Scheduling Of The Crystal Sugar Production System in Sugar Factory Using Max-Plus Algebra

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***Abstract*—Sugar is the main trading commodity besides as basic human needs and be a source of energy and mostly traded in the form of solid crystals of sucrose or crystal sugar with cane as raw materials. Sugar production process is very complicated because it had to pass through various stages that require considerable time. The number of machines used in production system affects the complexity in the calculation of production scheduling. In addition, if there are errors in analyzing the operating time that is different for each product, it will cause a chaos in the production scheduling. These conditions encourage us to conduct a study on the production flow or flow lines with buffer. The buffer is used on multiple processors as a placeholder for semi-finished material before it is processed in the next processors. Buffers are used in the form of vessels with varying volume. In this study, the max-plus algebra is the method used to obtain crystal sugar production scheduling system in the sugar factory. From the flow lines that have been made then we derive a model of max-plus algebra to obtain a production schedule that starts with the milling process to obtain crystal sugar. Based on the max-plus algebra model, we also obtained sugar output schedule and some kind of waste. In addition, we obtained two periodicities of each processor, that is from milling processor until sulfitation of thick juice processor with periodicity 177.64 minutes and from vacuum pan A processor until mixer D_2 processor with periodicity 1592.63 minutes, from these periodicities, we obtain a periodic production schedule for each processor.**

***Index Terms*—Crystal sugar production system, max-plus algebra, periodic scheduling.**

I. Introduction

SUGAR is the basic human need, besides rice, corn, vegetables, fruits, and other basic necessities, and as the main trading commodity traded in the form of solid crystals of sucrose or crystal sugar. Raw material of sugar is cane with five production stations. The production stations are mill station, purification station, evaporation station, crystallization station, rounds and drying station. Sugar production process is very complicated because a lot of machines are used, which is the production sequence and the operation time are different in each processor, so the production time needed is not a little. Especially if there are errors in analyzing the operating time, which is different for each product. It will cause a chaos in the production scheduling. In addition, in the implementation of the sugar production of PG (sugar factory) Z, they still use manual scheduling. This means currently PG Z does not apply any special method to optimize the production time. Thus, there should be a study on sugar production scheduling.

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Max-plus algebra is used because of the ease in completing the synchronization process and to optimize the crystal sugar production schedule so that the production time can be used effectively and efficiently. Synchronization in max-plus algebra has several advantages, which can be used successfully to model and analyze algebraically network problems, such as scheduling and analyzing the dynamic behavior of the system [1], [2], [3]. Max-Plus algebra can also be applied to construct a general form of production system model with or without buffer [4]. A study on the application of max-plus Algebra to the production system model Of crystal sugar in a sugar factory has been studied in [5]. In [5], the authors only constructed max-plus algebra model of crystal sugar production system without computing the schedule.

In this paper, we construct flow lines of crystal sugar production system as discussed in [5] but in accordance with the conditions in PG Z. Furthermore, we derive the max-plus algebra model and construct a schedule of the crystal sugar production system. Based on the obtained schedule, we obtain a periodic schedule for each processor.

II. Methods

A. Max-Plus Algebra

Let $\mathbb{R}_\varepsilon := \mathbb{R} \cup \{\varepsilon\}$, where $\mathbb{R}$ is the set of real numbers and $\varepsilon := -\infty$. We define the following two operations over $\mathbb{R}_\varepsilon$

$$x \oplus y := \max\{x, y\}, \quad x \otimes y := x + y,$$

for each $x, y \in \mathbb{R}_\varepsilon$. It has been shown in [6] that $(\mathbb{R}_\varepsilon, \oplus, \otimes)$ is a semiring with neutral element ε and unit element $e = 0$.

