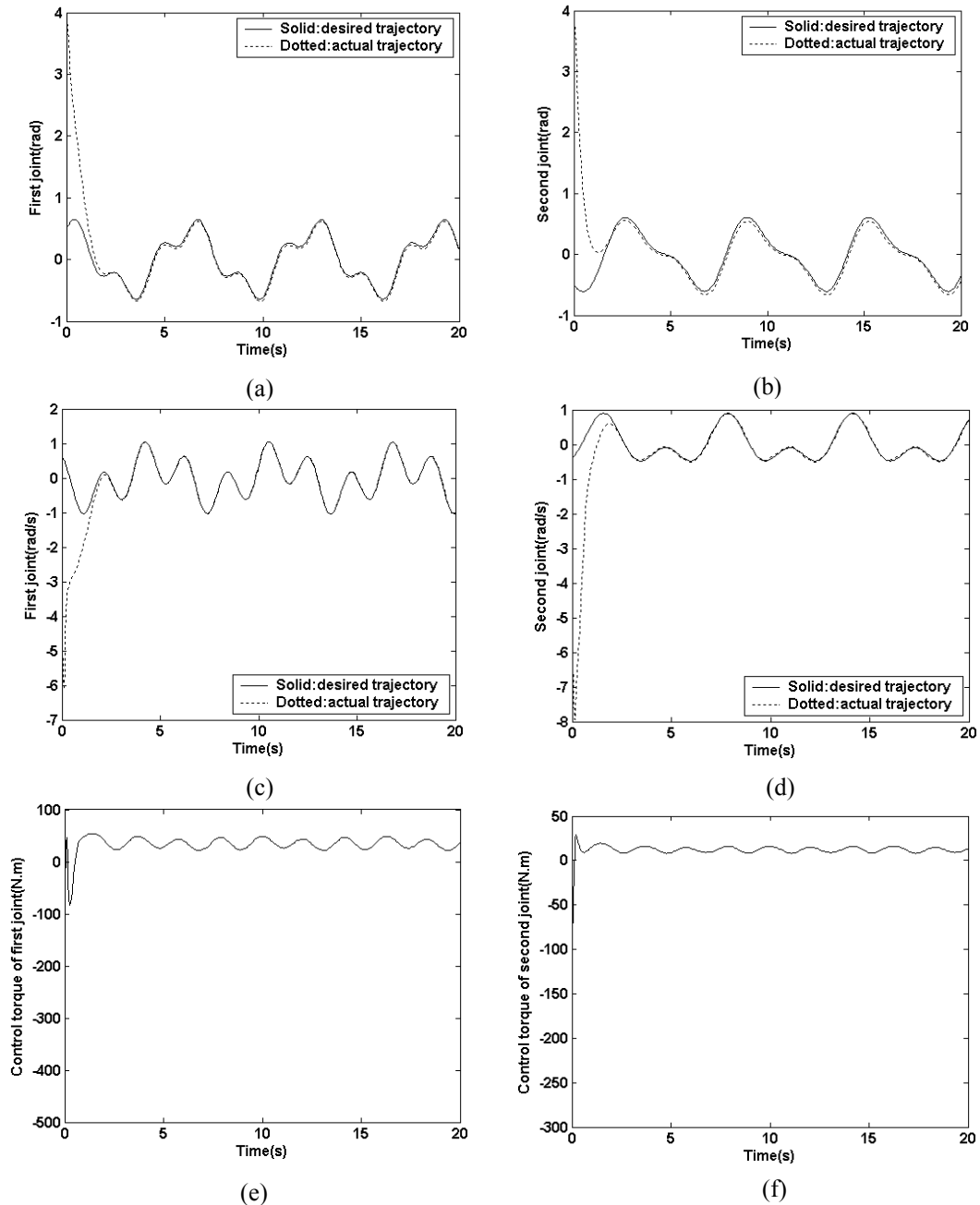


Figure 6. Tracking performances when the proposed robust adaptive compensation control scheme is applied to robot manipulators with uncertainties and disturbances.

Besides that the adaptive fuzzy control scheme is used to approximate the structured uncertainties in both this case and the third case, the exclusive difference between the third case and this case is that a nonlinear robust H_∞ controller in this case is employed to approximate the unstructured uncertainties. As can be seen from tracking performances in Figs.5 and 6, the tracking errors in this case are less than the third case. Hence, the proposed robust adaptive compensation control scheme can effectually and separately eliminate the effects of the structured and unstructured uncertainties on the control performances.

4.2 Comparison results between FBC-based and FFC-based control schemes

In order to extensively investigate the tracking control performances of the FBC-based and FFC-based control schemes, numerical simulation studies are still conducted on the two-link rotary robot manipulator, and the following tracking error equations over one training cycle of a trajectory are also defined to measure the tracking control performances.

$$E_p = \frac{1}{N} \sum_{i=1}^N \|q_{di} - q_i\|^2 \quad (\text{rad})^2 \quad E_v = \frac{1}{N} \sum_{i=1}^N \|\dot{q}_{di} - \dot{q}_i\|^2 \quad (\text{rad/sec})^2 \quad (32)$$

where E_p is the tracking error of the position; E_v is the tracking error of the velocity; N is the number of the position vectors; q_{di}, q_i denote the desired and actual trajectories, respectively. The tracking errors of the two control schemes after convergence are summarized in Table 1.

Table 1. Tracking errors after convergence.

Control schemes	Errors			
	First joint		Second joint	
	$E_p(\text{rad})^2$	$E_v(\text{rad/sec})^2$	$E_p(\text{rad})^2$	$E_v(\text{rad/sec})^2$
FFC-based control scheme	0.0009324	0.00034113	0.0029166	0.0008082
FBC-based control scheme	0.0029552	0.0015551	0.025762	0.007903
Uncompensated	0.024389	0.009591	0.30657	0.11172

As listed in Table 1, there are a large differences among the three control schemes. Compared with the uncompensated case, the two robust adaptive compensation control schemes exhibit extremely good control performances. Moreover, the FFC-based robust adaptive compensation control scheme performs slightly better in the circular trajectory than the FBC-based control scheme.

5. CONCLUSION

This paper addresses trajectory tracking problems of robot manipulators with the structured and unstructured uncertainties. A novel control design philosophy is presented to separately take into account the structured and unstructured uncertainties. On the basis of computed-torque controller as a nominal controller, two types of novel composite robust adaptive compensation control schemes incorporated with adaptive fuzzy control algorithm and a nonlinear H_∞ tracking controller are designed to handle inevitable uncertainties. The adaptive fuzzy control algorithm as a compensator is employed to approximate structured uncertainties, and the nonlinear robust H_∞ controller is designed to eliminate the effects of the unstructured uncertainties and approximation errors on the control performances.

Comparison studies of their control performances with the conventional computed-torque controllers in the presence of model uncertainties are carried out. Comparative results demonstrate that the two types of composite robust adaptive compensation control schemes are very effective in improving control performances in terms of uncertainties. The validity of the two types of robust adaptive compensation control schemes is demonstrated by numerical simulations of a two-link rotary robot manipulator. Simulation results demonstrate the proposed control schemes are the most effective and superiority. Further research should be carried out to test the proposed control scheme in real-time robot manipulator motion control.

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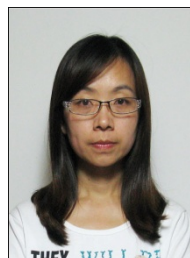
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