The set of matrices of size $n \times n$ over max-plus algebra is denoted by $\mathbb{R}^{n\times m}_{\max}$, where $n, m \in \mathbb{N}$. We define $\underline{n} := \{1, 2, \ldots, n\}$. A matrix of size $n \times m$ is written as follows

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1m} \\ a_{21} & a_{22} & \ldots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nm} \end{bmatrix}.$$

Sometimes, the entry of A at i-th row and j-th column is denoted by $[A]_{ij}$ for $i \in \underline{n}$ and $j \in \underline{m}$.

The max-plus matrix addition is denoted by $A \oplus B$ and defined as follows

$$[A \oplus B]_{ij} = a_{ij} \oplus b_{ij} = \max\{a_{ij}, b_{ij}\},$$

where $i \in \underline{n}$ and $j \in \underline{m}$.

The max-plus multiplication between a scalar and a matrix $\alpha \otimes A$ is defined as follows

$$[\alpha \otimes A]_{ij} = \alpha \otimes a_{ij},$$

where $i \in \underline{n}$ and $j \in \underline{m}$.

The max-plus matrix multiplication $A \otimes B$ is defined as follows

$$[A \otimes B]_{ij} = \max\{a_{ik} + b_{kj}\},$$

where $A \in \mathbb{R}_{\max}^{n\times p}$, $B \in \mathbb{R}_{\max}^{p\times m}$, $i \in \underline{n}$ and $j \in \underline{m}$.

B. Matrix and Graph

A directed graph $G = (\mathcal{N}, \mathcal{D})$ in the max-plus algebra can be represented as a matrix, for example a graph containing n nodes is represented as matrix $A \in \mathbb{R}_{\max}^{n\times n}$. As mentioned, the graph has n nodes and the set of nodes is denoted by $\mathcal{N}$. The set of arcs is denoted by $\mathcal{D}$. There is an arc from j to i, namely $(j,i) \in \mathcal{D}$, if $a_{ij} \neq \varepsilon$. The weight of arc (j,i) is denoted by $w(j,i) = a_{ij} \in \mathbb{R}_{\max}$.

For matrix $A \in \mathbb{R}_{\max}^{n\times n}$, we define $A^+ = \oplus_{k=1}^{\infty} A^{\otimes k}$ and $A^* = E \oplus A^+ = \oplus_{k\geq 0} A^{\otimes k}$, where $A^{\otimes k}$ is the k-th max-plus power of A, E is the max-plus identity matrix. The diagonal elements of the max-plus identity matrix are e, whereas the nondiagonal elements are ε.

According to [3], the following equation

$$X = A \otimes X \oplus B \otimes U$$

has a solution

$$X = A^* \otimes B \otimes U$$

where X is a vector of size $n \times 1$, U is the input vector of size $m \times 1$, A is a square matrix of size $n \times n$ and B is a matrix of size $n \times m$.

C. Modeling of Production Systems using Max-Plus Algebra

In this study, derivation of the Max-Plus algebra model refers to [4]. Let G be a directed graph that represents a flow line of production systems. A point on G represents the input, processing and output units in the flow line. The lines on G represent the displacement of materials. Suppose the set of points $V = \{U_1, \ldots, U_m, M_1, \ldots, M_p, Y_1, \ldots, Y_m\}$. The points $U_1, \ldots, U_m$ correspond to input units. The points $M_1, \ldots, M_p$ correspond to processing units. The points $Y_1, \ldots, Y_m$ correspond to output units. The constant $t_{j,i}$ represents the weight from M_j to M_i. The constant $t_{U_a,i}$ represents the weight from U_a to M_i. The constant t_{j,Y_s} represents the weight from M_j to Y_s. The constant d_j is the duration of process in M_j. The constant F_l is the buffer capacity in M_l. In the previous definitions, $i, j \in \underline{p}$; $a \in \underline{m}$; $s \in \underline{n}$.

We define matrices $A, B \in \mathbb{R}_{\max}^{p\times p}$, $C \in \mathbb{R}_{\max}^{n\times p}$, and $D \in \mathbb{R}_{\max}^{p\times m}$ where the entries are as follows

$$[A]_{i,j} = \begin{cases} d_j \otimes t_{i,j}, & \text{if there is a line from } M_j \text{ to } M_i \\ \varepsilon, & \text{if there is no line from } M_j \text{ to } M_i \end{cases}$$

$$[B]_{i,j} = \begin{cases} d_i, & \text{if } i = j \\ \varepsilon, & \text{if } i \neq j \end{cases}$$

$$[C]_{s,j} = \begin{cases} d_j \otimes t_{i,Y_s}, & \text{if there is a line from } M_j \text{ to } Y_s \\ \varepsilon, & \text{if there is no line from } M_j \text{ to } Y_s \end{cases}$$

$$[D]_{i,a} = \begin{cases} t_{U_a,i}, & \text{if there is a line from } U_a \text{ to } M_i \\ \varepsilon, & \text{if there is no line from } U_a \text{ to } M_i \end{cases}$$

The state variable $X_i(k+1)$ represents the time instant when the i-th processing unit starts for the k-th time. The output variable $Y_s(k)$ represents the time instant when the product of type s leaves the system for the k-th time. The input variable $U_a(k+1)$ represents the time instant when the raw material of type a enters the system for the $k+1$-th time. The dynamics are given by

$$\begin{aligned} X_i(k+1) = {} & \left(\bigoplus_{j=1}^{p} [A]_{i,j} \otimes X_j(k+1)\right) \oplus [B]_{i,i} \otimes X_i(k) \\ & \oplus \left(\bigoplus_{a=1}^{m} [D]_{i,a} \otimes U_a(k)\right) \\ & \oplus \left(\bigoplus_{l=1,[A]_{l,i}\neq\varepsilon,F_l\not<\infty}^{m} -t_{i,l} \otimes X_l(k-F_l)\right) \quad (1) \end{aligned}$$

$$Y_s(k+1) = \bigoplus_{j=1}^{p} [C]_{s,j} \otimes X_j(k) \quad (2)$$

Equations (1) and (2) can be written as follows

$$\begin{aligned} X(k+1) = {} & A \otimes X(k+1) \oplus B \otimes X(k) \oplus D \otimes U(k) \\ & \oplus \left(\bigoplus_{l=1}^{p} A_l \otimes X(k-F_l)\right) \quad (3) \\ Y(k) = {} & C \otimes X(k) \quad (4) \end{aligned}$$

where $[A_l]_{i,j} = \begin{cases} -t_{i,l}, & \text{if } [A]_{j,i} \neq \varepsilon, j = l, \text{ and } F_l \not< \infty \\ \varepsilon, & \text{otherwise} \end{cases}$.

From (1) and (2), then (3) and (4) can be written as follows

$$\begin{aligned} X(k+1) = {} & \hat{A} \otimes X(k) \oplus \hat{B} \otimes U(k) \\ & \oplus \left(\bigoplus_{l=1}^{p} \widehat{AB}_l \otimes X(k-F_l)\right) \quad (5) \\ Y(k) = {} & C \otimes X(k) \quad (6) \end{aligned}$$

where $X = \begin{bmatrix} X_1 & X_2 & \ldots & X_p \end{bmatrix}^T$, $\hat{A} = A^* \otimes B$, $\hat{B} = A^* \otimes D$ and $\widehat{AB}_l = A^* \otimes A_l$ for $l \in \underline{p}$. Because the $\oplus$-operation on max-plus algebra is commutative, (5) and (6) can be written as follows

$$\begin{aligned} X(k+1) = {} & \hat{A} \otimes X(k) \oplus \left(\bigoplus_{l=1}^{p} \widehat{AB}_l \otimes X(k-F_l)\right) \\ & \oplus \hat{B} \otimes U(k) \quad (7) \\ Y(k) = {} & C \otimes X(k) \quad (8) \end{aligned}$$

By using matrix notation, (7) and (8) can be written as follows

$$\tilde{X}(k+1) = \tilde{A} \otimes \tilde{X}(k) \oplus \tilde{B} \otimes U(k) \quad (9)$$

$$Y(k) = \tilde{C} \otimes \tilde{X}(k) \quad (10)$$

where $\tilde{A} \in \mathbb{R}_{\max}^{\tilde{p}\times\tilde{p}}$, $\tilde{B} \in \mathbb{R}_{\max}^{\tilde{p}\times m}$, $\tilde{C} \in \mathbb{R}_{\max}^{n\times\tilde{p}}$, $\tilde{p} = p(\max\{F_1, F_2, \ldots, F_p\} + 1)$,

$$\tilde{X}(k) = \begin{bmatrix} X(k)^T & \ldots & X(k - \max\{F_1, F_2, \ldots, F_p\})^T \end{bmatrix}^T$$

$$\tilde{A}(k) = \begin{bmatrix} \tilde{A} & [\tilde{A}]_{1,2} & \dots & [\tilde{A}]_{1,t-1} & [\tilde{A}]_{1,t} \\ E_{p,p} & \varepsilon_{p,p} & \dots & \varepsilon_{p,p} & \varepsilon_{p,p} \\ \varepsilon_{p,p} & E_{p,p} & \dots & \varepsilon_{p,p} & \varepsilon_{p,p} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \varepsilon_{p,p} & \varepsilon_{p,p} & \dots & E_{p,p} & \varepsilon_{p,p} \end{bmatrix}$$
$$\tilde{B}(k) = \begin{bmatrix} X(k)^T & \varepsilon_{p,m}^T & \varepsilon_{p,m}^T & \dots & \varepsilon_{p,m}^T \end{bmatrix}^T$$
$$\tilde{C}(k) = \begin{bmatrix} C & \varepsilon_{n,p} & \varepsilon_{n,p} & \dots & \varepsilon_{n,p} \end{bmatrix}$$

$E_{p,p}$ is the max-plus identity matrix of size $p \times p$, $\varepsilon_{p,p}$ is a null matrix of size $p \times p$, i.e. a matrix where all entries are ε. Finally $[\tilde{A}]_{1,b}$ is defined as follows

$$[\tilde{A}]_{1,b} = \bigoplus_{l=1, F_l=-1}^{p} \widehat{AB}_l$$

where $b \in \{2, 3, \dots, t\}$ and $t = \max\{F_1, F_2, \dots, F_p\} + 1$.

D. General Crystal Sugar Production Systems

Figure 1 describes the flow of general processing of cane into crystal sugar.

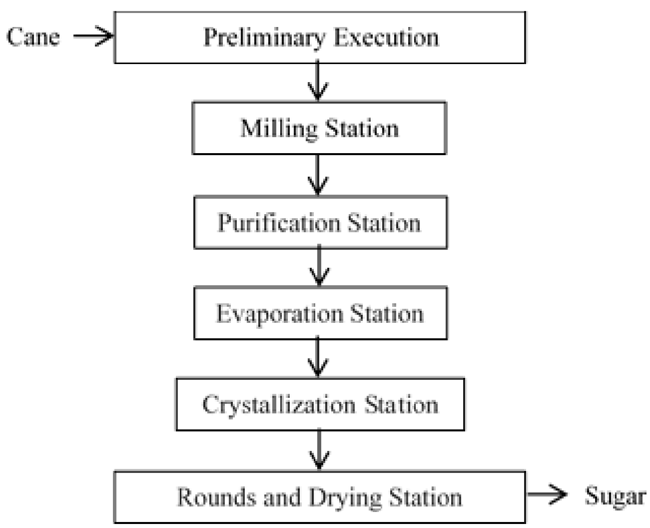

Fig. 1. Flow Chart of General Crystal Sugar Production System.

In the remainder of this paper, we discuss the flow line of crystal sugar production flows at PG Z as given in Fig. 1. Then we use the flow line to derive the max-plus algebra model. Finally, we use the model to obtain a schedule of crystal sugar production systems in PG Z.

III. Results and Discussions

A. Modeling of Crystal Sugar Production System with Buffer using Max-Plus Algebra

Based on Fig. 2, we obtain the following max-plus algebra model.

$$\begin{aligned} X_1(k+1) &= 1.2 \otimes X_1(k) \oplus 0.34 \otimes U_1(k) \\ &\quad \oplus -0.4 \otimes X_6(k-1) \\ X_2(k+1) &= 1.5 \otimes X_1(k+1) \oplus 2.4 \otimes X_3(k) \oplus 1.3 \otimes X_2(k) \\ &\quad \oplus -0.4 \otimes X_6(k-1) \\ X_3(k+1) &= 1.5 \otimes X_2(k+1) \oplus 2.4 \otimes X_4(k) \oplus 1.5 \otimes X_3(k) \\ X_4(k+1) &= 1.75 \otimes X_3(k+1) \oplus 2.5 \otimes X_5(k) \oplus 1.5 \otimes X_4(k) \\ &\quad \oplus 0.14 \otimes U_2(k) \\ X_5(k+1) &= 1.75 \otimes X_4(k+1) \oplus 1.6 \otimes X_5(k) \oplus 0.14 \otimes U_2(k) \\ X_6(k+1) &= 1.6 \otimes X_1(k+1) \oplus 1.7 \otimes X_2(k+1) \\ &\quad \oplus 14.03 \otimes X_{17}(k) \oplus 3 \otimes X_6(k) \\ &\quad \oplus -0.25 \otimes X_7(k-1) \\ &\vdots \\ X_{40}(k+1) &= 5.02 \otimes X_{39}(k+1) \oplus 5 \otimes X_{40}(k) \oplus 0 \otimes U_7(k) \\ &\quad \oplus -1.88 \otimes X_{26}(k-7) \end{aligned}$$

$$\begin{aligned} Y_a(k) &= Y_1(k) = 2.6 \otimes X_5(k) \\ Y_b(k) &= Y_2(k) = 15 \otimes X_{17}(k) \\ Y_g(k) &= Y_3(k) = 36 \otimes X_{30}(k) \\ Y_t(k) &= Y_4(k) = 5.02 \otimes X_{37}(k) \end{aligned}$$

where $Y_a(k) = Y_1(k)$ represents the output of bagase, $Y_b(k) = Y_2(k)$ represents the output of blotong, $Y_g(k) = Y_3(k)$ represents the output of crystal sugar, $Y_t(k) = Y_4(k)$ represents the output of syrup.

Then we obtain the following max-plus algebra model:

$$\begin{aligned} X(k+1) &= A \otimes X(k+1) \oplus B \otimes X(k) \oplus D \otimes U(k) \\ &\quad \oplus \left(\bigoplus_{l=1}^{p} A_l \otimes X(k - F_l) \right) \\ &= A \otimes X(k+1) \oplus B \otimes X(k) \oplus D \otimes U(k) \\ &\quad A_6 \otimes X(k-1) \oplus A_7 \otimes X(k-1) \\ &\quad \oplus A_{17} \otimes X(k-1) \oplus A_{18} \otimes X(k-1) \\ &\quad \oplus A_{25} \otimes X(k-8) \oplus A_{26} \otimes X(k-7) \\ &\quad \oplus A_{31} \otimes X(k-4) \oplus A_{35} \otimes X(k-8) \end{aligned}$$

where the size of A is 40×40, the size of B is 40×40, the size of C is 1×40, and the size of D is 40×7.

Based on [4], we obtain the following equations:

$$\begin{aligned} X(k+1) &= \hat{A} \otimes X(k) \oplus \hat{B} \otimes U(k) \\ &\quad \oplus \left(\bigoplus_{l=1}^{p} \widehat{AB}_l \otimes X(k - F_l) \right) \\ &= \hat{A} \otimes X(k) \oplus \hat{B} \otimes U(k) \oplus \widehat{AB}_6 \otimes X(k-1) \\ &\quad \oplus \widehat{AB}_7 \otimes X(k-1) \oplus \widehat{AB}_{17} \otimes X(k-1) \\ &\quad \oplus \widehat{AB}_{18} \otimes X(k-1) \oplus \widehat{AB}_{25} \otimes X(k-8) \\ &\quad \oplus \widehat{AB}_{16} \otimes X(k-7) \oplus \widehat{AB}_{31} \otimes X(k-4) \\ &\quad \oplus \widehat{AB}_{35} \otimes X(k-8) \\ Y(k) &= C \otimes X(k) \end{aligned}$$

where $\hat{A} = A^* \otimes B$, $\hat{B} = A^* \otimes D$ and $\widehat{AB}_l = A^* \otimes A_l$, $l = 1, 2, 3, \dots, 40$.

We then obtain

$$\begin{aligned} \tilde{X}(k+1) &= \tilde{A} \otimes \tilde{X}(k) \oplus \tilde{B} \otimes U(k) \\ Y(k) &= \tilde{C} \otimes \tilde{X}(k) \end{aligned}$$

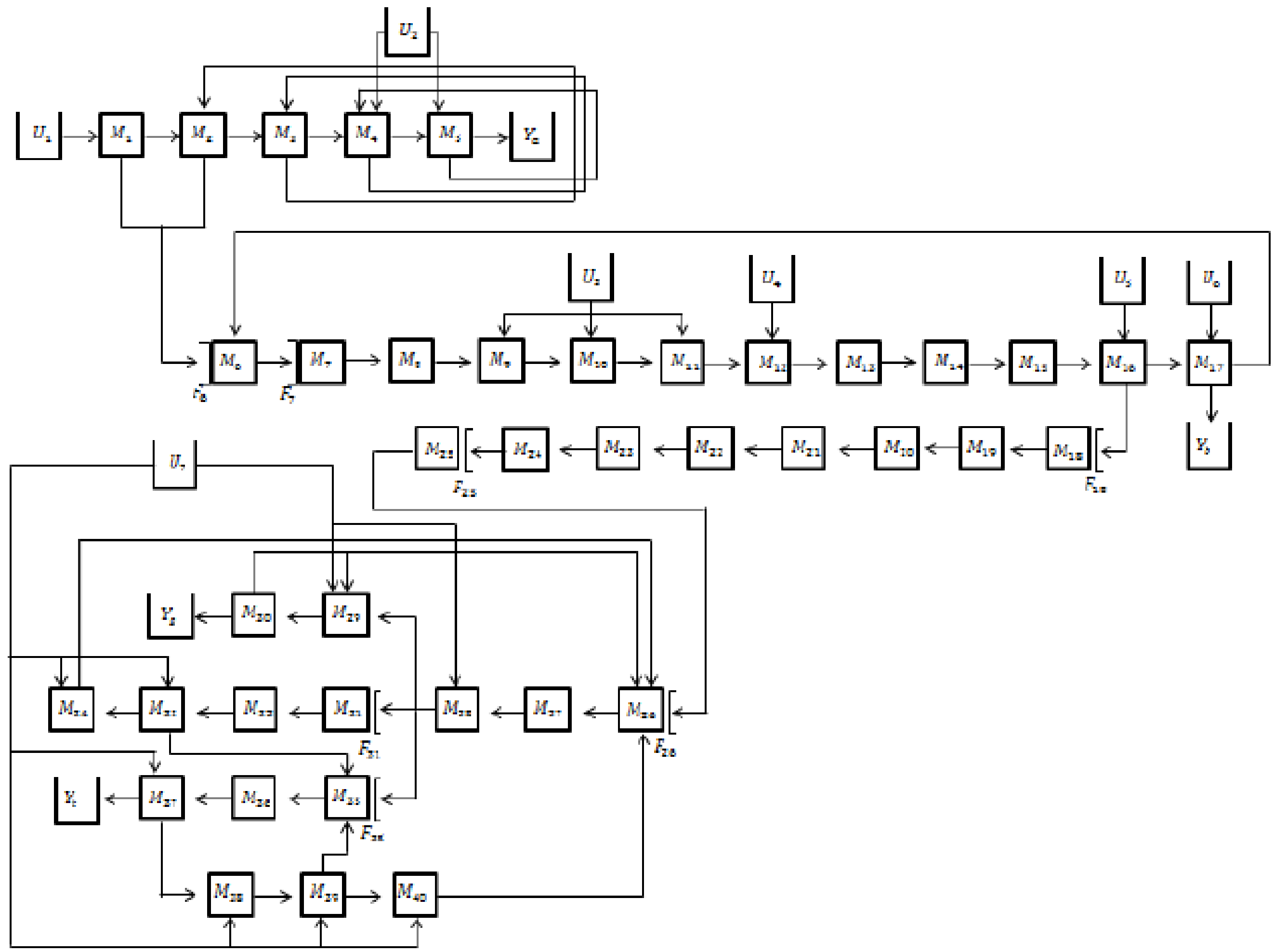

Fig. 2. The Flow Line of Crystal Sugar Production Systems containing Five Stations Process.

where

$$\tilde{X} = \begin{bmatrix} X(k) \\ X(k-1) \\ X(k-2) \\ X(k-3) \\ X(k-4) \\ X(k-5) \\ X(k-6) \\ X(k-7) \\ X(k-8) \end{bmatrix}, \text{ for } X(k+i) = \begin{bmatrix} X_1(k+i) \\ X_2(k+i) \\ X_3(k+i) \\ \vdots \\ X_{39}(k+i) \\ X_{40}(k+i) \end{bmatrix}$$

$$\tilde{A} = \begin{bmatrix} a & b & c & c & d & e & c & c & f \\ g & c & c & c & c & c & c & c & c \\ c & g & c & c & c & c & c & c & c \\ c & c & g & c & c & c & c & c & c \\ c & c & c & g & c & c & c & c & c \\ c & c & c & c & g & c & c & c & c \\ c & c & c & c & c & g & c & c & c \\ c & c & c & c & c & c & g & c & c \\ c & c & c & c & c & c & c & g & c \end{bmatrix}$$

$$\tilde{B} = \begin{bmatrix} \hat{B} \\ \varepsilon(40\times 4) \\ \varepsilon(40\times 4) \\ \varepsilon(40\times 4) \\ \varepsilon(40\times 4) \\ \varepsilon(40\times 4) \\ \varepsilon(40\times 4) \\ \varepsilon(40\times 4) \\ \varepsilon(40\times 4) \end{bmatrix}, \text{ and } \tilde{C} = \begin{bmatrix} \hat{C} \\ \varepsilon(4\times 40) \\ \varepsilon(4\times 40) \\ \varepsilon(4\times 40) \\ \varepsilon(4\times 40) \\ \varepsilon(4\times 40) \\ \varepsilon(4\times 40) \\ \varepsilon(4\times 40) \\ \varepsilon(4\times 40) \end{bmatrix}$$

where $a = \hat{A}$, $b = \widehat{AB}_6 \oplus \widehat{AB}_7 \oplus \widehat{AB}_{17} \oplus \widehat{AB}_{18}$, $c = \varepsilon(40 \times 40)$, $d = \widehat{AB}_{31} \oplus \widehat{AB}_{35}$, $e = \widehat{AB}_{31} \oplus \widehat{AB}_{35}$, $f = \widehat{AB}_{25}$ and $g = E(40 \times 40)$.

B. Scheduling of the Crystal Sugar Production

Based on calculations by the Scilab software 5.5.2 and max-plus toolbox with initial value $U(0) = 0$, $X(0) = 0$ and with ten processes ($k = 10$), we obtain matrix $\tilde{X}$ of size 360×11. However, the results that indicate the processing time are on the first until fourtieth row. From the calculation results, each processor has different time period: the first processor (M_1) until 25-th processor (M_{25}) has 177.64 minutes time period, the 26-th processor (M_{26}) until 40-th processor (M_{40}) has

TABLE I
INITIAL STATE OF THE ACTIVE SYSTEM.

Minute-	Conversion to Days and Hours
0	Monday, 06:00:00
1.5	Monday, 06:01:30
3	Monday, 06:03:00
4.75	Monday, 06:04:45
6.5	Monday, 06:06:30
355.68	Monday, 11:55:40
358.93	Monday, 11:58:55
371.46	Monday, 12:11:27
380.24	Monday, 12:20:14
384.27	Monday, 12:24:16
386.8	Monday, 12:26:48
388.2	Monday, 12:28:12
394.67	Monday, 12:34:40
405	Monday, 12:45:00
416.47	Monday, 12:56:28
428.49	Monday, 13:08:29
519.29	Monday, 14:39:17
519.34	Monday, 14:39:20
525.17	Monday, 14:45:10
533.47	Monday, 14:53:28
550.5	Monday, 15:10:30
567.53	Monday, 15:27:31
585.06	Monday, 15:45:03
596.49	Monday, 15:56:29
608.79	Monday, 16:08:47
6273.79	Friday, 14:33:47
6483.81	Friday, 18:03:48
6724.31	Friday, 22:04:18
6729.33	Friday, 22:09:19
6734.63	Friday, 22:14:37
6729.52	Friday, 22:09:31
6981.54	Saturday, 02:21:32
7223.42	Saturday, 06:23:25
7228.44	Saturday, 06:28:26
7230.3	Saturday, 06:30:18
7602.32	Saturday, 12:42:19
7844.2	Saturday, 16:44:12
7849.22	Saturday, 16:49:13
7854.52	Saturday, 16:54:31
7859.54	Saturday, 16:59:32

TABLE II
THE TIME WHEN PRODUCTS LEFT THE SYSTEM.

Minute-	Conversion to Days and Hours
9.1	Monday, 06:09:06
534.29	Monday, 14:54:17
6765.63	Friday, 22:50:37
7849.22	Saturday, 16:49:13

1592.63 minutes time period. Each processor has a time period to start the remaining process, that is when $k = 5$ or $\tilde{X}(5)$.

In preparing the schedule, we assume the production begins on Monday morning at 06.00. Thus we obtain the following initial state of the active system:

Based on Tables I and II, it can be seen that if a crystal sugar production started on Monday at 06:00:00, then the product, namely crystal sugar, will be ready on Friday at 22:50:37. In addition to the main product, that is crystal sugar, the manufacturing system also produces some other outputs, such as several kinds of waste that can still be used. For example, the baggase produced on Monday at 06:09:06 can be used as fuel for the boiler station. Output blotong produced on Monday 14:54:17 can be used as organic fertilizer and also as a substitute for firewood. Output in the form of syrup is produced on Saturday at 16:49:13 can be used for industrial fermentation, such as alcohol, MSG, spiritus, and animal feed.

IV. CONCLUSIONS

The flow line of crystal sugar production systems in a sugar factory or PG, in this case is PG Z, in Situbondo should represent the real conditions, comprising mill station, purification station, evaporation station, crystallization station, and rounds and drying station with assumption no interruption in the system.

Based on the max-plus algebra model, if a crystal sugar production started on Monday at 06:00:00, then the crystal sugar product will be produced on Friday at 22:50:37 and the other products, that are the output baggase produced on Monday at 06:09:06, output blotong on Monday 14:54:17 , and output syrup produced on Saturday at 16:49:13. In addition, we obtained two periodicity of each processor, that is from milling processor until sulfitation of thick juice processor with periodicity 177.64 minutes and from vacuum pan A processor until mixer D_2 processor with periodicity 1592.63 minutes.

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